SCATTERING OF ELECTROMAGNETIC WAVES BY DIELECTRIC GRATINGS WITH ELLIPTICALLY LAYERED MEDIA

Tsuneki YAMASAKI*, Takashi HINATA**, and Toshio HOSONO**

* Department of Industrial Technology, Junior College, Nihon University,
1-24-1, Narashino-Dai, Funabashi City, 274-8501 Chiba, Japan
Tel. & Fax: +81-474-69-5533, E-mail: yamasaki@elec.jcn.nihon-u.ac.jp
** Department of Electrical Engineering, College of Science and Technology,
Nihon University

1. Introduction

Dielectric gratings have found applications in various areas such as integrated optics[1] and acousto-optics, optical filters, and holography. Recently, the refractive index can easily be controlled to make the periodic structures such as fiber grating and photonic crystal waveguide by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures have been proposed. In the multilayer method[2]~[5], as the inhomogeneous region is divided into an assembly of stratified thin layers with modulated index, the order of the matrix depends on the number of layers. However, in our approach, the order of characteristic matrix equation depends on the modal truncation number, but does not depend on the number of layers. Therefore the range of applicability to periodic structures is much wider[6] than that of other method, and our method can be applied easily to the guiding problems, such as planar slanted gratings[7],[8].

In this paper, the scattering of electromagnetic waves by dielectric gratings with elliptically layered media[9] are analyzed using the combination of improved Fourier series expansion method[10] and the multilayer method[11].

Numerical results are given for the transmitted scattered characteristics for the case of incident angle and frequency by dielectric gratings whose shape of grating is an elliptic cylinder, and whose interior distribution of permittivity is an elliptically layered medium for both TM and TE waves. The influences of the incident angle and frequency of the transmitted power are compared between circular cylinder and elliptic cylinder.

2. Method of Analysis

We consider the dielectric grating with elliptically layered media as shown in Fig.1(a). The grating is uniform in the y-direction and the permittivity $e(x,z)$ is an arbitrary periodic function of z with
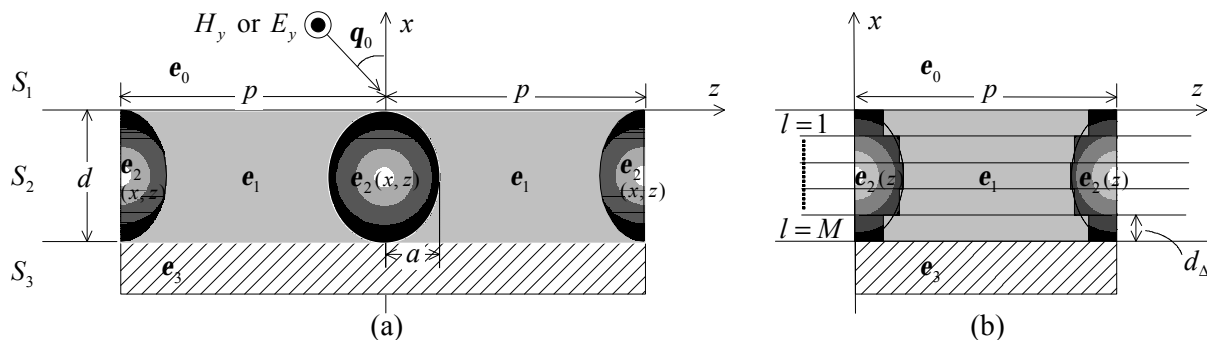

Fig.1 Structure of the dielectric grating with elliptically layered media. (a) Coordinate system, (b) Approximated inhomogeneous layers.

period p. Fig.1(a) shows the configuration whose shape of the grating is an elliptic cylinder with the cross section of $a \times d/2$, and whose interior distribution of permittivity $\boldsymbol{e}(x,z)$ is elliptically layered medium. The permeability is assumed to be $\boldsymbol{m}_0$. The time dependence is $\exp(-iwt)$ and suppressed throughout. In the formulation, the TM (the magnetic field has only the y-component) case is discussed. For the TE (the electric field has only the y-component) case, only numerical results are presented.

When the plane wave is assumed to be incident from $x > 0$ at the angle $\boldsymbol{q}_0$, the scattering fields in the regions $S_1 \, (x \geq 0)$ and $S_3 \, (x \leq -d)$ are expressed[6] as

$$\underline{S_1 (x \geq 0)} \; : \quad H_y^{(1)} = e^{ik_1 (z \sin \boldsymbol{q}_0 - x \cos \boldsymbol{q}_0)} + e^{ik_1 z \sin \boldsymbol{q}_0} \sum_{n=-N}^{N} r_n^{(1)} e^{i \left(k_n^{(1)} x + 2 \boldsymbol{p} n z / p \right)} , \tag{1}$$

$$\underline{S_3 (x \leq -d)} \; : \quad H_y^{(3)} = e^{ik_1 z \sin \boldsymbol{q}_0} \sum_{n=-N}^{N} t_n^{(3)} e^{-i \left\{ k_n^{(3)} (x+d) - 2 \boldsymbol{p} n z / p \right\}} , \tag{2}$$

$$E_z^{(j)} = \left(-iw\boldsymbol{e}_j \right)^{-1} \left(\partial H_y^{(j)} / \partial x \right) , \tag{3}$$

$$k_n^{(j)} \triangleq \sqrt{ k_0^2 \boldsymbol{e}_j / \boldsymbol{e}_0 - \left(k_1 \sin \boldsymbol{q}_0 + 2 \boldsymbol{p} n / p \right)^2 } \; ; \; k_1 \triangleq w \sqrt{\boldsymbol{e}_1 \boldsymbol{m}_0} \; , \; k_0 \triangleq 2 \boldsymbol{p} / \boldsymbol{l} \; , \; j = 1, 3 ,$$

where $\boldsymbol{l}$ is the wavelength in free space and $r_n^{(1)}$ and $t_n^{(3)}$ are unknown coefficients to be determined from boundary conditions.

In inhomogeneous grating region $S_2 \, (-d < x < 0)$, the elliptic cylinder is divided into thin layers $d_\Delta (= d/M)$, as shown in Fig.1(b). The dielectric distribution of thin layer is approximated to step index profile $\boldsymbol{e}^{(l)}(z) [\triangleq \boldsymbol{e}((l+0.5)d_\Delta, z) ; l = 1 \sim M]$. In each layer ($l = 1 \sim M$), the magnetic field can be written as $H_y^{(l,2)} = H^{(l)}(z) \cdot e^{ih^{(l)} x}$, where $h^{(l)}$ is the propagation constant in the x-direction, and $H^{(l)}(z)$ must satisfy the following wave equation[6]

$$\frac{d^2 H^{(l)}(z)}{dz^2} - \frac{1}{\boldsymbol{e}^{(l)}(z)} \frac{d\boldsymbol{e}^{(l)}(z)}{dz} \frac{dH^{(l)}(z)}{dz} + \left[k_0^2 \boldsymbol{e}^{(l)} / \boldsymbol{e}_0 - \left\{ h^{(l)} \right\}^2 \right] H^{(l)}(z) = 0 . \tag{4}$$

Taking into account the Floquet's theorem, $H^{(l)}(z)$ can be approximated by the finite Fourier series as

$$H^{(l)}(z) = e^{ik_1 z \sin \boldsymbol{q}_0} \sum_{n=-N}^{N} u_n^{(l)} e^{i2\boldsymbol{p} n z / p} . \tag{5}$$

If $\boldsymbol{e}^{(l)}(z)$ is decomposed as $\boldsymbol{e}^{(l)}(z) \triangleq f^{(l)}(z) / g^{(l)}(z)$, for example $\boldsymbol{e}^{(l)}(z) = \boldsymbol{e}_0 \mathrm{sech}(1 + z/p) = 2\boldsymbol{e}_0 / (e^{1+z/p} + e^{-(1+z/p)})$, the analysis is made easy to put $f^{(l)}(z) = 2\boldsymbol{e}_0$ and $g^{(l)}(z) = (e^{1+z/p} + e^{-(1+z/p)})$ rather than that of reference[12] so that, substituting Eq.(5) into Eq.(4) and multiplying both sides by $g^{(l)}(z) f^{(l)}(z) \cdot e^{-i2\boldsymbol{p} n z / p}$, and rearranging after integrating with respect to z in the interval $0 \leq z < p$, we get the following eigenvalue equation in regard to $h^{(l)}$ [6]

$$\Lambda_1 \mathbf{U}^{(l)} = \left\{ h^{(l)} \right\}^2 \Lambda_2 \mathbf{U}^{(l)} \; ; \; \Lambda_1 \triangleq \left[\boldsymbol{h}_{m,n}^{(l)} \right], \; \Lambda_2 \triangleq \left[\boldsymbol{z}_{m,n}^{(l)} \right], \; l = 1 \sim M , \tag{6}$$

where

$$\mathbf{U}^{(l)} \triangleq \left[u_{-N}^{(l)}, \cdots, u_0^{(l)}, \cdots u_N^{(l)} \right]^T , \; T \; : \; \text{transpose} ,$$

$$\boldsymbol{z}_{n,m}^{(l)} \triangleq k_0^2 \boldsymbol{x}_{n,m}^{(l)} - \boldsymbol{g}_n^{(l)} \left\{ \boldsymbol{g}_n^{(l)} \boldsymbol{h}_{n,m}^{(l)} + 2\boldsymbol{p}(n-m)\boldsymbol{h}_{n,m}^{(l)} / p - \boldsymbol{j}_{n,m}^{(l)} \right\} ,$$

$$\boldsymbol{h}_{n,m}^{(l)} \triangleq \frac{1}{p} \int_0^p \left\{ f^{(l)}(z) g^{(l)}(z) \right\} e^{i2\boldsymbol{p}(n-m)z/p} dz , \; \boldsymbol{j}_{n,m}^{(l)} \triangleq \frac{2i}{p} \int_0^p \left\{ f^{(l)}(z) \frac{d\left\{ g^{(l)}(z) \right\}}{dz} \right\} e^{i2\boldsymbol{p}(n-m)z/p} dz ,$$

$$\boldsymbol{x}_{n,m}^{(l)} \triangleq \frac{1}{p} \int_0^p \left\{ f^{(l)}(z) \right\}^2 e^{i2\boldsymbol{p}(n-m)z/p} dz , \; \boldsymbol{g}_n^{(l)} \triangleq \left(k_1 \sin \boldsymbol{q}_0 + 2\boldsymbol{p} n / p \right), \; m,n = \left(-N, \cdots, 0, \cdots N \right) .$$

However, $f^{(l)}(z)$ or $g^{(l)}(z)$ contain discontinuity such as the step function, $(n-m)\boldsymbol{h}_{n,m}^{(l)}$ does not converge, because $\boldsymbol{h}_{n,m}^{(l)}$ is $O(1/|n-m|; n \neq m)$ as $|n-m| \to \infty$ [$|\boldsymbol{h}_{n,m}^{(l)}|$ is less than $K/|n-m|$, where K is independent of $|n-m|$], so that the solution of the Eq.(6) also does not converge to the correct value. Therefore, the method cannot directly be applied to the grating with step distribution[10].

To solve this difficulty, the function containing the discontinuity is approximated by a Fourier se-

ries of N_f terms[10].

We have experienced that $N = 1.5 N_f$ is sufficient when N_f is related to the modal truncation number N[10].

The electromagnetic fields in each layer are expressed as

$S_2 (-d < x < 0)$:

$$H_y^{(l,2)} = \sum_{n=1}^{2N+1} \left[A_n^{(l)} e^{ih_n^{(l)}\{x+(l-1)d_\Delta\}} + B_n^{(l)} e^{ih_n^{(l)}(x+ld_\Delta)} \right] \cdot f_n^{(l)}(z) , \quad d_\Delta = d/M , \quad l = 1 \sim M , \tag{7}$$

$$E_z^{(l,2)} = \left\{ -iw e^{(l)}(z) \right\}^{-1} \partial H_y^{(1,2)} / \partial x \; ; \; f_n^{(l)}(z) \triangleq e^{ik_1 z \sin q_0} \sum_{n=-N}^{N} u_{n,n}^{(l)} e^{i2pnz/p} , \tag{8}$$

where $A_n^{(l)}$, $B_n^{(l)}$ are unknown coefficients to be determined by boundary conditions.

From the boundary conditions at $x = 0$, $x = -l \cdot d_\Delta$ $(l = 1 \sim M - 1)$, and $x = -d$, we get the following homogeneous matrix equation in regard to $A^{(M)}$ by matrix algebra[11].

$$\mathbf{W} \cdot \mathbf{A}^{(M)} = \mathbf{F} , \quad \mathbf{W} \triangleq \left[\mathbf{Q_1 S_1 + Q_2 S_3} - (\mathbf{Q_1 S_2 + Q_2 S_4}) \mathbf{Q_4^{-1} Q_3} \right] , \tag{9}$$

where the elements of matrix $\mathbf{W}$ and $\mathbf{F}$ are obtained by reference[6],[9]

The mode power transmission coefficients $| T_n^{(TM)} |^2$ is given by

$$| T_n^{(TM)} |^2 \triangleq e_1 \, \mathrm{Re} \left\{ k_n^{(3)} \right\} | t_n^{(3)} |^2 / \left(e_3 k_0^{(1)} \right) , \tag{10}$$

where superscript (TM) indicates TM wave case.

3. Numerical Analysis

We consider the following structure of grating:

(1) Grating shape ; elliptic cylinder

$$[(x + d/2)/(d/2)]^2 + (z/a)^2 = 1 \tag{11}$$

(2) Distribution of permittivity $e(x,z)$; elliptically layered media

$$e(x,z) \triangleq \begin{cases} e_2 [1 - b\{(2(x+d/2)/d)^2 + (z/a)^2\}] & ; b = 1 - e_1/e_2 \quad : \text{inside of elliptic cylinder} \\ e_1 & : \text{outside of elliptic cylinder} \end{cases} \tag{12}$$

For the above case, we put $g^{(l)}(z) = 1$ and $f^{(l)}(z) = e(x,z)$, and the values of parameters chosen are $e_1 = e_3 = e_0$, $e_2/e_0 = 3$ and $d/p = 2/3$. For the TM wave, the results to check the validity of our method are computed with $N = 9$ ($N_f = 6$) and $M = 30$. For the TE wave, the results are computed with $N = 10$ and $M = 20$. In this case, the relative error are less than about 0.1% and the energy error is less than about 10^{-3} for both TM and TE waves[12].

Figures 2(a) and 2(b) show $| T_0^{(TM)} |^2$ and $| T_0^{(TE)} |^2$ for various values of incident angle q_0 at $2a/d = 0.8, 1.0$ and 1.2 for $p/l = 1.0$. The case of circular cylinder is $2a/d = 1.0$. From in Figs.2(a), the effect of grating shape is seen clearly at the minimum of coupling resonance $| T_n^{(TM)} |^2 \cong 0$, and it moves toward larger q_0 as $2a/d$ increases. On the other hand, for TE case, $| T_0^{(TE)} |^2$ has a symmetric shape around $q_0 = 30°$. Therefore, we note that the q_0 dependence at coupling resonance is more significant for the TE case than for the TM case. It is interest the peak of $| T_0^{(TE)} |^2$ at $2a/d = 1.0$ moves toward at $q_0 \cong 90°$.

Figures 3(a) and 3(b) show $| T_0^{(TM)} |^2$ and $| T_0^{(TE)} |^2$ for various values of normalized frequency (p/l) at $q_0 = 30°$ with the same parameters as in Fig.2. Comparing the TM case with the TE case, we note that the characteristic tendencies for the effect of grating shape are approximately same at $p/l < 0.7$, but for about $p/l > 0.7$, the effect of the grating shape is more significant for TE case.

4. Conclusions

In this paper, we have analyzed the scattering of electromagnetic waves by dielectric gratings with elliptically layered media using improved Fourier series expansion method and multilayer method. Numerical results are given for the transmitted scattered characteristics for the case of incident angle and frequency for both TM and TE cases. The influences of the incident angle, and the frequency on the transmitted power for grating shape are discussed. Finally, the authors would like to thank Mr. Ryuji Terada at graduate student of Nihon University for help with drawing graphics in this work.
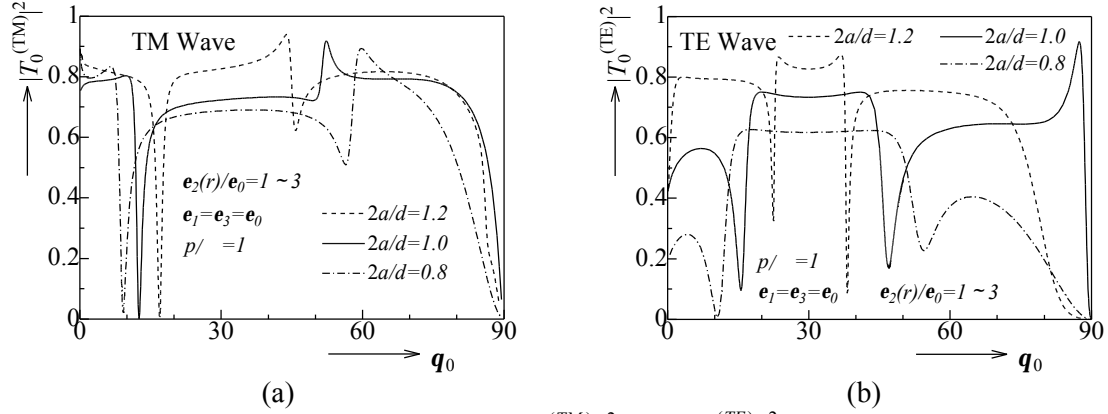
Fig.2 Mode power transmission coefficients $|T_0^{(TM)}|^2$ and $|T_0^{(TE)}|^2$ versus incident angle q_0 for the case of [0]th-mode with elliptical layered media. (a) TM wave, (b) TE wave.
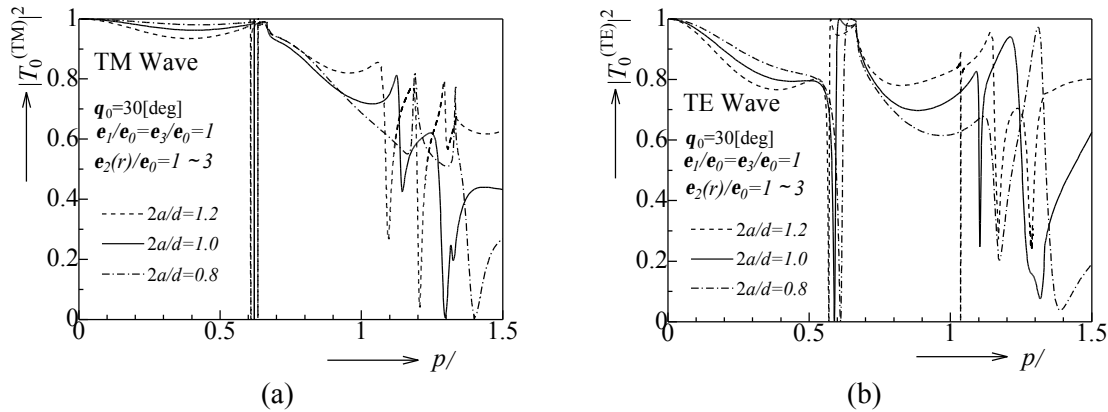

Fig.3 Mode power transmission coefficients $|T_0^{(TM)}|^2$ and $|T_0^{(TE)}|^2$ versus a normalized frequency $p/1$ for the case of [0]th-mode with elliptical layered media. (a) TM wave, (b) TE wave.

References

[1] *Tamir, T., ed.,* Integrated Optics. Springer-Verlag, - p.110-118, 1979.

[2] *Lifeng, L.,* Multilayer modal method for diffraction gratings of arbitrary profile, depth, and permittivity, J. Opt. Soc. Am. A, Vol.10, No.12. p.2581-2591, 1993.

[3] *Cotter, N. P. K., Preist, T. W. and Sambles, R.,* Scattering-Matrix approach to multilayer diffraction, J. Opt. Soc. Am. A, vol.12(5), pp.1097-1103, 1995.

[4] *Montiel, F. and Neviere, M.,* Differential Theory of Gratings: extension to deep gratings of arbitrary profile and permittivi ty through the R-matrix propagation algorithm, J. Opt. Soc. Am. A, vol.11(12), pp.3241-3250, 1994.

[5] *Neviere, M.,* Bragg-Fresnel multilayer gratings: Electromagnetic theory, J. Opt. Soc. Am. A, vol.11, 6, pp. 1835-1845, 1994.

[6] *Yamasaki, T., Tanaka, T., Hinata, T. and Hosono, T.,* Analysis of Electromagnetic Fields Inhomogeneous Dielectric Grat ings with Periodic Surface Relief, Radio Science, Vol.31, No.6, pp.1931-1939, 1996.

[7] *Yamasaki, T. and Tanaka, T.,* Scattering of electromagnetic waves by a dielectric grating with planar slanted-fringe , IEICE Trans. in Japan, Vol. E76-C, No.10. p.1435-1442, 1993.

[8] *Tanaka, H., Yamasaki, T. and Hosono, T.,* Propagation characteristics of dielectric waveguides with slanted grating struc ture, IEICE Trans. in Japan, Vol. E77-C, No.11. p.1820-1827, 1994.

[9] *Yamasaki, T., Hinata T., and Hosono T.,* Scattering of Electromagnetic Waves by Columnar Dielectric Grating with Inho mogeneous Media, Tech. Rep. Electromagnetic Theory, I.E.E., Japan, Vol. EMT-99-114. p.45-50, 1999 (in Japanese).

[10] *Yamasaki, T., Hinata, T. and Hosono, T.,* Electromagnetic field analysis of planar gratings with periodic distribution of dielectric constant, IECE Trans. in Japan., Vol.J69-B, No.1. p.125-132, 1985 (in Japanese). [translated Scripta Technica, INC., Vol.69, April. p.75-84, 1986.]

[11] *Yamasaki, T., Hosono, T. and Kong, J. A.,* Propagation characteristics of dielectric waveguides with periodic surface-relief, IEICE Trans., in Japan., Vol.E74, No.9. p.2839-2847, 1991.

[12] *Peng, S. T., Bertoni, H. L. and Tamir, T.,* Theory of periodic dielectric waveguides, IEEE Trans. Microwave Theory Tech., Vol. MTT-23, No.1. p.123-133, 1975.