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THE METHOD OF THE ORTHOGONAL POLYNOMIALS IN WAVE DIFFRACTION THEORY BY THIN CYLINDRICAL SCREEN

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INTRODUCTION

The E-polarized wave scattering by two-dimensional unclosed cylindrical screen of arbitrary smooth profile is investigated. Our aim is the development of the simple effective method for solution this problem. This diffraction problem has been treated by some authors [1-4].

Applying the idea of Chebyshev polynomial (Ch.P.) expansions [4,5], we suggest the Ch.P. of the complicated argument as the basic functions of expansions the unknown functions for integral equations (I.E.) with 2π -periodic difference logarithmic kernel. Such kernel appears in the case if the screen contour is topologically equivalent of a unit circle and some continuous functions of the parametrization of this contour exist. We consider that the application of the unit circle for the parametrization of the screen contour instead of the unit straight segment [4] is more preferable for scatterers having interior resonant region.

FORMULATION OF PROBLEM AND SOLUTION

Let a two-dimensional conducting unclosed cylindrical screen be described by a curve C in the xy-plane (Fig.1a). An E- polarized electromagnetic wave is scattered by the screen L. The wave is monochromatic with angular frequency ω and the time dependence $\exp(-i\omega t)$ is omitted in the following.

After that for unknown current density $j(\vec{q})$ we can write the following integral equation :

$$\frac{i}{4}\int_L j(\vec{q})H_0^{(1)}(k|\vec{p}-\vec{q}|)dl_q = -E_z^{inc}(\vec{p}), \quad \vec{p}\in L \tag{1}$$

Here $|\vec{p}-\vec{q}| = R(p,q)$ is the distance between the points on the contour L, $H_0^{(1)}(x)$ is the Hankel function of the first kind. Let L be parametrized by:

$$q(\alpha) = (x_q(\alpha), y_q(\alpha)); \quad q(\alpha)\in L, \quad \alpha < |\theta|$$

Then we can rewrite eqn.1 as

$$\frac{i}{4}\int_{-\theta}^{\theta}\rho(\alpha)H_0^{(1)}(kR(\tau,\alpha))d\alpha = -F(\tau); \quad \tau\in[-\theta,\theta] \tag{2}$$

where $\rho(\alpha) = j(\alpha)l(\alpha); \quad l(\alpha) = \sqrt{(x'(\alpha))^2 + (y'(\alpha))^2}$;

$$R(\tau,\alpha) = \sqrt{(x(\tau)-x(\alpha))^2 + (y(\tau)-y(\alpha))^2}.$$

Knowing the behavior of the Hankel function under $x \to 0$, we factorize the I.E. kernel on the singular and regular parts:

$$H_0^{(1)}(kR(\tau,\alpha)) = \frac{2i}{\pi}\ln|2\sin\frac{\alpha-\tau}{2}| + N(kR(\tau,\alpha)), \tag{3}$$

where $N(kR(\tau,\alpha))$ is a continuous function. Representing functions $\rho(\alpha)$, $F(\tau)$ and $H_0^{(1)}(kR(\tau,\alpha))$ as the sum of even and odd terms and using the representation (3) for the kernel we obtain the following coupled system of integral equations:

$$\begin{cases} \frac{i}{8}\int_{-\theta}^{\theta}\rho^+(\alpha)[\frac{4i}{\pi}\ln|2\sin\frac{\alpha-\tau}{2}| + N^+(kR(\tau,\alpha))]d\alpha \\ +\frac{i}{8}\int_{-\theta}^{\theta}\rho^-(\alpha)N^-(kR(\tau,\alpha))d\alpha = -F^+(\tau); \qquad \tau\in[-\theta,\theta], \\ \frac{i}{8}\int_{-\theta}^{\theta}\rho^+(\alpha)N^-(kR(\tau,\alpha))d\alpha + \\ +\frac{i}{8}\int_{-\theta}^{\theta}\rho^-(\alpha)[\frac{4i}{\pi}\ln|2\sin\frac{\alpha-\tau}{2}| + N^+(kR(\tau,\alpha))]d\alpha = -F^-(\tau); \quad \tau\in[-\theta,\theta], \end{cases} \tag{4}$$

where $\rho^{\pm}(\alpha) = \rho(\alpha) \pm \rho(-\alpha); F^{\pm}(\tau) = F(\tau) \pm F(-\tau)$;

$$N^{\pm}(kR(\tau,\alpha)) = N(kR(\tau,\alpha)) \pm N(kR(-\tau,-\alpha)).$$

(For $R(\tau,\alpha) = R(-\tau,-\alpha), N^-(kR(\tau,\alpha)) = 0$).

Assuming that unknown functions satisfies the condition $\rho^{\pm}(\alpha) \underset{\alpha\to\pm\theta}{\sim} (\theta^2-\alpha^2)^{-1/2}$ we propose the following expressions for its:

$$\rho^+(\alpha) = \frac{\cos(\alpha/2)}{\sqrt{2(\cos(\alpha)-\cos(\theta))}}\sum_{n=0}^{\infty} x_{2n}T_{2n}(\frac{\sin(\alpha/2)}{\sin(\theta/2)}; \qquad \alpha\in[-\theta,\theta], \tag{5'}$$

$$\rho^-(\alpha) = \frac{1}{\cos(\alpha/2)\sqrt{2(\cos(\alpha)-\cos(\theta))}}\sum_{n=1}^{\infty} x_{2n-1}T_{2n-1}(\frac{\tan(\alpha/2)}{\tan(\theta/2)}; \qquad \alpha\in[-\theta,\theta], \tag{5"}$$

The form of expansions (5',5") is caused by the fact that Chebyshev polynomials $T_{2n}(\frac{\sin(\alpha/2)}{\sin(\theta/2)})$ and $T_{2n-1}(\frac{\tan(\alpha/2)}{\tan(\theta/2)})$ are eigen functions of I.O. corresponding to the singular part of the I.E. kernel (4). Using the spectral expressions and orthogonality conditions proposed in [6] we obtain two coupled infinite systems of linear algebraic equations for seeking unknown coefficients $\{x_{2k}\}_{k=0}^{\infty}$ and $\{x_{2k-1}\}_{k=1}^{\infty}$:

$$\begin{cases} x_{2k} + \sum_{n=0}^{\infty} x_{2n}A_{2k,2n}^{+} + \sum_{n=1}^{\infty} x_{2n-1}A_{2k,2n-1}^{-} = f_{2k}^{+} \\ x_{2k-1} + \sum_{n=0}^{\infty} x_{2n}A_{2k-1,2n}^{-} + \sum_{n=1}^{\infty} x_{2n-1}A_{2k-1,2n-1}^{+} = f_{2k-1}^{-} \end{cases} \tag{6}$$

Here we have denoted

$$A^+_{2k,2n} = a_{2k} \int_0^\pi \int_0^\pi N^+(\psi,\varphi)\cos(2n\varphi)\cos(2k\psi)d\varphi d\psi;$$

replacements in

$$N^+(kR(\tau,\alpha)) : \alpha = 2\arcsin(\cos\varphi\sin(\theta/2)), \tau = 2\arcsin(\cos\psi\sin(\theta/2)),$$

$$A^-_{2k,2n-1} = a_{2k} \int_0^\pi \int_0^\pi N^-(\psi,\varphi)\cos((2n-1)\varphi)\cos(2k\psi)d\varphi d\psi;$$

replac. in $N^-(kR(\tau,\alpha)) : \alpha = 2\arctan(\cos\varphi\tan(\theta/2)),\ \tau = 2\arcsin(\cos\psi\sin(\theta/2))$,

$$A^+_{2k-1,2n-1} = a_{2k-1} \int_0^\pi \int_0^\pi N^+(\psi,\varphi)\cos((2n-1)\varphi)\cos((2k-1)\psi)d\varphi d\psi; \qquad (7)$$

replac. in $N^+(kR(\tau,\alpha)) : \alpha = 2\arctan(\cos\varphi\tan(\theta/2)), \tau = 2\arctan(\cos\psi\tan(\theta/2))$,

$$A^-_{2k,2n} = a_{2k-1} \int_0^\pi \int_0^\pi N^-(\psi,\varphi)\cos(2n\varphi)\cos((2k-1)\psi)d\varphi d\psi;$$

replac. in $N^-(kR(\tau,\alpha)) : \alpha = 2\arcsin(\cos\varphi\sin(\theta/2)), \tau = 2\arctan(\cos\psi\tan(\theta/2))$, where $a_{2k} = \dfrac{i\beta_{2k}}{4\pi\sigma_{2k}}; a_{2k-1} = \dfrac{i\beta_{2k-1}}{4\pi\sigma_{2k-1}}$;

$$\sigma_{2k} = \begin{cases} \ln(1/\sin(\theta/2)) \\ \dfrac{1}{2k} \end{cases}; \sigma_{2k-1} = \frac{1}{(2k-1)\cos(\theta/2)}; \beta_{2k} = \begin{cases} 1, k = 0; \\ 2, k \neq 0. \end{cases}$$

$$f^+_{2k} = b_{2k} \int_0^\pi F^+(\psi)\cos(2k\psi)d\psi; \quad \tau = 2\arcsin(\cos\psi\sin(\theta/2));$$

$$f^-_{2k-1} = b_{2k-1} \int_0^\pi F^-(\psi)\cos((2k-1)\psi)d\psi; \quad \tau = 2\arctan(\cos\psi\tan(\theta/2)). \qquad (8)$$

To compute the integral in matrix elements (7,8) with trigonometric functions we can use the quadrature formula of Filon . The solution of infinite systems (6) may be obtained with any given prefixed accuracy by the truncation method.

SOME NUMERICAL RESULTS AND DISCUSSION

For the sake of convenience we consider two numerical examples for circular cylindrical screen. Here the case of the plane wave scattering is presented only. In the first example for $\theta = 60$ (Fig.1,2) the plane wave is scattered by simple antenna reflector

. In the second one for $\theta = 120$ the screen has resonant volume (Fig.3,4). We also fulfilled numerical comparisons with previous studies and received good agreement with Shestopalov [1] and Tuchkin [3]. The comparisons with the Ziolkovski and Grant [2] gave something small discrepancy.

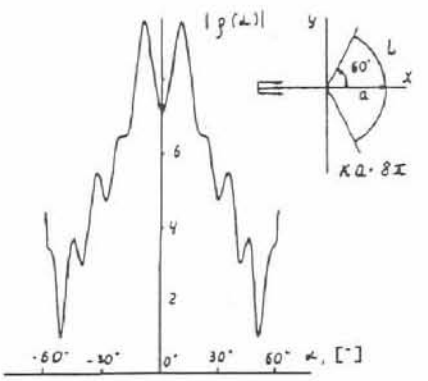

Fig.1

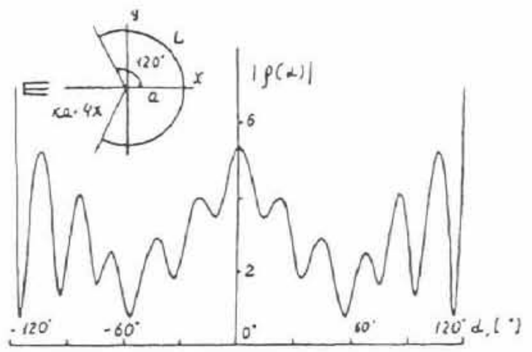

Fig 3

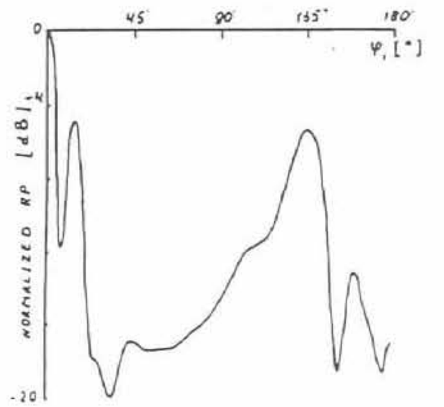

Fig.2

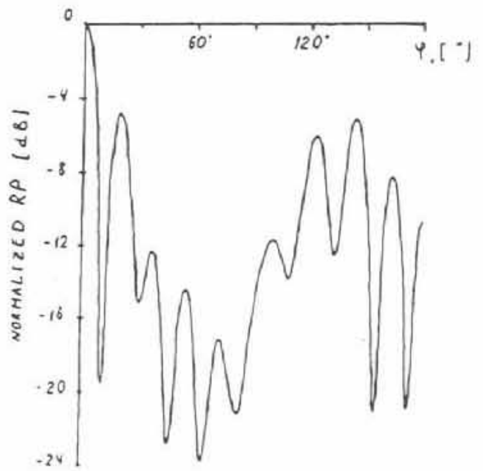

Fig.4

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