Dyadic Green's Functions in Cylindrically Multilayered Gyroelectric Chiral Media

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Abstract

This paper presents a novel eigenfunction expansion of the electric-type dyadic Green's functions for both a unbounded gyroelectric chiral medium and a cylindrically-multilayered gyroelectric chiral medium in terms of the cylindrical vector wave functions. The unbounded and scattering Green dyadics are formulated based on the principle of scattering superposition for the electromagnetic waves, namely, the direct wave and scattered waves. First, the unbounded dyadic Green's functions are correctly derived and some mistakes occuring in the literature are pointed out. Secondly, the scattering dyadic Green's functions are formulated and their coefficients are obtained from the boundary conditions at each interface. These coefficients are expressed in a compact form of recurrence matrices; coupling between TE and TM modes are considered and various wave modes are decomposed one from another. Finally, three cases, where the impressed current source are located in the first, the intermediate, and the last regions respectively, are taken into account in the mathematical manipulation of the coefficient recurrence matrices for the dyadic Green's functions.

I Introduction

The dyadic Green's functions (DGFs) play an important role in electromagnetic theory and its applications to practical analysis of various electromagnetic boundary value problems. The eigenfunction expansion of the dyadic Green's functions has been well developed and applied over the last several decades [1–3], despite the ever increasing demand for numerical methods. The vector wave functions have found versatile applications in the formulation of the dyadic Green's functions, as can be seen from the work done [1–7]. Although the DGFs in isotropic media have been well-studied in the last three decades, complete formulation of the DGFs in various anisotropic media using the eigenfunction expansion technique has not been achieved so far. Since 1970's, the dyadic Green's functions in anisotropic media have been derived [8,9], using (1) the Fourier transform technique, (2) the method of angular spectrum expansion, and (3) the transmission matrix method. The DGFs and fields in gyroelectric media have also been formulated [10,11].

There have been some results for bianisotropic media or gyroelectric chiral media available nowadays, however most of them are basically valid for unbounded media only while some of them are not correct, for instance, the results in [11] commented by [12]. Thus, the motivation of this work is quite appearant. Different from the existing work, this paper aims at (1) the direct development of the unbounded dyadic Green's functions in an unbounded gyroelectric chiral medium where the cylindrical vector wave expansion technique is employed and mistakes in the existing work are pointed out; (2) the formulations of the scattering dyadic Green's functions and their coefficients in a cylindrically multilayered gyroelectric chiral medium where each layer is assumed to be the gyroelectric chiral medium and its results can be reduced to those of the isotropic media, and where the source is assumed to have an arbitrary 3-dimensional distribution and can be located anywhere while the field point can also be arbitrarily located in the multilayers; and (3) the rigorous derivation of the irrotational part of the dyadic Green's functions which were not always provided in the existing work. Due to the different geometries of the multilayered gyroelectric chiral media, the formulation of the dyadic Green's functions differs one from another. The work included in the present paper is a further extension of the previous work where the dyadic Green's functions have been represented for the planar-multilayered gyroelectric chiral media.

II DGFs For Unbounded Gyroelectric Chiral Media

Throughout the paper, a time dependence $e^{-i\Psi t}$ is assumed and suppressed in the analysis. A homogeneous gyroelectric chiral medium with the time harmonic excitation can be characterized by a set of constitutive relations [11]

$$\begin{aligned} \boldsymbol{D} &= \bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + i\xi_c \boldsymbol{B}, \\ \boldsymbol{H} &= i\xi_c \boldsymbol{E} + \boldsymbol{B}/\mu, \end{aligned} \quad \text{where} \quad \bar{\boldsymbol{\epsilon}} = \begin{bmatrix} \epsilon & -ig & 0 \\ ig & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{1}$$

Substituting (1) into the source-incorporated Maxwell's equations leads to

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - 2\omega\mu\xi_c \boldsymbol{\nabla} \times \boldsymbol{E} - \omega^2 \mu_0 \bar{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} = i\omega\mu_0 \boldsymbol{J}. \tag{2}$$

A General Formulation of Unbounded DGFs

The electric field can thus be expressed in terms of the DGF and electric source distribution as

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu \int_{V'} \overline{\boldsymbol{G}}_e(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')\, dV', \tag{3}$$

where V' denotes the volume occupied by the exciting current source. Similarly, substituting (3) into (2) leads to

$$\nabla \times \nabla \times \overline{\boldsymbol{G}}_e(\boldsymbol{r},\boldsymbol{r}') - 2\omega\mu\xi_c \nabla \times \overline{\boldsymbol{G}}_e(\boldsymbol{r},\boldsymbol{r}') - \omega^2\mu_0 \overline{\epsilon} \cdot \overline{\boldsymbol{G}}_e(\boldsymbol{r},\boldsymbol{r}') = \overline{\boldsymbol{I}}\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{4}$$

where $\overline{\boldsymbol{I}}$ and $\delta(\boldsymbol{r}-\boldsymbol{r}')$ denotes the dyadic identity and Dirac delta function, respectively.

According to the well-known Ohm-Rayleigh method, the source term in (4) can be expanded in terms of the solenoidal and irrotational cylindrical vector wave functions in cylindrical coordinate system. Thus,

$$\overline{\boldsymbol{I}}\delta(\boldsymbol{r}-\boldsymbol{r}') = \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_{n\lambda}(h)\boldsymbol{A}_{n\lambda}(h) + \boldsymbol{N}_{n\lambda}(h)\boldsymbol{B}_{n\lambda}(h) + \boldsymbol{L}_{n\lambda}(h)\boldsymbol{C}_{n\lambda}(h)\right], \tag{5}$$

where $\boldsymbol{M}_{n\lambda}(h)$ & $\boldsymbol{N}_{n\lambda}(h)$ are the solenoidal, and $\boldsymbol{L}_{n\lambda}(h)$ is the irrotational, cylindrical vector wave functions while λ and h are the spectral longitudinal and radial wave numbers, respectively. The solenoidal and irrotational cylindrical vector wave functions are defined as [11]

$$\boldsymbol{M}_{n\lambda}(h) = \nabla \times [\Psi_{n\lambda}(h)\hat{\boldsymbol{z}}], \quad \boldsymbol{N}_{n\lambda}(h) = \frac{1}{k_\lambda}\nabla \times \boldsymbol{M}_{n\lambda}(h), \quad \boldsymbol{L}_{n\lambda}(h) = \nabla[\Psi_{n\lambda}(h)], \tag{6a}$$

where $k_\lambda = \sqrt{\lambda^2+h^2}$, and the generating function is given by $\Psi_{n\lambda}(h) = J_n(\lambda\rho)e^{i(n\phi+hz)}$. The vector expansion coefficients, $\boldsymbol{A}_{n\lambda}(h)$, $\boldsymbol{B}_{n\lambda}(h)$, and $\boldsymbol{C}_{n\lambda}(h)$ in (5), are to be determined from the orthogonality relationships among the cylindrical vector wave functions. Therefore, by taking the scalar product of (5) with $\boldsymbol{M}_{-n',-\lambda'}(-h')$, $\boldsymbol{N}_{-n',-\lambda'}(-h')$ and $\boldsymbol{L}_{-n',-\lambda'}(-h')$ each at a time, the vector expansion coefficients are given by:

$$\boldsymbol{A}_{n\lambda}(h) = \frac{1}{4\pi^2\lambda}\boldsymbol{M}'_{-n,-\lambda}(-h), \quad \boldsymbol{B}_{n\lambda}(h) = \frac{1}{4\pi^2\lambda}\boldsymbol{N}'_{-n,-\lambda}(-h), \quad \boldsymbol{C}_{n\lambda}(h) = \frac{\lambda}{4\pi^2(\lambda^2+h^2)}\boldsymbol{L}'_{-n,-\lambda}(-h), \tag{7}$$

where the prime notation of the cylindrical vector wave functions denotes the expressions at the source point $\boldsymbol{r}'$. The dyadic Green's function can thus be expanded as [11]:

$$\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_{n\lambda}(h)\, \boldsymbol{a}_{n\lambda}(h) + \boldsymbol{N}_{n\lambda}(h)\, \boldsymbol{b}_{n\lambda}(h) + \boldsymbol{L}_{n\lambda}(h)\, \boldsymbol{c}_{n\lambda}(h)\right], \tag{8}$$

where the vector expansion coefficients $\boldsymbol{a}_{n\lambda}(h)$, $\boldsymbol{b}_{n\lambda}(h)$ and $\boldsymbol{c}_{n\lambda}(h)$ are obtained by substituting (8) and (5) into (4), which the dyadic Green's function must satisfy, and noting the instinct properties of the vector wave functions,

$$\boldsymbol{M}_n(h,\lambda) = \frac{1}{k_\lambda}\nabla \times \boldsymbol{N}_n(h,\lambda), \quad \boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda}\nabla \times \boldsymbol{M}_n(h,\lambda), \quad \nabla \times \boldsymbol{L}_n(h,\lambda) = 0, \tag{9}$$

we end up with

$$\begin{aligned}
&\int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \Big\{\left[k_\lambda^2 \overline{\boldsymbol{I}} - \omega^2\mu_0\overline{\epsilon}\right] \cdot \left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda) + \boldsymbol{N}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right] \\
&\qquad -2k_\lambda\omega\mu_0\xi_c\left[\boldsymbol{N}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda) + \boldsymbol{M}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right] - \omega^2\mu_0\overline{\epsilon} \cdot \boldsymbol{L}_n(h,\lambda)\boldsymbol{c}_n(h,\lambda)\Big\} \\
&= \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{A}_n(h,\lambda) + \boldsymbol{N}_n(h,\lambda)\boldsymbol{B}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\boldsymbol{C}_n(h,\lambda)\right].
\end{aligned} \tag{10}$$

The above approach shows an important fact that the afore-assumed unknowns, $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, and $\boldsymbol{c}_n(h,\lambda)$, may not be the same as those coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$ although they were assumed to be the same in [11].

By taking the anterior scalar product of (10) with the vector wave equations, respectively, and by performing the integration over the entire space, we can formulate the equations satisfied by the unknown vectors and the known scalar and vector parameters in a matrix form as given below:

$$[\boldsymbol{\Phi}]_{3\times3}[\boldsymbol{X}]_{3\times1} = [\boldsymbol{\Theta}]_{3\times1}, \quad \text{where} \quad [\boldsymbol{\Phi}]_{3\times3} = [\boldsymbol{\Phi}_1\boldsymbol{\Phi}_2\boldsymbol{\Phi}_3] \tag{11}$$

with

$$\mathbf{\Phi}_1 = \begin{bmatrix} k_\lambda^2 - \omega^2\mu_0\epsilon \\ -\omega\mu_0\left(2\xi_c k_\lambda + \omega g\frac{h}{k_\lambda}\right) \\ -i\omega^2\mu_0 g\frac{\lambda^2}{k_\lambda^2} \end{bmatrix}, \quad \mathbf{\Phi}_2 = \begin{bmatrix} -\omega\mu_0\left(2\xi_c k_\lambda + \omega g\frac{h}{k_\lambda}\right) \\ k_\lambda^2 - \frac{\omega^2\mu}{k_\lambda^2}(h^2\epsilon + \lambda^2\epsilon_z) \\ -\frac{ih\lambda^2}{k_\lambda^3}\omega^2\mu_0(\epsilon - \epsilon_z) \end{bmatrix}, \quad \mathbf{\Phi}_3 = \begin{bmatrix} i\omega^2\mu_0 g \\ \frac{ih}{k_\lambda}\omega^2\mu_0(\epsilon - \epsilon_z) \\ -\frac{\omega^2\mu_0}{k_\lambda^2}\left(\lambda^2\epsilon + h^2\epsilon_z\right) \end{bmatrix}, \tag{12}$$

and $[\boldsymbol{X}]$ and $[\mathbf{\Theta}]$ are two column vectors given respectively by

$$[\boldsymbol{X}] = \begin{bmatrix} \boldsymbol{a}_n(h,\lambda) \\ \boldsymbol{b}_n(h,\lambda) \\ \boldsymbol{c}_n(h,\lambda) \end{bmatrix}, \quad [\mathbf{\Theta}] = \begin{bmatrix} \boldsymbol{A}_n(h,\lambda) \\ \boldsymbol{B}_n(h,\lambda) \\ \boldsymbol{C}_n(h,\lambda) \end{bmatrix}; \quad \text{so that} \quad [\boldsymbol{X}] = \frac{1}{\Gamma}\begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix}[\mathbf{\Theta}] \tag{13}$$

where

$$\begin{aligned} \Gamma = {} & k_\lambda^2(h^2\epsilon_z + \epsilon\lambda^2) - \mu\omega^2\left[2h^2\epsilon\epsilon_z - \lambda^2(g^2 - \epsilon^2 - \epsilon\epsilon_z) + 4\mu\xi^2(h^2\epsilon_z + \epsilon\lambda^2)\right] \\ & -4gh\epsilon_z\mu^2\xi_c\omega^3 + \epsilon_z\mu^2\omega^4(\epsilon^2 - g^2) \end{aligned} \tag{14}$$

and the coupling coefficients are

$$\alpha_1 = h^2\epsilon_z + \lambda^2\epsilon - \omega^2\mu_0\epsilon\epsilon_z, \quad \beta_1 = \alpha_2 = \frac{\omega\mu_0}{k_\lambda}\left[gh\epsilon_z\omega + 2\xi_c(h^2\epsilon_z + \epsilon\lambda^2)\right], \tag{15a}$$

$$\beta_2 = h^2\epsilon_z + \epsilon\lambda^2 - \omega^2\mu_0\epsilon(h^2\epsilon_z + \lambda^2\epsilon) + \omega^2\mu_0 g^2\lambda^2, \quad \gamma_1 = i\left[2\omega\mu h\xi_c(\epsilon - \epsilon_z) + g(k_\lambda^2 - \omega^2\mu\epsilon_z)\right], \tag{15b}$$

$$\gamma_1 = -\frac{k_\lambda^2}{\lambda^2}\alpha_3, \quad \gamma_2 = -\frac{k_\lambda^2}{\lambda^2}\beta_3 = i\frac{1}{k_\lambda}\left[h(k_\lambda^2 - \mu\omega^2\epsilon)(\epsilon - \epsilon_z) + \omega\mu_0 g(2k_\lambda^2\xi_c + gh\omega\right], \tag{15c}$$

$$\gamma_3 = \frac{1}{\omega^2\mu}\left\{-k_\lambda^4 + \omega^2\mu_0\left[2h^2\epsilon + \lambda^2(\epsilon + \epsilon_z) + 4k_\lambda^2\mu\xi_c^2\right] + 4gh\xi_c\mu^2\omega^3 + \frac{\omega^4\mu^2}{k_\lambda^2}\left[h^2(g^2 - \epsilon^2) - \epsilon\epsilon_z\lambda^2\right]\right\}. \tag{15d}$$

It should be noted that the substitution of (8) and (5) into (4) to give (10) is based upon the condition that one can interchange the summation on n and the integrals on h, λ. This condition can be justified if one notes that the terms in the square brackets of (5) and (8) are continuous with respect to h and λ, simultaneously. Now, it becomes quite clear that each of the coefficients, $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, and $\boldsymbol{c}_n(h,\lambda)$, is actually a linear combination of those known coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$. In other words, the coupling of the TE and TM modes from source distribution, and to the field expression, exists. Thus, the results obtained in [11] are not correct at all because such coupling is absent there in the formulation. In terms of the cylindrical vector wave functions for the gyroelectric chiral media, this paper presents a correct form of the unbounded dyadic Green's functions.

B Analytical Evaluation Of The λ Integral

Obviously, this irrotational Green's dyadic of the gyroelectric chiral media can be simply reduced to that of an isotropic medium by letting $\epsilon_z = \epsilon$. It is observed that this irrotational term is only a function of the permittivity ϵ_z only. This is simply because the anisotropic medium has an axial direction of $\widehat{\boldsymbol{z}}$. The second integration term can be evaluated by making use of the residue theorem in λ-plane (appendix). This term contributes from the solenoidal vector wave functions. Hence after some mathematical manipulations, we arrived at the final unbounded dyadic Green's function for a gyroelectric chiral medium which is suitable for further analysis of a cylindrically multilayered structure (for $\rho \gtrless \rho'$):

$$\begin{aligned} \overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = {} & -\frac{1}{\omega^2\mu_0\epsilon_z}\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}') + \frac{i}{4\pi}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty}\frac{1}{2(k_1^2 - k_2^2)}\sum_{j=1}^{2}\frac{(-1)^{j+1}}{\lambda_j^2} \\ & \times\left\{\begin{bmatrix} \boldsymbol{M}_{n,h}^{(1)}(\lambda_j)\boldsymbol{P}'_{-n,-h}(-\lambda_j) \\ \boldsymbol{M}_{n,h}(-\lambda_j)\boldsymbol{P}'^{(1)}_{-n,-h}(\lambda_j) \end{bmatrix} + \frac{k_{\lambda j}}{\epsilon}\begin{bmatrix} \boldsymbol{Q}_{n,h}^{(1)}(\lambda_j)\boldsymbol{M}'_{-n,-h}(-\lambda_j) \\ \boldsymbol{Q}_{n,h}(-\lambda_j)\boldsymbol{M}'^{(1)}_{-n,-h}(\lambda_j) \end{bmatrix}\right. \\ & \left. + \frac{k_{\lambda j}^2}{h^2\Psi^2\mu_0\epsilon}\begin{bmatrix} \boldsymbol{U}_{n,h}^{(1)}(\lambda_j)\boldsymbol{N}'_{-nt,-h}(-\lambda_j) \\ \boldsymbol{U}_{n,h}(-\lambda_j)\boldsymbol{N}'^{(1)}_{-nt,-h}(\lambda_j) \end{bmatrix} + \frac{k_{\lambda j}^2}{\lambda_j^2\Psi^2\mu_0\epsilon}\begin{bmatrix} \boldsymbol{V}_{n,h}^{(1)}(\lambda_j)\boldsymbol{N}'_{-nz,-h}(-\lambda_j) \\ \boldsymbol{V}_{n,h}(-\lambda_j)\boldsymbol{N}'^{(1)}_{-nz,-h}(\lambda_j) \end{bmatrix}\right\}, \end{aligned} \tag{16}$$

where the superscript (1) of the vector wave functions denotes the first-kind cylindrical Hankel function $H_n^{(1)}(\lambda\rho)$. The vector wave functions $\boldsymbol{P}'_{-n,-h}(-\lambda_j)$, $\boldsymbol{Q}_{n,h}(\lambda_j)$, $\boldsymbol{U}_{n,h}(\lambda_j)$ and $\boldsymbol{V}_{n,h}(\lambda_j)$ are given respectively by

$$\boldsymbol{P}'_{-n,-h}(-\lambda_j) = (\lambda_j^2 + \frac{\epsilon_z}{\epsilon}h^2 - \epsilon_z\omega^2\mu)\boldsymbol{M}'_{-n,-h}(-\lambda_j) + \frac{k_{\lambda j}}{\epsilon}\left[\frac{g}{h}(\epsilon_z\omega^2\mu - \lambda_j^2) + 2\epsilon_z\omega\mu\xi_c\right]$$

$$\times \boldsymbol{N}'_{-nt,-h}(-\lambda_j) + \frac{k_{\lambda j}}{\epsilon}(gh + 2\epsilon\,\omega\mu\xi_c)\boldsymbol{N}'_{-nz,-h}(-\lambda_j), \tag{17a}$$

$$\boldsymbol{Q}_{n,h}(\lambda_j) = \left[\frac{g}{h}(\epsilon_z\omega^2\mu - \lambda_j^2) + 2\epsilon_z\omega\mu\xi_c\right]\boldsymbol{N}_{nt,h}(\lambda_j) + (gh + 2\epsilon\,\omega\mu\xi_c)\boldsymbol{N}_{nz,h}(\lambda_j), \tag{17b}$$

$$\begin{aligned}\boldsymbol{U}_{n,h}(\lambda_j) = {} & \left[(k_{\lambda j}^2 - \epsilon\,\omega^2\mu)(\epsilon_z\omega^2\mu - \lambda_j^2) + 4\lambda_j^2\omega^2\mu^2\xi_c^2\right]\boldsymbol{N}_{nt,h}(\lambda_j) \\ & + h\left[h(k_j^2 - \epsilon\,\omega^2\mu) - 2\omega^2\mu^2\xi_c(2h\xi_c + g\omega)\right]\boldsymbol{N}_{nz,h}(\lambda_j),\end{aligned} \tag{17c}$$

$$\begin{aligned}\boldsymbol{V}_{n,h}(\lambda_j) = {} & \lambda_j^2\left[(k_{\lambda j}^2 - \epsilon\,\omega^2\mu) - \frac{2\omega^2\mu^2\xi_c}{h}(2h\xi_c + g\omega)\right]\boldsymbol{N}_{nt,h}(\lambda_j) + \frac{1}{\epsilon}\left\{k_{\lambda j}^2\left[\lambda_j^2\epsilon + h^2(\epsilon_z - \epsilon)\right]\right. \\ & + \omega^2\mu\left[g^2\lambda_j^2 + h^2(2\epsilon - \epsilon_z)(\epsilon + 4\mu\xi_c^2) - k_{\lambda j}^2\epsilon(\epsilon_z + 4\mu\xi_c^2)\right] + 4gh\Psi^3\mu^2\xi_c(\epsilon - \epsilon_z) \\ & \left. + \omega^4\mu^2(g - \epsilon)(g + \epsilon)(\epsilon - \epsilon_z)\right\}\boldsymbol{N}_{nz,h}(\lambda_j).\end{aligned} \tag{17d}$$

Now, we obtained the rigorous expression of the unbounded dyadic Green's functions for the gyroelectric chiral medium. This form differs from the existing representations of the dyadic Green's functions for the gyroelectric chiral medium or some more generalized media. The agreement can be obtained in no ways between the present form of DGFs and other forms given in the literature elsewhere. This is because (1) the dyadic Green's functions obtained elsewhere in the literature using different approaches take quite different forms as ours and the direct comparison among these theoretical results are almost impossible; (2) the dyadic Green's functions obtained using the same approach in [11] are incorrect as indicated and shown in [12]; and (3) the dyadic Green's functions given using the similar approach in some other work by him are not rigorously correct as the irrotational parts of the DGFs were missing in the presentations (where the mistakes are due to the ignorance of the Jordan lemma conditions [1]). The scattering DGF's are also formulated in this work. Detailed formulation of the scattering DGF's and derivation of their scattering coefficients are shown in the presentation, but are suppressed here due to space constraint.

III Conclusion

A complete eigenfunction expansion of the dyadic Greens' functions for a unbounded gyroelectric chiral medium and a cylindrically multilayered gyroelectric chiral medium is presented in this paper. The unbounded dyadic Green's function in the gyroelectric chiral medium is first obtained, based on the Ohm-Raleigh method. The scattered dyadics are constructed with the principle of scattering superposition for a multilayered medium. By the use of the boundary conditions at each interface, the scattering coefficients of the dyadic Green's functions are represented in the form of compact recurrence matrices. Further analysis is performed for three cases, i.e. the source excitation located in the first, the intermediate and the last regions, respectively. From the formulation of the generalized dyadic Green's functions, it is seen that (1) the general form of DGFs for the cylindrically multilayered gyroelectric chiral medium can be reduced to those DGFs for less complex media, such as chiral media, anisotropic media, and isotropic media; (2) the wave mode splitting is observed from the formulation of the DGFs; and (3) as a result, the dyadic Green's functions can be easily decomposed using the aforementioned modes. In summary, the present paper contributes to (1) a correct formulation of the unbounded dyadic Green's function in a gyroelectric chiral medium as compared with the published work [11] and [12], (2) a detailed formulation of the irrotational part of the dyadic Green's functions which was quite often ignored in the recent publications such as in , (3) a formulation of the dyadic Green's functions in a cylindrically multilayered gyroelectric chiral medium, and (4) the compact matrix expression of the scattering coefficients of the dyadic Green's functions. Application of the present work can be made to problems of electromagnetic wave propagation through and scattering by, and antenna radiation in, cylindrically multilayered gyroelectric chiral media.

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