Analysis of an X-shaped Cavity-backed Wide Slot 2×2-element Sub-array by Hybrid MoM/FEM with Numerical Eigenmode Basis Functions

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1. Introduction

Application of the method of moments (MoM) with eigenmode expansion to a waveguide slot array enables fast analysis even though the number of the slots is large [1] because of small analysis scale, that is, the unknowns are sources placed only in the slot apertures. However, the method is applicable typically to slot shapes whose eigenmodes are analytically derivable. The hybrid MoM/FEM [2] was proposed for analysis of an array of slots with arbitrary shapes. The eigenmodes of a waveguide with the cross section of the slot shape are numerically derived by the finite element method (FEM) for the basis functions of the magnetic currents in the slot in MoM. In this paper, we apply the hybrid MoM/FEM to an X-shaped cavity-backed wide slot 2×2-element sub-array (Fig. 1) in the corporate-feed double-layer hollow-waveguide slot array antenna [3] as shown in Fig. 2. The 2×2-element sub-array is the radiation unit and fed in equal amplitude and phase by a coupling slot placed at each end of the feed waveguide in the antenna.

2. Analysis method

2.1 Analysis model and application of method of moments

The analysis model and its dimensions are shown in Fig. 1 and Table 1, respectively. The excitation coefficients of the four radiating slots are assumed to be equal because of the symmetric structure of the cavity shape. Based on the field equivalent theorem, all the apertures: the bottom and the top surfaces of the coupling and the radiating slots (denoted as S_1, S_2, S_3 and S_4 respectively), are replaced by perfect electric conductors (PEC) with equivalent magnetic currents $\mathbf{M}_i$, $i = 1,...,4$. The model is divided into five regions; the feeding waveguide, the coupling slot, the cavity, the radiating slot and the external region (denoted as region I, II, III, IV and V, respectively). The external region is a shorted waveguide with two pairs of periodic boundaries in the side walls to include the mutual coupling in the two-dimensional infinite array. The magnetic current $\mathbf{M}_i$ in each slot aperture is expanded by entire-domain basis functions $\mathbf{m}_{u\alpha\beta}$ and $\mathbf{m}_{v\alpha'\beta'}$ as follows.

$$\mathbf{M}_i = \sum_{\alpha=1}^{A}\sum_{\beta=0}^{B} V_{u\alpha\beta}\mathbf{m}_{u\alpha\beta} + \sum_{\alpha'=0}^{A'}\sum_{\beta'=1}^{B'} V_{v\alpha'\beta'}\mathbf{m}_{v\alpha'\beta'} \quad \left(\text{on } S_i, i = 1,...,4\right),$$

$$\mathbf{m}_{u\alpha\beta} = \sqrt{\frac{\epsilon_{\alpha\beta}}{lw}} \sin\left\{\frac{\alpha\pi}{l}\left(u + \frac{l}{2}\right)\right\} \cos\left\{\frac{\beta\pi}{w}\left(v + \frac{w}{2}\right)\right\} \hat{\mathbf{u}},$$

$$\mathbf{m}_{v\alpha'\beta'} = \sqrt{\frac{\epsilon_{\alpha'\beta'}}{lw}} \cos\left\{\frac{\alpha'\pi}{l}\left(u + \frac{l}{2}\right)\right\} \sin\left\{\frac{\beta'\pi}{w}\left(v + \frac{w}{2}\right)\right\} \hat{\mathbf{v}}$$

where $V_{u\alpha\beta}$ and $V_{v\alpha'\beta'}$ are the weights to be solved. u and v are local coordinate system whose origin is each slot center. u- and v-axis represent the directions along the length and the width, respectively. The basis functions $\mathbf{m}_{u\alpha\beta}$ and $\mathbf{m}_{v\alpha'\beta'}$ have the components along the length and the width, respectively and α, β, α' and β' are their orders. For the narrow coupling slot, only the lowest-order u-direction basis function $\mathbf{m}_{u10}$ is assumed. l and w are the length and the width of the slot, respectively. Normalized constant $\epsilon_{\alpha\beta}$ is two when either α or β is zero and four otherwise. The continuity conditions of the tangential component of the magnetic field on the apertures and the application of Galerkin's MoM gives the following system of liner equations.

$$\begin{bmatrix} [Y_{11}^{\mathrm{I}}]+[Y_{11}^{\mathrm{II}}] & -[Y_{12}^{\mathrm{II}}] & 0 & 0 \\ -[Y_{21}^{\mathrm{II}}] & [Y_{22}^{\mathrm{II}}]+[Y_{22}^{\mathrm{III}}] & -4[Y_{23}^{\mathrm{III}}] & 0 \\ 0 & -[Y_{32}^{\mathrm{III}}] & 4[Y_{33}^{\mathrm{III}}]+[Y_{33}^{\mathrm{IV}}] & -[Y_{43}^{\mathrm{IV}}] \\ 0 & 0 & -[Y_{43}^{\mathrm{IV}}] & [Y_{44}^{\mathrm{IV}}]+[Y_{44}^{\mathrm{V}}] \end{bmatrix} \begin{bmatrix} [V_1] \\ [V_2] \\ [V_3] \\ [V_4] \end{bmatrix} = \begin{bmatrix} -[i^{(+)}] \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where Y_{ij}^{n} is the reaction between the basis functions $\mathbf{m}_i$ and $\mathbf{m}_j$ in the region n and defined as

$$Y_{ij}^{n} = \int_{S_i} \mathbf{m}_i \cdot \int_{S_j} \bar{\mathbf{G}}^{n} \cdot \mathbf{m}_j dS_s dS_o,$$

where $\bar{\mathbf{G}}^{n}$ is the dyadic Green's function in the region n. $i^{(+)}$ is the mutual reaction between the incident magnetic field and the basis function $\mathbf{m}_1$. The reactions in the region I, II, IV and V can be derived in analytical form since each region has a canonical shape: rectangular. However, the reactions in the cavity region can not be derived in analytical form because of the X-shaped cross section. The reaction in the cavity region is expressed as

$$Y_{ij}^{\mathrm{III}} = \sum_{p} -\alpha_p^{ij} \int_{S_j} \mathbf{m}_j \cdot \mathbf{h}_p dS_o \int_{S_i} \mathbf{h}_p \cdot \mathbf{m}_i dS_s \quad (i, j = 2, 3),$$

$$\alpha_p^{ij} = \coth(\gamma_p t_c) \quad (i = j), \quad \frac{1}{\sinh(\gamma_p t_c)} \quad (i \neq j),$$

where p is the mode number including both TE and TM modes, $\mathbf{h}_p$ is the eigenmode of the waveguide with the cross section of the cavity and γ_p is the corresponding propagation constant of eigenmode [4]. The eigenmodes are numerically derived by FEM for calculation of the reactions in the cavity region.

2.2 Eigenmode analysis of the waveguide with the cross section of the cavity by edge-based FEM

The eigenmodes are derived by the edge-based FEM [5]. The analysis shape is shown in Fig. 3 (a). In order to exclude unnecessary asymmetric eigenmodes for the equal excitation of the four radiating slots in the cavity, the quarter of the cavity are analyzed with assumption of PEC and perfect magnetic conductor (PMC) boundaries. The analysis shape is divided by rectangular elements and first-order elements [5] are adopted as vector basis functions.

3. Analysis results

The computed electrical and magnetic fields of the lowest-order TE and TM eigenmodes are shown in Fig. 3 (b) and (c), respectively. The analysis frequency is from 55 GHz to 70 GHz. The number of the elements is 5567 and RMS edge length l_0 is 0.04 mm. Contributions by eigenmodes whose cutoff wavelength is longer than 10 l_0 are summed in consideration of field representation limitation by the first-order basis functions and the finite number of the elements. In this case, 194 TE and 174 TM modes are considered. The number of the basis functions for the radiating slot is 420 (A = 10, B = 20, A' = 20, B' = 10). The frequency characteristic of reflection coefficient is shown in Fig. 4 in comparison with the HFSS result. The both agree each other very well. The difference may result from three different factors; the insufficient numbers of the basis functions and the modes for the dyadic Green's function expansion and the assumption that the excitation of the radiating slots are symmetry.

The computation consists of eigenmode derivation of the X-shaped cavity and calculation of reflection coefficient at each frequency. The eigenmode derivation depends only on the shape of X-shaped cavity and the divided elements. It is not dependent on the frequency. The eigenmode derivation results in generalized sparse eigenvalue problem which can be fast solved by MATLAB function `sptarn`. The function uses Arnoldi algorithm with spectral transformation. The reflection calculation time depends on the numbers of the element and the basis functions. The eigenmode derivation takes 310 seconds and the calculation of the reflection coefficient for one frequency takes 74 seconds by a personal computer with 2.80-GHz dual core CPU and 8-GB memory.

Elimination of unnecessary basis functions and higher order eigenmodes can speed up the analysis sustaining certain degree of accuracy. The normalized excitation coefficients of the basis functions at 61.5 GHz are shown in Fig. 5. Among the u- and v-component basis functions, the excitation coefficients corresponding to $\alpha = 1$ and $\beta' = 1$ are higher than the others, respectively. Elimination of weakly excited basis functions such as $\alpha \geq 3$ or $\beta' \geq 3$ may not affect the reflection coefficient. Analyzed reflection coefficients using smaller number of basis functions are shown in Fig. 6. Even though the number of basis functions is reduced up to 57 ($A = 3$, $B = 10$, $A' = 5$, $B' = 4$), almost equivalent results are obtained. However, further reduction of the basis functions produces largely different results with HFSS one. Next, the reflection coefficients with a smaller number in the eigenmode expansion are shown in Fig. 7 in the case that the number of the basis functions is 57 ($A = 3$, $B = 10$, $A' = 5$, $B' = 4$). Difference of reflection coefficient with the 54 TE and 43 TM modes used case is negligible small. The difference with the 35 TE and 26 TM modes used case is acceptable small in the sense of antenna design. In the case that a few number of the eigenmode expansion such as 10 TE and 6 TM modes is used, reflection coefficient also has similar tendency. The analysis time is reduced up to 2 seconds for the eigenmode derivation and 0.7 seconds for the calculation of reflection coefficient in the case of the 57 basis functions and 35 TE and 26 TM modes used with sufficient accuracy for antenna design.

4. Conclusion

We have applied the hybrid MoM/FEM to the X-shaped cavity-backed wide slot 2×2-element sub-array. In order to analyze the sub-array with wide slots, the basis functions with the components along the slot length and width are introduced. The analysis result by the method agrees with that by HFSS and the analysis without unnecessary basis functions and higher order eigenmodes gives adequate accuracy with fast computation time.

References

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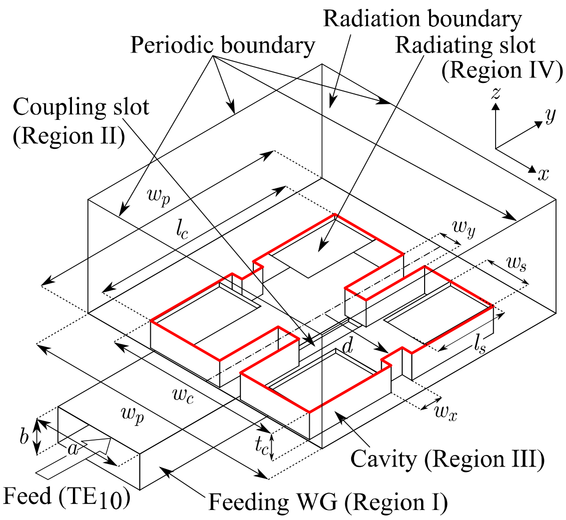


Figure 1: Analysis model of the 2×2-element sub-array.

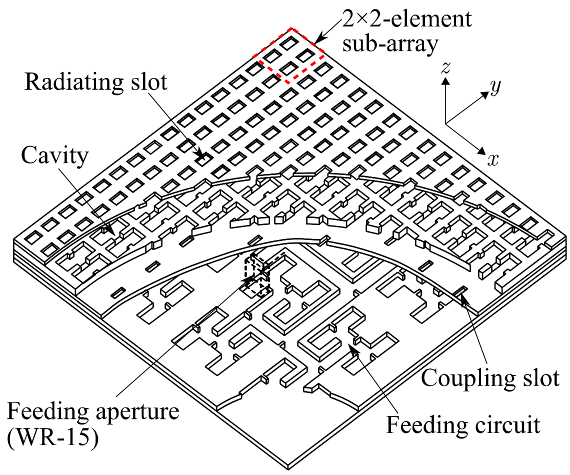


Figure 2: 16×16-elment double-layer full-corporate-feed hollow-waveguide slot array antenna.

Table 1: DIMENSIONS OF THE ANALYSIS MODEL

Component	Parameter	Unit: mm
Feeding waveguide	$a \times b$	2.94 × 1.2
Coupling slot	$l_{cp} \times w_{cp} \times t_{cp}$	2.74 × 1.0 × 0.3
Cavity	$l_c \times w_c \times t_c$	6.94 × 5.95 × 1.2
Walls in the cavity	w_x , w_y , d	0.75, 0.75, 2.3
Radiating slot	$l_s \times w_s \times t_s$	2.74 × 1.75 × 0.3
Periodic boundary wall	$w_p \times w_p$	8.4 × 8.4

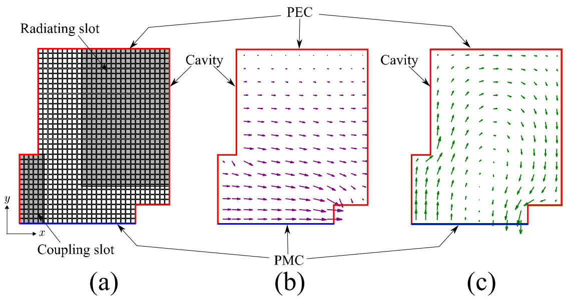
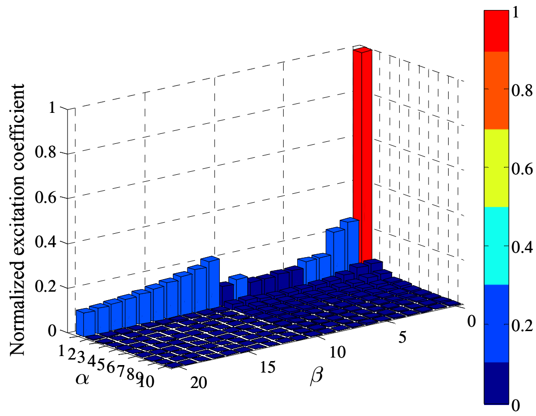
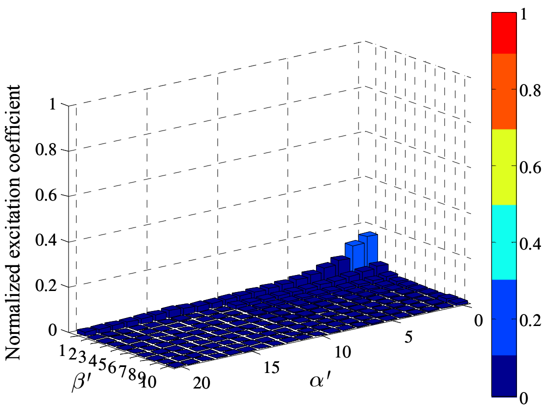


Figure 3: Quater cavity. (a) Analysis model. (b), (c) E- and H-field of the lowest TE and TM mode



(a) u-component.



(b) v-component.

Figure 5: Normalized excitation coefficients of basis functions of radiating slot.

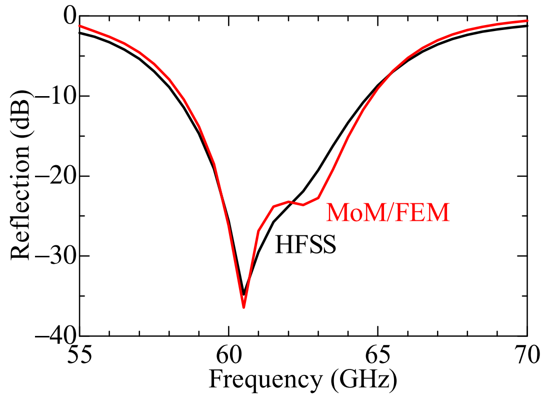


Figure 4: Comparison of frequency characteristic of reflection coefficient with HFSS.

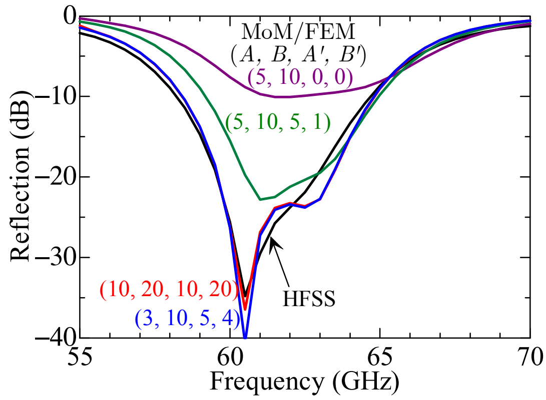


Figure 6: Analysis result with different.number of basis functions.

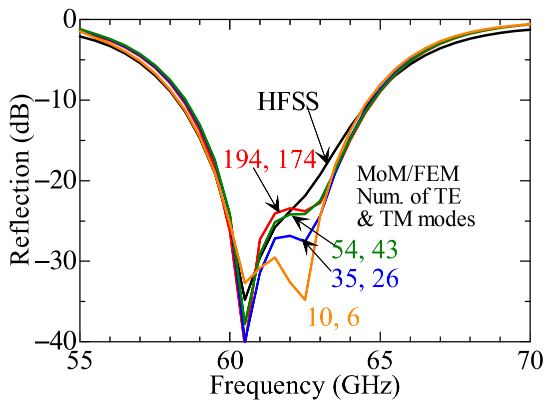


Figure 7: Analysis result with different number in the eigenmode expansion.