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CONSTRAINED OPTIMIZATION OF PARASITIC ARRAYS

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For arrays of identical antennas where the element currents are independent variables the solution to the optimization problem has been known for some time. The pioneering work was done by Uzkov[1], recently a number of cases has been treated by Lo et.al.[2], who also consider cases with constraints on the Q-factor. A good account of recent work is given in ref.3.

For an array with only one excited element and the remaining elements parasitic the situation is quite different, and it does not seem possible to express the gain or Q as a ratio of Hermitian forms in the independent variables. The approach in this paper is therefore to use a strictly numerical technique where available computer-optimization procedures are used to find the solution. The method is only applicable for small arrays, say 15-20 elements, and is therefore well suited for Yagi-Uda antennas.

The particular case considered here is a linear array of infinitely thin half-wave dipoles with equidistant spacing. The parasitic dipoles are loaded at their centers with arbitrary reactances which are the independent variables for the array. This array simulates to some extent a Yagi-Uda array where the reactance variation is achieved by variation of element lengths. The problem is solved by inversion of the impedance matrix, the matrix equation being

$$\begin{pmatrix} 0 \\ 0 \\ V_{N_x} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} Z_{11}+jX_1 & & (Z_{mn}) \\ & Z_{22}+jX_2 & \\ (Z_{mn}) & & \ddots \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ \\ I_N \end{pmatrix} \qquad (1)$$

Element No. N_x is the excited one, $X_1, X_2, \dots$ are the loading reactances, $I_1, I_2, \dots$ the unknown currents. Z_{mn} are the known mutual impedances which are constant for a given spacing. Since X_{N_x} does not enter in gain calculations the N-element array poses a N-1-dimensional optimization problem. For a given set of X_i, the current vector is found from (1) and knowing the currents the remaining antenna parameters, gain, Q, impedance and pattern are easily found.

Rosenbrock's method of optimization[4] has been applied, it has the advantage of allowing a number of additional functions to be constrained in specified ranges. In the present case it may be used to find maximum unconstrained gain for a given number of elements, and maximum gain with a constraint on Q.

Some results for gain optimization are presented in Fig. 1, which shows gain relative to a half-wave dipole for a N-element array as a function of element spacing. For comparison the maximum gain for an arbitrary current vector is also shown, these results are taken from ref. 3.

Some of the conclusions to be drawn are the following. For a given number of elements there is an optimum spacing which increases with the number of elements. At least for a small number of elements the maximum gain is close to the value obtainable with free excitation of the currents. The excited element should be close to the center of the array for macimum gain, although the difference is small (compare (N,Nx) = (7,2), (7,3), (7,4)).

Maximum gain with a constraint on Q is also shown for N = 3 and N = 4. For the unconstrained cases the Q-values are about 10, in the constrained cases the Q-values are less than 5 with only a moderate gain reduction.

References:

[1] Uzkov, A.I., "An approach to the problem of optimum directive antenna design", Compt. Rend. (Doklady) Acad. Sci. USSR, Vol. 53, pp. 35-38, 1946.

[2] Lo, Y.T., S.W. Lee, and Q.H. Lee, "Optimization of directivity and signal-to-noise ratio of an arbitrary antenna array", Proc. IEEE, Vol. 54, pp. 1033-1045, August 1966.

[3] Collin, R.E. and F.J. Zucker (eds.), "Antenna Theory, Part I", Ch. 5, McGraw-Hill Book Co., 1969.

[4] Rosenbrock, H.H., "An automatic method for finding the greatest or least value of a function", Computer J., Vol. 3, pp. 175-184, October 1966.

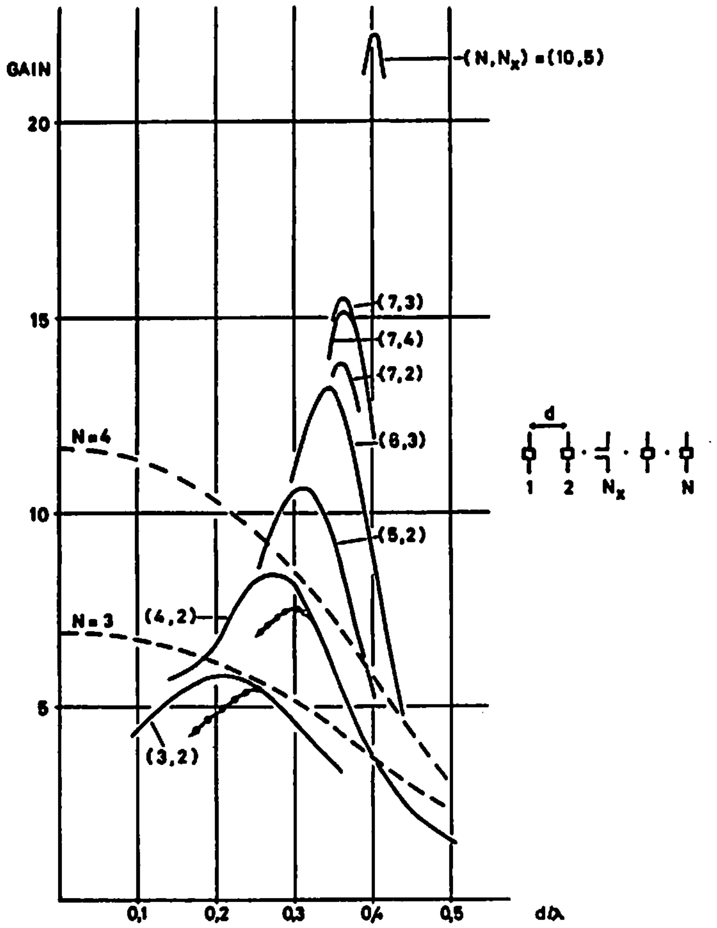

Fig. 1. ——— Maximum relative endfire gain of an N-element array of loaded half-wave dipoles. Element No. N_x is excited, remaining elements are parasitic.
------- Maximum relative endfire gain of an N-element array with all elements excited. (Ref. 3).
.... Constrained optimization, $Q \leq 5$.