B—1—3

GENERAL THEORY OF NEAR-FIELD SCANNING

Paul F. Wacker
National Bureau of Standards
Boulder, Colorado 80303, U.S.A.

Near-field scanning on planar, circular cylindrical, and spherical surfaces has many advantages including the possibility of accuracies seldom equalled on far-field ranges or in anechoic chambers. The same is true of the author's extrapolation method [1] for gain and effective area, which is the most accurate method of calibrating standard gain horns. Since the data processing is based upon the Fast Fourier "Transform," supplemented by matrix multiplication in the spherical case, the rigorous three-dimensional techniques described here are extremely fast. Tens or hundreds of thousands of complex coefficients of exact global solutions (modes) of the appropriate differential equation(s) (Maxwell's in the electromagnetic case) and the associated total far field are computed in minutes or even seconds. (The various proposed techniques involving small angle, Kirchhoff diffraction, scalar, asymptotic, and/or defocussing approximations are usually not as fast.) Further, these methods provide a very comprehensive, rigorous, and detailed theory of near-field analysis and measurement, with minimal assumptions, idealizations, and approximations, particularly in the formulation described here.

The planar, cylindrical, and spherical analyses were originally developed in different laboratories and the treatments particularized for each surface. However, due to the complexity of the spherical analysis, it is highly desirable to have a panoramic view of near field scanning and a common notation for the various scanning surfaces and physical systems. Then both general and detailed comparisons can be made, facilitating (a) the transfer of understanding from one surface or physical system to another, (b) the translation of computer programs from one physical system to another (say EM to linearized acoustic in a gas, liquid, or solid), (c) making sure that all available computational efficiencies are utilized, (d) emphasis upon fundamentals rather than a morass of detail, (e) generalization and extension to other scanning surfaces and physical systems, and (f) defining the limits upon the techniques. (Ad hoc studies of particular systems have lead to various erroneous statements in the literature concerning limitations of near-field techniques.) To achieve these ends, a generalized theory of near-field scanning is presented, with spherical, planar, and cylindrical scanning and electromagnetic, acoustic, and heat flow systems treated as mere special cases; in particular, this theory includes the author's scheme for spherical data reduction [2], the only existing practical technique with probe correction, adopted by both the NBS and the Technical University of Denmark. Further, it provides background for the author's extrapolation technique.

The antenna or other transducer is treated as a black box as far as the formal analysis is concerned, with no consideration of its shape or current distribution; however, known or assumed symmetry of the transducer may be used in reducing the measurement and computational effort, and qualitative information concerning the pattern or design is useful in choosing measurement conditions and the modes used to represent the data.

The theory is based upon generalized scattering-matrix theory and symmetry analysis of the medium and differential equation(s), e.g., relativistic and gauge invariance. It thus assumes mathematical linearity and invariance of the medium with respect to certain translations, rotations, and/or reflections. Actually, the theory is easily extended to non-linear transmitting transducers operating at fixed input levels. The scattering matrix theory includes all the transmitting, receiving, and scattering properties of the transducers, and multiple reflections are included to all orders in the extrapolation method. However, to simplify the data reduction for scanning on a surface, multiple reflections between the transducers are minimized experimentally and neglected in the scanning computations. Nevertheless, multiple reflections between each transducer and its source or load are treated to all orders. There are two parallel treatments of scanning; one assumes an ideal probe and the other corrects for the effect of the probe pattern upon the measured signal. Attention is here confined to the (more complicated) probe-correction case.

For a given position and orientation of the probe with respect to the test transducer, the complex received signal is given by

$$W(\underline{R}_o) = \sum_{\underline{N}} \sum_{\underline{N}'} P_{\underline{N}'} \, G_{\underline{N}'\underline{N}}(\underline{R}_o) \, Q_{\underline{N}}, \tag{1}$$

where W is an experimental quantity defined by

$$W(\underline{R}_o) \equiv {}_r b_0(\underline{R}_o) \, (1 - {}_r\Gamma \, {}_r\Gamma_L)/{}_t a_0, \tag{2}$$

${}_t a_0$ and ${}_r b_0$ are the complex traveling-wave voltages of the input to the transmitting transducer and the output from the receiving transducer, respectively, and ${}_r\Gamma$ and ${}_r\Gamma_L$ are the complex reflection coefficients of the receiving transducer, looking from the load to the transducer and from the transducer to the load, respectively. The operator $\underline{R}_o$ expresses the separation of the two transducers and their relative orientations (for each measurement) in terms of translations, rotations, and/or reflections which take the test-transducer coordinate system into coincidence with the coordinate system attached to the probe. The P's and Q's are elements of the scattering matrices of probe and test-transducer, respectively, corresponding to the modes indicated by the subscripts; specifically they are elements of the transmitting and receiving submatrices (or vice versa, if the probe receives rather than transmits). Since the modes constitute linearly-independent mathematically-complete sets of exact global solutions of the appropriate differential equation(s), they and their coefficients completely describe the needed transmitting and receiving properties of the two transducers. The modes are completely specified by the subscripts, including such factors as direction of propagation, kind of Bessel function, TE,TM character, polarization (ellipticity), and angular frequency ω. A prime is used to indicate the coordinate system of the probe and the absence of such a mark indicates the coordinate system of the test transducer. The summation sign is used to indicate summation and/or integration, e.g., integration with respect to k_x and k_y.

For the familiar planar, circular cylindrical, and spherical modes, we express $\underline{N}$ (for the test-transducer coordinate system) by $(\omega,\delta,s),k_x,k_y,k_z;-;-$, $(\omega,\delta,s),k_z,k_R;m,h;-$, and $(\omega),\delta,k_r;n,m,s,h;-$, respectively, where the k's are the indicated components of the propagation vector, m occurs in $\exp(im\phi)$, n occurs in $P_n^m(\cos\theta)\exp(im\phi)$, h is +1 and -1 for the first and second kinds of Hankel functions, respectively, δ indicates spin (0 for a scalar field and unity for an electromagnetic field (photon)), and s is

1 for a TM mode and 2 for a TE mode, transverse being defined with respect to z = constant planes for Cartesian planar and cylindrical scanning and r = constant spheres for spherical scanning. The TE,TM index s is of course deleted for the scalar case. Note that the P's and Q's in (1) are for the coordinate systems of the probe and test transducer, respectively. The G's express the transformation of the modal coefficients due to change of the coordinate system ($\underline{R}_o$'s) during scanning and are based upon modal addition theorems, like Graf's addition theorem for Bessel functions. Equations (1) and (2) are independent of the physical system, whether the probe transmits or receives, and the nature of the medium, provided only mathematical linearity and negligible multiple reflections between the transducers. Neither of the transducers need be a probe; these equations express the complex received signal for cosite interference or the extrapolation method as a function of transducer separation and relative orientations. Reciprocity of the transducers is not relevant; they may be dissimilar arrays with ferrite phase shifters and isolators. Similarly, the medium may be anisotropic, non-reciprocal, inhomogeneous, etc. and of course lossy, provided only the G's and modes are known.

To obtain practical explicit expressions for the G's and the modes, some assumptions must be made concerning the medium and form(s) of the differential equation(s). If they are invariant with respect to a set of $\underline{R}_o$'s which form a group (i.e., a set which includes the identity element, all the inverses, and all the ordered products of the elements), we write $\underline{N} = \underline{n};\underline{m};\underline{p}$, where the partitioning is into major, minor, and subsidiary indices, respectively. The $\underline{n}$'s and $\underline{m}$'s are symmetry indices, like even and odd, and the $\underline{p}$'s distinguish between linearly independent modes of the same symmetry. (In the terminology of group representations, the major indices designate an irreducible representation and the minor indices designate a row of that representation.) As indicated by the dashes in the explicit forms of the last paragraph, there are no subsidiary indices for the familiar modes in rectangular, circular cylindrical, or spherical coordinates, i.e., the modes (including the definitions of the functions involved) are determined by symmetry alone. For analytic simplicity, we choose the modes for different $\underline{m}$'s (but the same $\underline{n}$ and $\underline{p}$) to be partner functions, like sin mø and cos mø. Then for the group of $\underline{R}_o$'s,

$$G_{\underline{N}'\underline{N}}(\underline{R}_o) = \delta_{\underline{n}'\underline{n}} \; \delta_{\underline{p}'\underline{p}} \; \mathcal{D}^{(\underline{n})}_{\underline{m}'\underline{m}}(\underline{R}_o^{-1}), \tag{3}$$

where $\delta_{\underline{n}'\underline{n}}=1$ if $\underline{n}'=\underline{n}$ but is otherwise zero. For fixed $\underline{R}_o$ and $\underline{n}$, the $\mathcal{D}$'s are matrices whose elements are (possibly complex) numbers; all probe-compensated data reductions for planar, circular cylindrical, and spherical scanning are based, at least implicitly, upon them. The $\mathcal{D}$'s are determined by symmetry alone, express the transformations of functions of given symmetries under symmetry operations, and are available for many groups. The familiar expressions for the transformations of even and odd functions of x under reflection in the x=0 plane are elementary examples. The fact that there is no $\underline{m}$ index for rectangular coordinates is responsible for the diagonal character of the G matrix for planar scanning, resulting in much of the simplicity of the analysis; in a lossless medium, the $\mathcal{D}$ matrices may be chosen unitary, so for planar scanning, the G matrix is not only diagonal, but the diagonal elements are roots of unity and so merely indicate phase shift.

Thus, we have explicit expressions for the interaction between a pair of transducers as a function of separation and relative orientations. Further, for each measured W value, there is a complex equation, resulting in a set of

simultaneous equations for the desired unknowns (Q's). (The P's are determined in advance from measurements in the far field of the probe.) The finite set of significant Q's expressing the pattern of the test transducer may be determined from measurements (W's) made at essentially arbitrary positions and orientations of the probe in the near and/or far field, provided only that the W's determine the Q's in either the ordinary or least squares sense. However, mechanical simplicity and efficient data reduction may be achieved by particular choices of the measurement lattice ($\underline{R}_o$'s) and modes. Thus, it is convenient to confine measurements to a particular lattice on a coordinate surface. We divide $\underline{R}_o$ into two parts, namely operations $\underline{S}_o$ parallel to the measurement surface, preceded by the operation N_o normal to the measurement surface, yielding the expression

$$W(\underline{R}_o) = \sum_{\underline{N}^\circ} \sum_{\underline{N}} \delta_{\underline{n}^\circ \underline{n}} \, \delta_{\underline{p}^\circ \underline{p}} \, P_{\underline{N}^\circ} \, D^{(\underline{n})}_{\underline{m}^\circ \underline{m}}(\underline{S}_o^{-1}) \, Q_{\underline{N}}, \tag{4}$$

where

$$P_{\underline{N}^\circ} = P_{\underline{N}^\circ} = \sum_{\underline{N}'} \delta_{\underline{n}'\underline{n}^\circ} \, \delta_{\underline{p}'\underline{p}^\circ} \, \mathcal{D}^{(\underline{n})}_{\underline{m}'\underline{m}^\circ}(N_o^{-1}) \, P_{\underline{N}'}. \tag{5}$$

Transformation (5) is simpler than that for a general $\underline{R}_o^{-1}$ and, for a given probe, frequency, and scanning surface, may be carried out once and for all, yielding probe pattern coefficients in an intermediate coordinate system indicated by the degree sign; this is quite important for the radial transformation of spherical EM modes. Operations and transformations parallel to the measurement surface are indicated by Roman rather than script typefaces; the D's are simpler than the $\mathcal{D}$'s, far simpler in the spherical case.

If the measurement lattice is chosen such that the $\underline{S}_o$'s form a (discrete) group, application of orthonormalities of the <u>transformation coefficients</u> (D's) with respect to summation on the measurement lattice to (4) yields

$$P_{\underline{N}^\circ} \, Q_{\underline{N}} = q^{-1} \sum_{\underline{S}_o} W(\underline{R}_o) \, D^{(\underline{n})}_{\underline{m}^\circ \underline{m}}(\underline{S}_o^{-1})^*, \tag{6}$$

where * indicates complex conjugate, q is the ratio of the number of $\underline{S}_o$'s to the number of rows in the $D^{(\underline{n})}$ matrices, and the representation ($D^{(\underline{n})}$'s) is chosen to be unitary, which is possible even for a lossy medium. If there are more measurements than interim unknowns, least squares values are obtained. The D's for the translation and two-dimensional rotation groups have the forms $\exp(+ik_x x_o)$ and $\exp(+im\phi_o)$, the familiar Fourier transform, series, and DFFT being symmetry decompositions; hence, (6) represents a Fourier decomposition for planar and cylindrical scanning, modified for the spherical case. Numerical implementation of (6) of course requires truncation of the infinite set of modes (for the spherical case, neglect only of insignificant supergain modes), but no other approximation is involved, even in the use of the Discrete Fast Fourier "Transform." The DFFT, supplemented by matrix multiplication in the spherical case, also supplants computation of functions of angles, even in evaluating the far field. Explicit expressions will be given for the partner-function modes, $\mathcal{D}$'s, and D's for planar, circular cylindrical, and spherical scanning of scalar and electromagnetic fields.

[1] A. C. Newell, R. C. Baird, and P. F. Wacker, "Accurate Measurement of Antenna Gain and Polarization at Reduced Distances by an Extrapolation Technique," <u>IEEE Trans. Antennas Propagat.</u>, vol. AP-21, pp. 418-431, July, 1973.
[2] P. F. Wacker "Non-Planar Near-Field Measurements: Spherical Scanning," NBSIR 75-809, Natl. Bur. Stds., Washington, June 1975.