B—1—4 MEASUREMENTS OF A LARGE CASSEGRAIN ANTENNA AT A REDUCED DISTANCE BY DEFOCUSING THE SUBREFLECTOR

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1. Introduction

The performance of an earth-station antenna is required to be measured before launching a satellite so that satellite characteristics can be immediately measured after launch. The measurement techniques by use of radio stars which are employed at lower frequencies (e.g. 4GHz Intelsat antennas), however, cannot be directly applied to quasi-millimeter or millimeter wave bands because flux densities of available radio sources are lower and the absolute value of them are not so clear.

On the other hands, conventional far-zone measurements on the ground may be impractical because Fraunhofer range $R = 2D^2/\lambda$ may become very large. For example, for D=10m antenna and f=35GHz, the required R exceeds 23km.

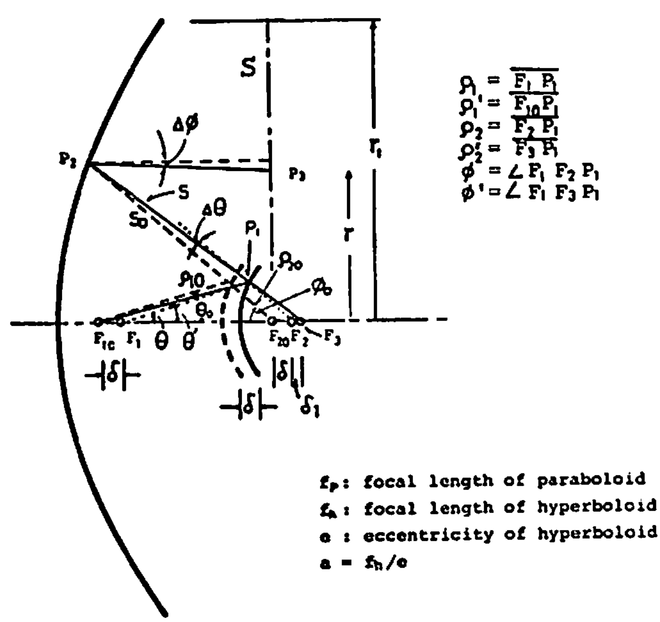
The need for the techniques of testing such antennas at reduced ranges is, therefore, both practical and urgent.

Cheng[1] studied first the defocusing method for a front-feed paraboloidal antenna in the Fresnel region measurement and later it was corrected by Chu[2].

For Cassegrain antennas, which are widely used for space communication, the analytical treatment of defocusing is rather complicated and it has not yet been done.

In this paper the author proposes the measurement of Cassegrain antennas by displacing the subreflector slightly in a direction away from the main reflector and the feed horn. Analysis based on geometrical optics on the optimum defocusing and the deviation of measured gain from the Frunhofer gain will be shown.

FIG. 1 GEOMETRY OF CASSEGRAIN ANTENNA WITH DEFOCUSED SUBREFLECTOR



2. Path-length differences over the aperture plane by defocusing subreflector

There will first be given the path-length differences from the primary source point F_{10} to all the points over the aperture plane S as a function of the amount of displacement of the subreflector δ as shown in Fig. 1.

Refering Fig. 1, the total path-length PL_t is given by

$$PL_t = \overline{F_{10}P_1} + \overline{P_1 P_2} + \overline{P_2 P_3} = \rho_1' - \rho_2' + \overline{F_3 P_2} + \overline{P_2 P_3} \ . \qquad (1)$$

In order to help the analysis, let's introduce two hypothetical rays, one is that arising from the hypothetical source point F_1 which is displaced δ from F_{10} and directing to the point P_1 and the other is the ray corresponding to the non-defocused antenna structure and passing point P_2 (the dotted line and the broken line in Fig. 1, respectively). Since δ may be very small compared with the dimensions of the antenna, i.e. the diameters and the focal lengths of the main- and sub-reflectors, the path-lengh in eq. (1) can be expressed with sufficient accuracy only by the first order term of δ and the ray for the non-defocused antenna structure.

For a small displacement of the subreflector , $\Delta\theta$ can be expressed as

$$\Delta\theta \simeq \sin\Delta\theta = \sin\theta\delta/\rho_1' \simeq \sin\theta_0 \cdot \delta/\rho_{10} \ , \qquad (2)$$

and ρ's can be written as

$$\rho_1' = \frac{\rho_1 \sin\theta}{\sin(\theta - \Delta\theta)} \simeq \rho_1 (1 + \cot\theta_0 \cdot \Delta\theta) \qquad (3)$$

$$\rho_2' = \frac{\rho_2 \sin\phi}{\sin(\phi - \Delta\theta)} \simeq \rho_2 (1 + \cot\phi_0 \cdot \Delta\theta) \ . \qquad (4)$$

From the geometrical property of a hyperboloidal subreflector and $\Delta\theta$ in eq. (2), the first order approximation of the difference between ρ_1' and ρ_2' becomes

$$\rho_1' - \rho_2' = 2a + \left(\cos\theta_0 - \frac{\rho_{20}}{\rho_{10}} \cot\phi_0 \ \sin\theta_0 \right)\delta = 2a + [\cos\theta_0 - \cos\phi_0 K^2(\cos\phi_0)]\delta \qquad (5)$$

In the above equation, a geometrical relation

$$\frac{\rho_{20}}{\rho_{10}} = \frac{\sin\theta_0}{\sin\phi_0} = \frac{e^2 - 1}{e^2 + 2e\cos\phi_0 + 1} = K(\cos\phi_0) \qquad (6)$$

where e is the eccentricity of the hyperboloid, is used.

The remaining term in eq. (1) is

$$\overline{F_3 P_2} + \overline{P_2 P_3} = s + \frac{s_0 \cos\phi_0}{\cos\Delta\phi} \simeq s + s_0 \cos\phi_0 \ . \qquad (7)$$

s can be also expressed as a perturbation from s_0 by the displacement, and eq. (7) becomes

$$\overline{F_3 P_2} + \overline{P_2 P_3} = 2f_p + \cos\phi_0 [1 + K^2(\cos\phi_0)]\delta \qquad (8)$$

where f_p is the focal length of the paraboloidal reflector.

Finally, from eqs. (5) and (8) the total path-length of the ray PL_t from the primary feed point to a point over the aperture plane S is obtained as

$$PL_t = 2(a + f_p) + \delta[\cos\theta_0 + \cos\phi_0] \ . \qquad (9)$$

θ_0 is determined by a given ϕ_0 and ϕ_0 can be written in terms of radius r over the aperture plane as

$$\cos\theta_0 = \frac{(e^2 + 1)\cos\phi_0 + 2e}{e^2 + 2e\cos\phi_0 + 1} , \qquad (10)$$

and

$$\cos\phi_o = \frac{(2f_p)^2 - r^2}{(2f_p)^2 + r^2}. \qquad (11)$$

Consequently, the path-length difference over the aperture by defocusing is given by

$$PL_d = 4\delta\left[\frac{r^2}{(2f_p)^2 + r^2} - \frac{er^2}{(e+1)^2 (2f_p)^2 + (e-1)^2 r^2}\right]. \qquad (12)$$

3. Optimum defocusing for measurements in the Fresnel region

When the field point lies in the Fresnel region, the path-length differences from the points over the aperture to the field point occur and are written by

$$PL_{Fresnel} = r^2/(2R_o), \qquad (13)$$

where R_o is the distance from the field point to the antenna. Defocusing the subreflector can approximately compensate these path-length differences and it provides that the measured radiation pattern may approach the true far-zone pattern. Since the functions in eqs. (12) and (13) are not exactly same, the perfect compensation cannot be expected, but if one makes the best choice of δ, the deviations of gain and radiation pattern from those measured in the Fraunhofer range will be sufficiently small.

In this section, the optimum amount of on-axis defocus δopt which maximizes the measured gain and the deviation in gain will be given for an ideal Cassegrain antenna with circularly symmetric aperture over which the distributions of both amplitude and phase are uniform.

The radiated field on the beam axis is obtained by integration of contribution from each point over the aperture taking account of the path-length difference between those in eqs. (12) and (13). After tranformation of variable, $r^2 = y$, it follows

$$E = \pi\int_{y_o}^{y_1} e^{jk\left\{y/2R_o - 4\delta[y/(y+y_f) - Ay/(y+Fy_f)]\right\}} dy = \pi\int_{y_o}^{y_1} e^{jk\Delta(y)} dy, \qquad (14)$$

where $k=2\pi/\lambda$, $y_o = r_o^2$, $y_1 = r_1^2$, $y_f = (2f_p)^2$, $F=[(e+1)/(e-1)]^2$, and $A=e/(e-1)^2$. Since an assumption that $k\Delta(y) < 1$ is usually reasonable, the integrand in eq. (14) can be expressed by the first several terms of the series expansion with respect to $jk\Delta(y)$. Keeping the first three terms in the series, eq. (14) becomes

$$E = \pi\int_{y_o}^{y_1} [1 + jk\Delta(y) - k^2\Delta^2(y)/2]\, dy. \qquad (15)$$

By letting $E_o = \pi(y_1 - y_o) \simeq \pi y_1$ and neglecting the term involving $k^4\Delta^4(y)$, the fractional error of the square of electric field amplitude is calculated as

$$\frac{\Delta|E|^2}{|E_o|^2} = 1 - \frac{|E|^2}{|E_o|^2} = \frac{1}{y_1}\int_o^{y_1} k^2\Delta^2(y)\, dy - \frac{1}{y_1^2}\left[\int_o^{y_1} k\Delta(y)\, dy\right]^2. \qquad (16)$$

Substituting $\Delta(y)$ in eq. (14) into eq. (16) and performing the integration, we have

$$\Delta|E|^2/|E_o|^2 = a\delta^2 - 2b\delta + c, \qquad (17)$$

where

$$a = 16(k)^2\left\{\frac{A^2FP}{1+FP} + \frac{P}{1+P} - \frac{2AFP}{1-F}\left[\ln\left(1+\frac{1}{FP}\right) - \ln\left(1+\frac{1}{P}\right)\right] - \left[P\ln\left(1+\frac{1}{P}\right) - AFP\ln\left(1+\frac{1}{FP}\right)\right]^2\right\},$$

$$b = \frac{k^2 y_1}{R_o}\left[2P(AF-1) + P(2P+1)\ln\left(1+\frac{1}{P}\right) - AFP(1+2FP)\ln\left(1+\frac{1}{FP}\right)\right],$$

$c = k^2 y_1^2/48R_o^2$, and $P = y_f/y_1 = (2f_p)^2/(r_1)^2$.

The optimum amount of defocus δopt and the minimum value of $\Delta|E|^2/|E_o|^2$ are derived as

$$\delta_{opt} = \frac{b}{a} \quad , \tag{18}$$

and

$$\Delta|E|^2_{min}/|E_o|^2 = c - \frac{b^2}{a} \quad . \tag{19}$$

As predicted physically, δopt is independent of frequency.
The gain reduction factor η_G from the Fraunhofer gain G_o iswritten by

$$\eta_G = G/G_o = 1-\Delta|E|^2_{min}/|E_o|^2. \tag{20}$$

4. Numerical examples

Fig. 2 shows calculated examples for a Cassegrain antenna of 10m diameter. The parameters used in this calculation are as follows:

$r_1 = 5$m, f =11.5 GHz and 34.5 GHz, e = 1.5,
P = 1 (aperture angle = 180°) .

The broken line in Fig. 2 shows δopt and solid curves show the gain reduction from the Fraunhofer gain for both frequencies as a function of distance of measurement.

References

(1) D.K. Cheng,"On the simulation of Fraunhofer radiation patterns in the Fresnel region", IRE Trans. AP vol. AP-5, Oct. 1957

(2) T.S. Chu,"A note on simulating Fraunhofer radiation patterns in the Fresnel region", IEEE Trans AP, vol. AP-19, Sept.1971

FIG. 2 OPTIMUM DEFOCUSING AND GAIN REDUCTION FROM THE FRAUNHOFER GAIN