

NEAR FIELD SCATTERED BY AN AIR SPHERE  
EMBEDDED IN A DIELECTRIC MEDIUM

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1. Introduction

Locating high-contrast geological anomalies such as underground cylindrical cavity, the cross-borehole continuous wave electromagnetic probing[1] is superior to the geophysical tomography methods[2, 3] because the single frequency signal is free from the host medium dispersion which gives greater system dynamic range and effects of refraction and the diffraction by the cavity are dominant far beyond the correction level[4]. If the wavelength of the transmitted signal is about the radius of the cylindrical cavity, double dips in the amplitude pattern of the received signal appear at two locations corresponding to the top and the bottom boundaries of the empty cavity. These double dips of the total field for certain signal frequencies become double nulls in the near field region[5].

It is interesting to ask whether similar phenomenon occurs for an air sphere in a denser host medium. It is shown that the double nulls and dips are also formed and the nulls are shown to be formed when the scattered field equals the incident field with its phase reversed. Their corresponding parameters such as frequencies and field points are obtained.

2. Double nulls and dips

When a monochromatic plane wave  $E^i$ , whose electric vector is linearly polarized in the x-direction, is incident upon an air sphere of radius  $a$  embedded in a homogeneous dielectric medium of its relative dielectric constant  $\epsilon_r$ , as shown in Fig. 1, the total field  $E^t$  in the  $z = z'$  plane outside the sphere as a function of  $(\rho, \phi)$  may be obtained analytically by the sum of  $E^i$  and the scattered field  $E^s$  from the exact solution[6]. For the numerical calculation, the radius of the air sphere is chosen to be 1 m,  $\epsilon_r$  of the host medium to be 2, and the frequency of the transmitting signal to be from 50 MHz to 950 MHz.

A typical double dip pattern of the co-polarized total field  $E_x^t$  is shown in the top figure of Fig. 2. Amplitude and phase variation of the corresponding scattered field is shown in the bottom figure for 510 MHz,  $z' = 3$  m, and  $\phi = 0^\circ$ . Two dips occur at  $\rho = \rho_{a1}$  and  $\rho_{a2}$ , where the amplitude of the scattered field is equal to that of the incident field. Amplitudes of the scattered field at these points,  $\rho_{a1}$  and  $\rho_{a2}$ , are slightly larger than that of the incident field and the corresponding amplitude dips of the total field become about 22.3 dB below that of the incident field.

Two dips of the total field for 640 MHz occur at near  $\rho_{p1}$  and  $\rho_{p2}$ , where the phase of the scattered field differs by  $180^\circ$  from that of the incident field, as shown in Fig. 3. As the signal frequency increases from 510 MHz to 640 MHz, the equal amplitude point  $\rho_a$  becomes smaller than the  $180^\circ$  phase points  $\rho_p$  as shown in Fig. 3 while  $\rho_a$  is larger than  $\rho_p$  for 510 MHz as in Fig. 2. This crossing over two points,  $\rho_p$  and  $\rho_a$ , occur at frequency of 549.6043 MHz (the corresponding wavelength in the host medium is 0.386 m) for  $z' = 3$  m where the double nulls of the total field are formed at  $\rho = 0.670$  m, for  $\phi = 0^\circ$  and the distance between two nulls is about two-thirds the size of sphere, as shown in Fig. 4.

In the far field region, however, the amplitude of the scattered fields are smaller than that of the incident field and nulls in the amplitude pattern of the total field can not occur. In the case of the air sphere of 1 m radius and  $\epsilon_r = 2$  this critical distance  $z'$  are calculated to be 3.967 m, 4.119 m, and 4.267 m for  $\phi = 0^\circ, 45^\circ$ , and  $90^\circ$ , respectively.

One may find out signal frequencies where the double nulls occur by plotting 180 phase points( $\rho_p$ ) and equal amplitude points( $\rho_a$ ) versus frequency, as shown in Fig. 5. For  $z' = 3$  m, three frequencies, 422.180 MHz, 549.6043 MHz and 715.414 MHz, give  $\rho_p = \rho_a$  and the double nulls occur at these frequencies. One may show that other double nulls are also formed in the higher frequencies above 950 MHz. As  $\phi$  change from  $0^\circ$  to  $360^\circ$  and  $z'$  change from 2.1 m to 3 m, locations and frequencies where these double nulls occur are shown in Fig. 6.

#### References

- [1]. R. J. Lytle, E. F. Laine, D. L. Lager, and D. T. Davis, "Cross-borehole electromagnetic probing to locate high-contrast anomalies," *Geophysics*, vol. 44, pp. 1667-1676, 1979.
- [2]. K. A. Dines and R. J. Lytle, "Computerized geophysical tomography," *Proc. IEEE*, vol. 67, pp. 1065-1073, 1979.
- [3]. A. J. Devancy, "Geophysical diffraction tomography," *IEEE Trans. Geosci. Remote Sensing*, vol. GE-22, pp. 3-13, 1984.
- [4]. M. Slancy, A. C. Kak, and L. E. Larsen, "Limitation of imaging with first-order diffraction tomography," *IEEE Trans. Microwave Theory Tech.*, vol MTT-32, pp. 860-874, 1984.
- [5]. T. K. Lee, S. O. Park, J. W. Ra, and S. Y. Kim, "Near-field diffraction by an underground void of circular cylinder," *Microwave Optical Tech. Lett.*, vol. 2, pp. 197-183, 1989.
- [6]. D. S. Jones, *The Theory of Electromagnetism*. Pergamon Press. New York, 1964.

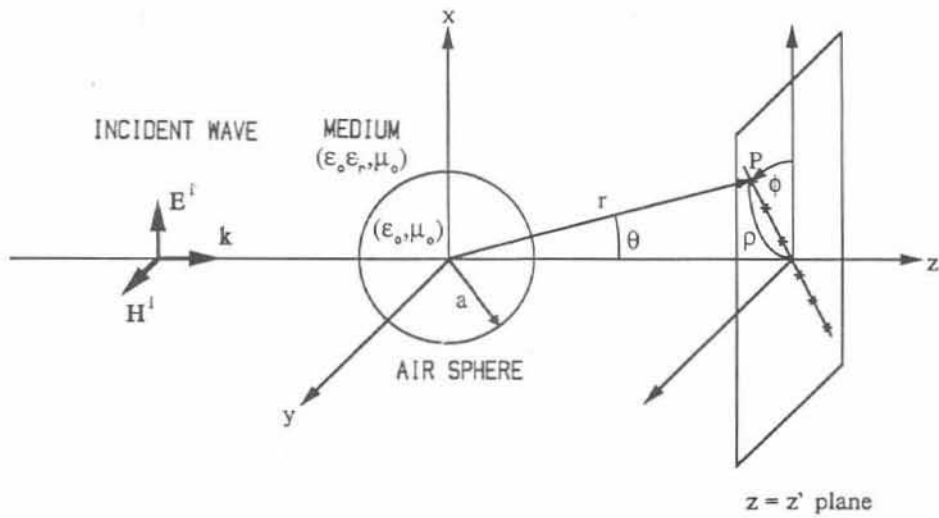


Fig. 1. Geometry for three dimensional scattering by an air sphere embedded in a dielectric medium.

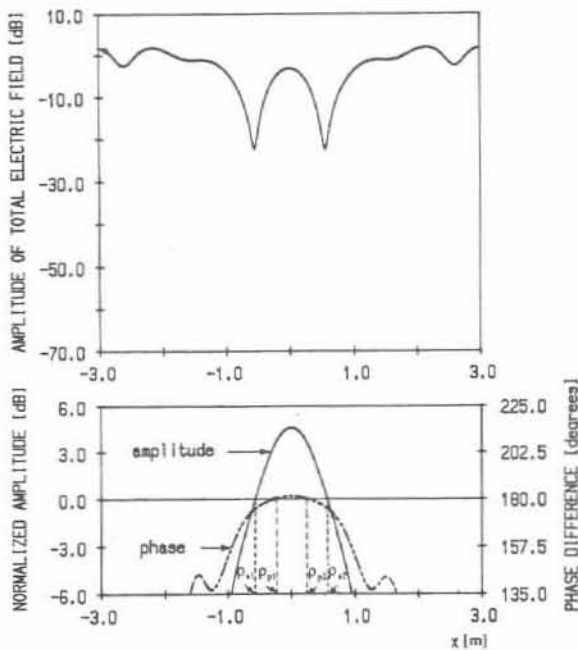


Fig. 2. Amplitude pattern of the co-polarized total field (top) and the amplitude and phase patterns of the scattered field normalized by the incident wave (bottom) for  $z' = 3$  m,  $\phi = 0^\circ$ , and  $f = 510$  MHz.

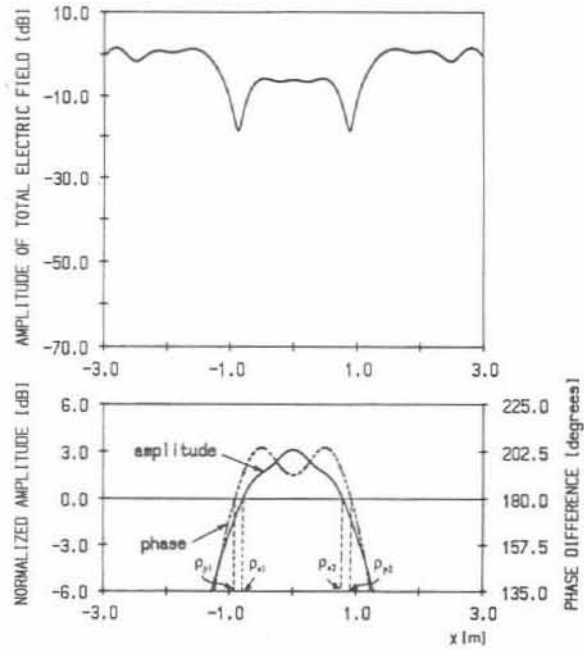


Fig. 3. Amplitude pattern of the total field and normalized amplitude and phase patterns of the scattered field for  $z' = 3$  m,  $\phi = 0^\circ$ , and  $f = 640$  MHz.

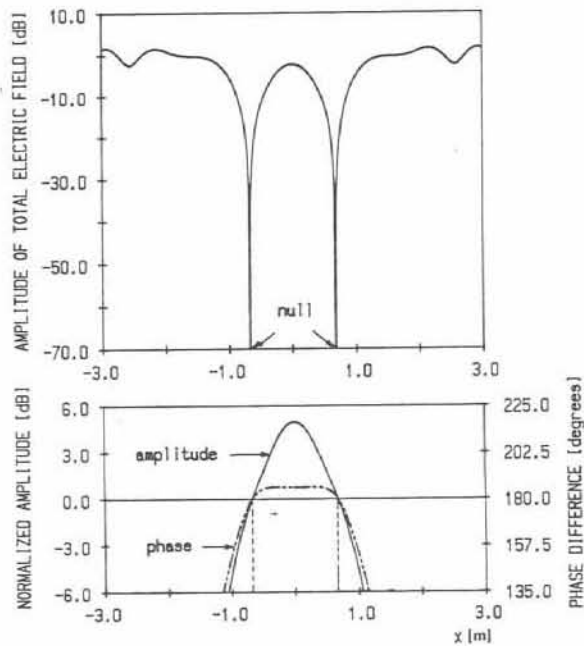


Fig. 4. Double nulls in the amplitude pattern of the total field and normalized amplitude and phase patterns of the scattered field for  $z' = 3$  m,  $\phi = 0^\circ$ , and  $f = 549.6043$  MHz.

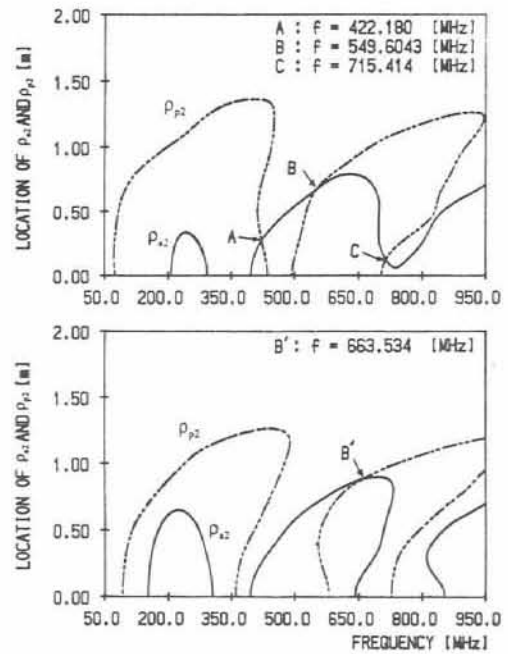


Fig. 5. Loci of equal-amplitude points  $p_{p2}$  and out-of-phase points  $p_{p2}$  as a function of the signal frequency from 50 MHz to 950 MHz for  $z' = 3$  m (top) and 2 m (bottom).

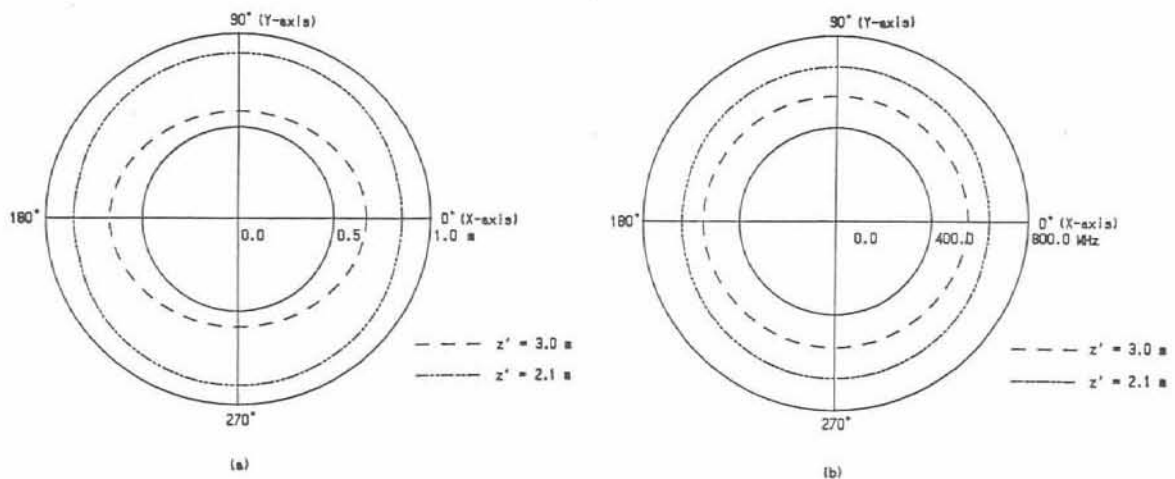


Fig. 6. Loci of the positions and frequencies where double nulls occur as a function  $\phi$  from  $0^\circ$  to  $360^\circ$  for  $z' = 2.1$  m and 3 m; (a) null positions vs.  $\phi$ , (b) null frequencies vs.  $\phi$ .