Application of Shadow Boundary Incremental Length Diffraction Coefficients to Bistatic Scattering from 3-D Bodies

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1. Introduction

With the availability of high speed computers and the development of efficient algorithms for calculating physical optics (PO) fields, computer codes can rapidly calculate the PO fields of scatterers thousands of wavelengths across. For backscattering, radar cross sections (RCS's) computed from the PO fields are usually quite accurate. For large-angle bistatic scattering, however, the RCS's for smooth perfectly electrically conducting (PEC) objects, computed from the PO fields, are often in error by several dB because the PO current terminates abruptly at the shadow boundary (SB) and poorly approximates the actual current close to the SB in both the shadow and lit regions. Shadow boundary incremental length diffraction coefficients (SBILDC's) provide a convenient and efficient high-frequency method for significantly improving upon the accuracy of the PO scattered fields of PEC three-dimensional (3-D) objects by closely approximating the fields radiated by the nonuniform (NU) currents (the difference between the actual and PO currents) and including them in the scattering calculations. Like other incremental length diffraction coefficients, any SBILDC is based on the use of a two-dimensional (2-D) canonical scatterer to locally approximate the surface of the 3-D scatterer to which it is applied. Circular cylinder SBILDC's are, to date, the only SBILDC's that have been obtained in closed form. In this paper these closed-form expressions are validated by applying them for the first time to a 3-D scatterer with varying radius of curvature – the prolate spheroid.

2. Shadow Boundary Incremental Length Diffraction Coefficients

SBILDC's are high-frequency fields designed to correct the errors in PO fields that result from the failure of the PO current approximation to accurately represent the actual currents in the vicinity of shadow boundaries on convex surfaces of PEC scatterers. The basic concept of SBILDC's is as follows. Consider a 3-D PEC scatterer illuminated by a wave, with a SB on a convex portion of the surface, and a thin strip of surface of the scatterer in the direction of the incident wave at a shadow boundary point P_{SB}. This surface strip can be approximated in the vicinity of P_{SB} by a locally conforming tangential thin strip of surface of a two-dimensional (2-D) canonical scatterer whose axis is parallel to the SB at P_{SB} and whose radius of curvature in the direction of the incident wave is equal to the radius of curvature of the 3-D scatterer at P_{SB} in the direction of the incident field. The field radiated by the NU current in the vicinity of P_{SB} on the surface strip of the 3-D object is approximated by the field radiated by the NU current that would be excited on the conforming strip of surface of the 2-D canonical scatterer – the SBILDC. The procedure is repeated for each point on the SB, and the NU current field of the 3-D scatterer then approximated by the integral of the SBILDC along the SB curve. The SBILDC can be obtained by substituting a closed-form expression, if available, of the canonical scatterer's 2-D NU current far fields in a general formula [1].

Yaghjian et al. [1] have derived closed-form expressions for circular cylinder SBILDC's that correct the PO SB errors for perfect conductors at all bistatic angles. Underlying the choice of the circular cylinder as the 2-D canonical scatterer for the SBILDC's is the fact that the NU SB current decays rapidly away from the SB. Hence the field radiated by the NU SB current on a thin strip of surface of a 3-D scatterer that is in the direction of the incident field can be approximated by the field radiated by the NU current that would be excited on a conforming thin strip of surface of a circular cylinder – the circular cylinder SBILDC. Even though a full description of the NU SB current, including the creeping-wave current, requires a knowledge of the radius of curvature over the surface of the 3-D scatterer, the only geometric parameters required by the circular cylinder SBILDC approximation are the SB curve, the normal vector to the surface along the SB, and the radius of curvature of the 3-D scatterer across the SB. These three parameters, however, suffice to account for the dominant contribution to the field from the NU SB current everywhere. This simplification is central to the computational efficiency of using the integral of the circular cylinder SBILDC along the SB to approximate the NU SB field of the 3-D scatterer. Since the circular cylinder SBILDC approximation is implemented by performing a linear integration of a closed-form expression along the SB, it does not add significantly to the computer time required to calculate the PO field.

3. Prolate Spheroid Plane Wave Bistatic Scattering

The primary purpose of the work presented here is to validate and test the effectiveness of the closed-form circular cylinder SBILDC's by applying them for the first time to a 3-D scatterer, the PEC prolate spheroid, whose radius of curvature normal to the SB varies over its surface. The prolate spheroid is the body of revolution (BOR) generated by revolving an ellipse around its major axis; see Fig. 1. The (x, y, z) axes have their origin at the center of the spheroid with (r, θ, ϕ) the associated spherical polar coordinates. The z axis is taken to be the axis of revolution. The propagation vector $\mathbf{k}^{inc}$ of the illuminating plane wave with amplitude E_0 is assumed to lie in the xz plane with $-\mathbf{k}^{inc}$ making an angle θ_0 with the positive z axis, $0 \leq \theta_0 \leq \pi/2$, and with $k_x^{inc} \leq 0$ so that $\mathbf{k}^{inc} = -k(\sin\theta_0\,\hat{\mathbf{x}} + \cos\theta_0\,\hat{\mathbf{z}})$ with $k = 2\pi/\lambda$.

The closed-form expression for the circular cylinder SBILDC in [1] is given in terms of a local coordinate system (LCS) centered on a SB point P_{SB}. The local y axis is in the direction of the external normal to the scatterer at P_{SB} and the local z axis is the tangent to the SB curve at P_{SB}. The integration of the circular cylinder SBILDC along the SB curve of the prolate spheroid, however, must be performed in the global coordinate system (GCS) of the spheroid (Fig. 1). Hence before integrating it is necessary to transform all quantities of the circular cylinder SBILDC (angles, unit vectors, distances, etc.) defined with reference to the LCS of the SBILDC's to the GCS of the spheroid. Full details of the procedure are given in [2].

4. Results

To correct the PO RCS of the prolate spheroid, the integral of the circular cylinder SBILDC was added to the PO field (obtained by numerical evaluation of the defining integral) and the resulting field was compared with a reference or "exact" scattered field. A dual-surface magnetic field integral equation (DSMFIE) BOR code [3],[4] was used to supply the "exact" field.

Each of the following figures shows the PO bistatic RCS, the PO + SBILDC RCS, and the "exact" RCS for a prolate spheroid with semi-major and semi-minor axes equal to $ka = 50$ and $kb = 25$, respectively. In Fig. 2 we show the H-plane plots for axial incidence. The large discrepancy between the PO and the exact RCS curve in the forward region is largely corrected by the SBILDC. Figure 3 shows the axial incidence E-plane plots. The SBILDC correction to the PO pattern is excellent throughout almost the entire range of θ. The SBILDC correction does not, however, accurately predict the small oscillations of the exact curve which are due to the interference between the specularly reflected fields and the fields of the creeping waves which travel deep into the shadow region of the spheroid and radiate to the far field. Figure 4 shows the $\phi = 0^o$ plane (H-plane) bistatic RCS pattern for a TE wave obliquely incident on the spheroid at 45°. Almost all the discrepancy between the PO and the exact RCS pattern is eliminated by the addition of the SBILDC field to the PO field.

These and similar results confirm the efficacy of circular cylinder SBILDC's in correcting the often substantial errors in large-angle bistatic PO RCS patterns. The use of the SB alone, instead of the entire shadow region, to calculate the circular cylinder SBILDC's, and the linear rather than surface integration required, greatly simplifies and accelerates the computation of the SBILDC correction to the PO field. Circular cylinder SBILDC's, in conjunction with a PO program, are therefore attractive candidates for an efficient general computer code to calculate high-frequency scattering from PEC objects, with a significantly greater accuracy than can be achieved with a PO code alone.

References

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[4] J.L. Schmitz,"Efficient solution for electromagnetic scattering using the dual-surface magnetic-field integral equation for bodies of revolution,"*IEEE Antennas Propagat. Soc. Int. Symp. Dig.*, Univ. Washington, Seattle, pp. 2318-2321, June 1994.

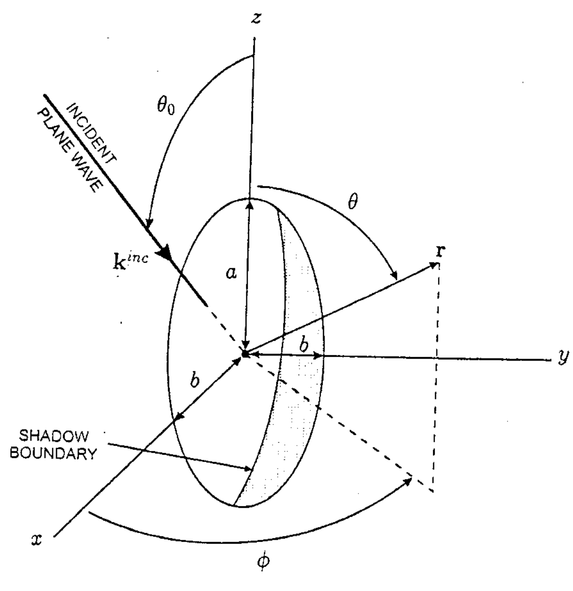

Figure 1. Prolate spheroid geometry and global coordinate system.

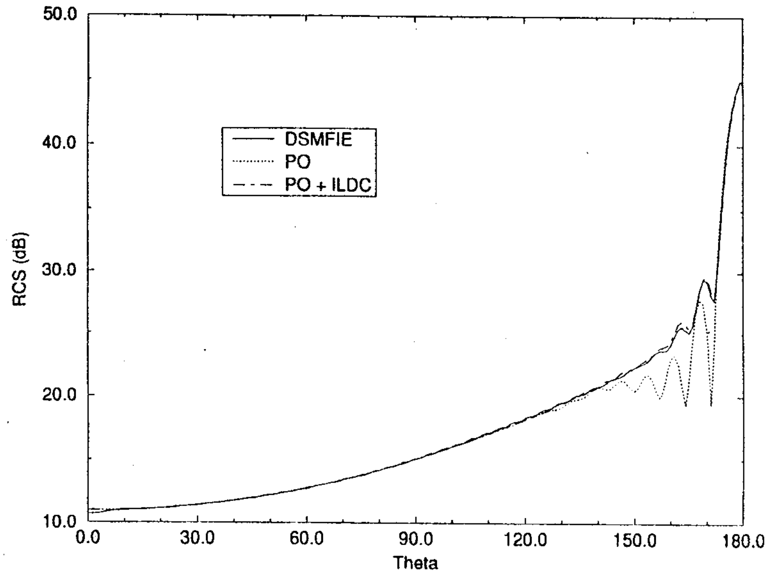

Figure 2. Prolate spheroid H-plane RCS for axial incidence; ka = 50, kb = 25.

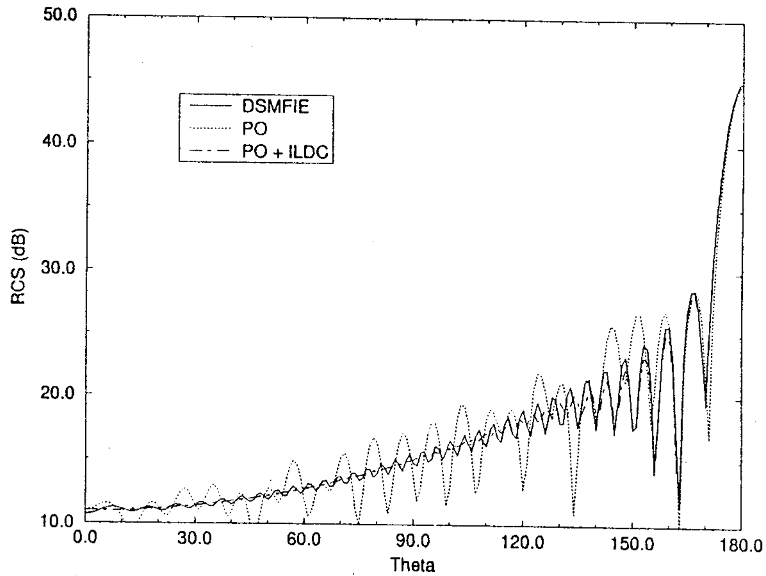

Figure 3. Prolate spheroid E-plane RCS for axial incidence; ka = 50, kb = 25.

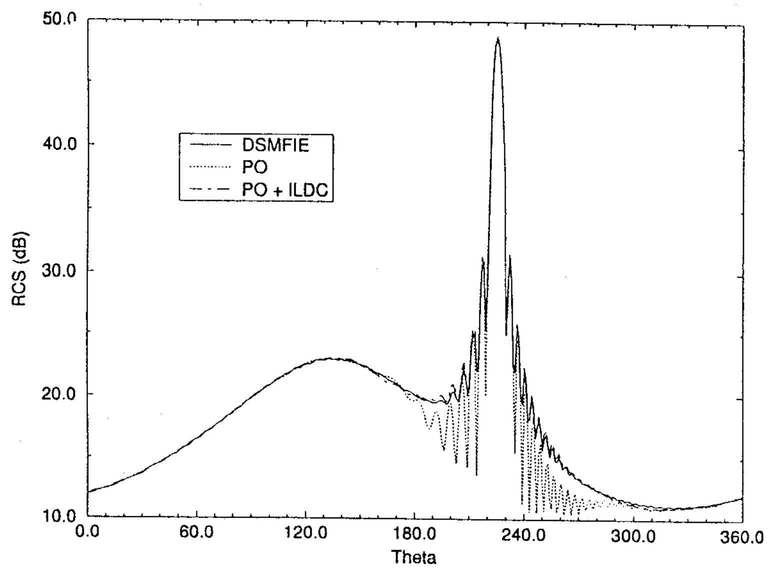

Figure 4. Prolate spheroid H-plane RCS for TE oblique incidence at 45°; ka = 50, kb = 25.