ANALYSIS OF ELECTROMAGNETIC WAVES DIFFRACTED BY A GROUNDED UNIAXIAL CHIRAL SLAB WITH AN INFINITE ARRAY OF STRIPS

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1 Introduction

Electromagnetic waves interacting with periodic structures have gained much attention because of its many applications as diffraction gratings, waveguides, couplers, filters and holography etc.[1]. Those have been studied involving various types of media such as dielectric, magnetic substance, plasmas etc. Chiral media have been considered to be novel type components of those devices due to their potential characteristics of electromagnetic waves caused by the two kinds of circularly-polarized eigen modes with different wave numbers in such media [2]-[3]. Most analyses of periodic chiral structures have been focused on chiral media with periodic shape or with periodically-modulated chirality [4],[5]. In microwave and millimeter wave regions however adding thin conducting arrays to chiral slabs is preferred in place of shaping them periodically or making periodicity in chirality itself considering the advantages of planar structures seen in conventional FSS and microstrip antennas etc.. In this work we present 4×4 matrix-based analysis to simulate electromagnetic waves scattered and diffracted by a grounded uniaxial chiral slab with an infinite array of thin strips. The direction of the principal axis of the uniaxial chirality considered is assumed to be arbitrary and same as that of dielectirc permittivity. In numerical examples the effects of the uniaxial chirality and other structural parameters on the characteristics of diffracted waves are shown and conditions about specific trasformation of polarization are found.

2 Description of the problem

The geometry of the problem being considered is shown in Fig.1. Semi-infinite regions 1 is an

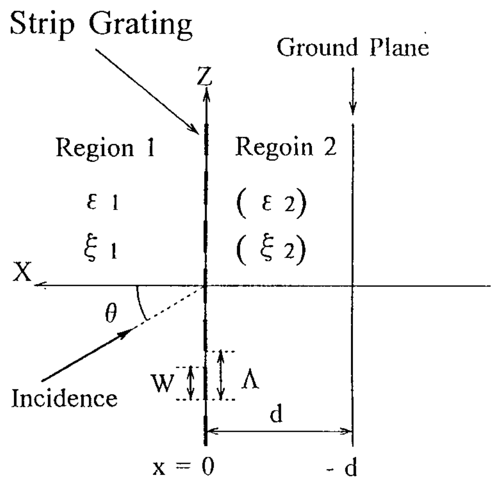

Figure 1: Geometry of the problem

isotropic chiral or achiral medium and region 2 with thickness d is a uniaxial anisotropic chiral medium which is backed by a ground plane. The tensors of chirality admittance and relative permittivity for region l, $(l = 1, 2)$ are generally expressed by 3×3 matrices (ξ_l) and (ϵ_l) and the elements of them are written as $\xi_{l,ij}$ and $\epsilon_{l,ij}$ $(i, j = x, y, z)$ respectively. (ξ_1) and (ϵ_1) are diagonal tensors where all the diagonal elements are equal to ξ_1, ϵ_1 respectively. As for region 2, each tensor can be given as

$$(\xi_2) = (T_{-\beta})(T_{-\alpha})(\xi_{dia})(T_\alpha)(T_\beta), \qquad (1)$$

$$(\epsilon_2) = (T_{-\beta})(T_{-\alpha})(\epsilon_{dia})(T_\alpha)(T_\beta) \qquad (2)$$

where (ξ_{dia}) and (ϵ_{dia}) are diagonal matrices whose diagonal elements are (ξ_2, ξ_2, ξ_{2e}) and $(\epsilon_2, \epsilon_2, \epsilon_{2e})$ respectively. ϵ_{2e} and ξ_{2e} are the values of relative permittivity and chiral admittance along the principal axis c of the uniaxial chiral medium and the remaining ϵ_2, ξ_2

are those along the axes a and b which is perpendicular to axis c. Cartesian coordinate (a,b,c) is defined by axes made by rotating the x,y and z-axis for angles α and β in the clockwise direction of screwing up toward +x and +b directions respectively. (T_α) and (T_β) are 3×3 matrices of rotational group for those transformations. Every region is lossless and has the same permeability μ_0. An infinite array of perfectly conducting strips with period Λ and width W is placed at the boundary between the region 1 and 2. The structure considered is assumed to be infinite in both y and z direction and have a plane of incidence in x-z plane where a TE or TM plane wave (magnetic and electric field vector is in x-z plane respectively) illuminates under the incidence angle θ. Electromagnetic fields with time harmonic dependence $exp(i\omega t)$ in each region $l\,(l = 1, 2)$ satisfy the Maxwell's equations and the Post-Jaggard type constitutive relations as follows [2]:

$$\boldsymbol{D} = \epsilon_0(\epsilon_l)\boldsymbol{E} - i(\xi_l)\boldsymbol{B},\ \ \boldsymbol{H} = -i(\xi_l)\boldsymbol{E} + \frac{1}{\mu_0}\boldsymbol{B}. \tag{3}$$

These equations enable us to derive the relations of electromagnetic fields with time dependence $exp(i\omega t)$, where the space variables are normalized by wave number in vacuum $k_0(=\omega_0\sqrt{\epsilon_0\mu_0})$, putting $k_0x \to x$, $k_0y \to y$ and $k_0z \to z$.

$$\begin{aligned}\overline{\mathrm{curl}}\sqrt{Y_0}\boldsymbol{E} &= -i\sqrt{Z_0}\boldsymbol{H} + (\tau_l)\sqrt{Y_0}\boldsymbol{E},\\ \overline{\mathrm{curl}}\sqrt{Z_0}\boldsymbol{H} &= i((\epsilon_l) + (\tau_l)^2)\sqrt{Y_0}\boldsymbol{E} - (\tau_l)\sqrt{Z_0}\boldsymbol{H}\end{aligned}$$

$$Y_0 = \frac{1}{Z_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}, (\tau_l) = (\xi_l)\sqrt{\frac{\mu_0}{\epsilon_0}} \tag{4}$$

where $\overline{\mathrm{curl}}$ and (τ_l) are the normalized operator curl and chiral admittance tensor respectively.

3 General solutions for fields

The total fields in each region are given by the sum of the current-independent primary fields and the scattered fields depending on the currents induced on the strips. Taking into account the phase matching at the boundaries for the incidence of a plane wave, the primary fields are given as

$$\sqrt{Y_0}E_i^p(x,z) = e_i^p(x)e^{-is_0z}, \tag{5}$$

$$\sqrt{Z_0}H_i^p(x,z) = h_i^p(x)e^{-is_0z}, \tag{6}$$

$$\text{where } i = x,y,z \text{ and } s_0 = \sqrt{\epsilon_1}\sin\theta. \tag{7}$$

The scattered fields in each region can be expanded in terms of Floquet modes as follows:

$$\sqrt{Y_0}E_i^s(x,z) = \sum_{m=-\infty}^{\infty} e_{im}^s(x)e^{-is_mz}, \tag{8}$$

$$\sqrt{Z_0}H_i^s(x,z) = \sum_{m=-\infty}^{\infty} h_{im}^s(x)e^{-is_mz} \tag{9}$$

$$\text{where } i = x,y,z \text{ and } s_m = s_0 + \frac{m}{\Lambda} \tag{10}$$

and the same phase matching as in Eq.(5) and Eq.(6) are considered. The coefficients $e_i^p(x)$, $h_i^p(x)$ in Eq.(5) and Eq.(6) and expansion coefficients $e_{im}^s(x)$, $h_{im}^s(x)$ in Eq.(8) and Eq.(9) $(i = x,y,z)$ are expressed here in vector form as

$$\boldsymbol{f}^p(x) = (e_y^p\ e_z^p\ h_y^p\ h_z^p)^t, \tag{11}$$

$$\boldsymbol{f}_m^s(x) = (e_{ym}^s\ e_{zm}^s\ h_{ym}^s\ h_{zm}^s)^t, \tag{12}$$

$$\boldsymbol{f}^{pn}(x) = (e_x^p\ h_x^p)^t,\quad \boldsymbol{f}_m^{sn}(x) = (e_{xm}^s\ h_{xm}^s)^t \tag{13}$$

Introducing the above scattered fields into Eq.(3), Eq.(4) derives the following coupled wave equation in matrix form about the expansion coefficients of the mth Floquet mode in region $l\,(l = 1, 2)$.

$$\frac{d}{dx}\boldsymbol{f}_m^s = i(R_{l,m})\boldsymbol{f}_m^s,\quad \boldsymbol{f}_m^{sn} = (K_{l,m})\boldsymbol{f}_m^s \tag{14}$$

where $(R_{l,m})$, $(K_{l,m})$ are 4×4 and 2×4 coupling matrices for region $l\,(l = 1, 2)$ whose elements are functions of elements of (τ_l) and (ϵ_l) and also of s_m (their forms are to be shown in the presentation of oral session). The solutions of Eq.(14) in region $l\,(l = 1, 2)$ are given in matrix form [6] as

$$\boldsymbol{f}_m^s(x) = (U_{l,m})\begin{pmatrix}(P_{l,m}^+(x - x_{g,l}^+))\boldsymbol{g}_{l,m}^{s+}\\(P_{l,m}^-(x - x_{g,l}^-))\boldsymbol{g}_{l,m}^{s-}\end{pmatrix} \tag{15}$$

where $(P_{l,m}^\pm(x))$ is a diagonal matrix whose diagonal elements are $(e^{i\kappa_{l,m}^{R\pm}x}, e^{i\kappa_{l,m}^{L\pm}x})$, $\boldsymbol{g}_{l,m}^{s\pm}$ is a column vector with elements $(g_{l,m}^{s,R\pm},\ g_{l,m}^{s,L\pm})$ and

$$(U_{l,m}) == (\boldsymbol{v}_{l,m}^{R+}\ \boldsymbol{v}_{l,m}^{L+}\ \boldsymbol{v}_{l,m}^{R-}\ \boldsymbol{v}_{l,m}^{L-}). \tag{16}$$

$\kappa_{l,m}^{R+}$, $\kappa_{l,m}^{L+}$, $\kappa_{l,m}^{R-}$, $\kappa_{l,m}^{L-}$ and $\boldsymbol{v}_{l,m}^{R+}$, $\boldsymbol{v}_{l,m}^{L+}$, $\boldsymbol{v}_{l,m}^{R-}$, $\boldsymbol{v}_{l,m}^{L-}$ in $(P_{l,m}^{\pm}(x))$ and $(U_{l,m})$ are eigenvalues and eigenvectors of matrix $(R_{l,m})$ respectively. The superscripts $R\pm$ and $L\pm$ denote right and left-hand elliptically-polarized eigenmodes propagating in + and − directions as for along x-axis respectively. These waves of eigenmodes propagating in differernt directions have different wave numbers because of the anisotropy of the medium. Each $\boldsymbol{g}_{l,m}^{s\pm}$ is an unknown column vector and is defined at $x = x_{g,l}^{\pm}$ in Eq.(15) where $\boldsymbol{g}_{l}^{p\pm}$ (appeared later) is also defined. The solutions of the primary field in region l $(l = 1, 2)$ are also derived in a similar mannar as

$$\boldsymbol{f}^{p}(x) = (U_{l,0}) \begin{pmatrix} (P_{l,0}^{+}(x - x_{g,l}^{+}))\boldsymbol{g}_{l}^{p+} \\ (P_{l,0}^{-}(x - x_{g,l}^{-}))\boldsymbol{g}_{l}^{p-} \end{pmatrix} \quad (17)$$

where $\boldsymbol{g}_{l}^{p\pm}$ is a column vector with elements $(g_{l}^{p,R\pm},\ g_{l}^{p,L\pm})$. The eigenmode fields in an isotropic chiral region can be expressed in closed forms and are separated into right and left-hand circularly-polarized modes with different wave numbers. In an isotropic achiral region, both separation of TE and TM linearly-polarized modes and that of modes with right and left-hand circular polarization are possible where every mode has an identical wave number [7]

4 Methods for solving

The current-independent primary fields satisfy the conditions of continuity of y and z components of fields at $x = 0$ and the condition of y and z components of electric fields being equal to zero at $x = -d$. Considering these conditions in the form of Eq.(17) yields linear equations by which the unknowns $\boldsymbol{g}_{1}^{p+}$, $\boldsymbol{g}_{2}^{p\pm}$ are determined assuming $\boldsymbol{g}_{1}^{p-} = (1, 0)^t$ and $(0, 1)^t$ for TE and TM incidence respectively. The scattered fields satisfy the conditions of y and z components of electric fields being continuous and equal to zero at $x = 0$ and $x = -d$ respectively and that of $-x$ components of wave vectors in region 1 being equal to zero. These fields depend on currents $J_y(0, z)$ and $J_z(0, z)$ on the strips at $x = 0$ which can be expanded in terms of Floquet modes with coefficients $c_{ym}(0)$ and $c_{zm}(0)$ in the same way as in Eq.(8) and Eq.(9). These unknown coefficients are related to those of magnetic fields $h_{ym}^{s}(0)$ and $h_{zm}^{s}(0)$ as

$$\begin{aligned} c_{ym}(0) &= h_{zm}^{s}(0)_{(\text{region1})} - h_{zm}^{s}(0)_{(\text{region2})} \\ c_{zm}(0) &= h_{ym}^{s}(0)_{(\text{region1})} - h_{ym}^{s}(0)_{(\text{region2})} \end{aligned} \quad (18)$$

because of the jump condition of magnetic fields. These conditions and Eq.(15) give linear equations which have solutions $\boldsymbol{g}_{l,m,i}^{s\pm}$ $(l = 1, 2.\ \ i = y, z.)$ corresponding the assumption that $(c_{ym}(0), c_{zm}(0)) = (1, 0)$ $(i = y)$ or $(c_{ym}(0), c_{zm}(0)) = (0, 1)$ $(i = z)$. Then, the unknowns $\boldsymbol{g}_{1,m}^{s+}$, $\boldsymbol{g}_{2,m}^{s\pm}$ can be expressed as

$$\boldsymbol{g}_{l,m}^{s\pm} = \boldsymbol{g}_{l,m,y}^{s\pm} + \boldsymbol{g}_{l,m,z}^{s\pm}, \quad (l = 1, 2.) \quad (19)$$

Combining Eq.(19) and Eq.(15) gives an expression of the scattered fields as follows:

$$\boldsymbol{f}_{m}^{s}(x) = \boldsymbol{G}_{my}(x)c_{ym}(0) + \boldsymbol{G}_{mz}(x)c_{zm}(0) \quad (20)$$

$$\boldsymbol{G}_{mt}(x) = (U_{l,m}) \begin{pmatrix} (P_{l,m}^{+}(x - x_{g,l}^{+}))\boldsymbol{g}_{l,m,t}^{s+} \\ (P_{l,m}^{-}(x - x_{g,l}^{-}))\boldsymbol{g}_{l,m,t}^{s-} \end{pmatrix} \quad (21)$$

$$(l = 1, 2.\ i = y, z).$$

The remaining unknowns that is currents on the strips can be approximated by

$$\sqrt{Z_0}J_i(0, z) = \sum_{k^i=1}^{N^i} C_{ik^i}\Phi_{ik^i}(z)e^{-i(q_0 y + s_0 z)}(i = y, z) \quad (22)$$

where $\Phi_{ik^i}(z)(k^i = 1, \ldots, N^i)$ form a set of linear independent basis functions defined on the strips. The primary fields Eq.(17), the scattered fields Eq.(20) with the above approximated currents form the boundary condition of perfect conductor on the strips. Applying the Galerkin's Method to this condition in the spectral domain yields a system of linear equation to determine the unknown coefficients of currents C_{ik^i} $(i = y, z.\ k^i = 1, 2, \ldots, N^i)$. in Eq.(22).

5 Numerical Results

For the numerical calculations the basis functions with the factors of the edge singularity are used here for the expansion in Eq.(22) [6]. Each summation of infinite number of Floquet

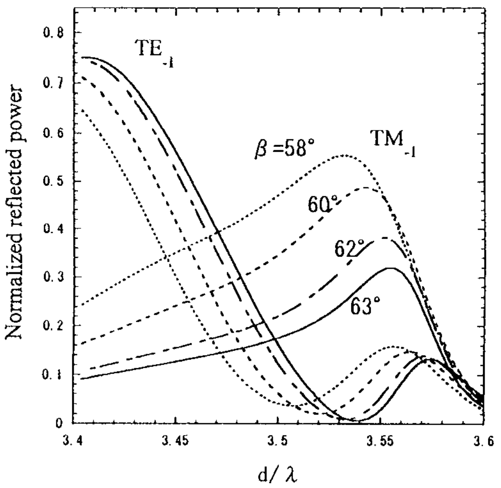

Figure 2: Reflected power of -1st diffraction, $\Lambda = 1.0\lambda$, $W = 0.3\Lambda$, TE incidence.

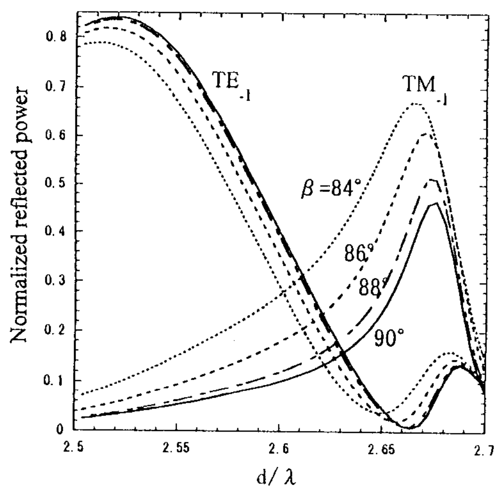

Figure 3: Reflected power of -1st diffraction, $\Lambda = 1.1\lambda$, $W = 0.4\Lambda$, TE incidence

modes in Eq.(8) and Eq.(9) is truncated at $m = \pm 90$ and the number of expansion in Eq.(22) is set $N^y = N^z = 30$ where graphical representation of results are possible. Calculations are performed under the conditions that $\epsilon_1 = 1.0$, $\epsilon_2 = 1.2$, $\epsilon_{2e} = 1.35$, $\xi_1 = 0(S)$, $\xi_2 = 1.0 \times 10^{-4}(S)$, $\xi_{2e} = 7.0 \times 10^{-4}(S)$ and $\theta = 5.4$ (degree). We assume that the axis b (described in chapter 2) is parallel to y axis that is $\alpha = 0$(degree) and the direction of the principal axis c of the uniaxial chirality can vary for angle β (degree). Figure 2 and 3 shows the reflected powers of -1th order diffracted TE and TM waves for the incidence of a TE wave. The periodicity and the width of strips are assumed to be $\Lambda = 1.0\lambda$, $W = 0.3\Lambda$ (Figure 2) and $\Lambda = 1.1\lambda$, $W = 0.4\Lambda$ (Figure 3). Both figures show that there are conditions under which the -1th order diffracted waves have polarization near TM which is different from the incident TE wave. The thickness d at which TM polarization largely appear varies centering around $d = 3.52\lambda$ (Figure 2) and $d = 2.66\lambda$ (Figure 3) according to the values of β. It is found that the periodicity and width of strips Λ and W also have strong effects on such conditions.

6 Conclusion

The 4×4 matrix-based analysis of the electromagnetic waves scattered and diffracted by a grounded uniaxial chiral medium with an infinite array of strips have been presented. The effects of the uniaxial chirality and other structural parameters on the diffracted waves have been investigated by numerical calculations. These studies will increase the possibility of novel type devices with useful phenomenon caused by the eigenmodes with different wave numbers in chiral media. The present analysis can be extended for the case of stratified strucures and of conducting patches (two-dimensional planar gratings) which are now being considered.

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