

MICRO-SCOPIC SCATTERING ANALYSIS OF X-RAY BEAMS
AT PARALLEL MULTI ATOM INTERFACES

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1. Introduction.

So far, X-ray has been mainly used in crystal optics, medical field and approximate methods geometrical and physical optics are discussed. Recently, generating technique of powerful X-ray such as SOR (Synchrotron Orbital Radiation) and super precision processing technique in atomic level have progressed. With these developments, new type X-ray optical devices such as X-ray fibers and functional devices have been studied and the field of new X-ray optics such as X-ray lithography, X-ray microscopy and X-ray laser has been developed. The main X-ray optical devices of these systems are reflection mirrors and so forth, but their characteristics are not sufficient to X-ray systems. X-ray optical fibers have not been studied. Therefore the analysis of scattering and reflection of X-ray from boundary surface of matter in atomic level is important problem for study and development of X-ray devices such as efficient X-ray mirrors, and fibers.

In the analysis of scattering and reflection of X-ray in atomic level, the boundary surface is considered as multilayer array of spheroids and cylinders, and in this paper, the scattering and reflection characteristics for a 2-dimensional Gaussian beam obliquely incident are studied. These analyses are carried out by using boundary element method (BEM) which can easily be applied to various cylindrical array forms under different conditions of the incident beam.

2. A Scattering Analysis
of Parallel Multi Cylinders.

Figure 1 shows coordinate system for the atomic boundary surface model and Gaussian beam. The coordinate systems for the parallel multi cylinders are (x,y,z) and ones for the incident beam are (x',y',z') . The cylinders is regarded as perfectly conductive bodies. The region Se is a semi-infinite free space.

The incident beam ϕ_z^{in} is a two-dimensional Gaussian beam of E-wave having the x-component of electric field E, or that of the H-wave having the x-component of magnetic field H. The incident beam wave is defined as (1) and (2).

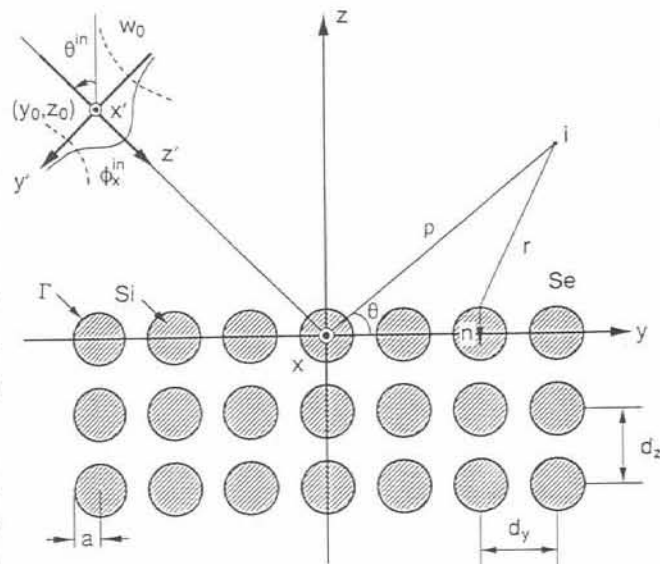


Fig.1 Coordinate system for the atomic boundary surface model and Gaussian beam

$$\phi_x^{\text{in}} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} u(y', z'). \quad (1)$$

$$u(y', z') = \sqrt{\frac{w_0}{w(z')}} \exp\{-j(kz' - \psi(z')) - (j\frac{2z'}{kw_0^2} + 1) \frac{y'^2}{w^2(z')}\} \quad (2a)$$

$$w^2(z') = w_0^2 \left\{ 1 + \left(\frac{2z'}{kw_0^2} \right)^2 \right\} \quad (2b)$$

$$\psi(z') = \frac{1}{2} \tan^{-1} \left(\frac{2z'}{kw_0^2} \right). \quad (2c)$$

The scattered wave ϕ_z^{sc} in the region Se satisfies the following Helmholtz's equation (3).

$$\nabla^2 \phi_x^{\text{sc}} + k^2 \phi_x^{\text{sc}} = 0 \quad (3)$$

The boundary conditions on the boundary Γ are given by

$$\phi_x^{\text{sc}} = -\phi_x^{\text{in}} \quad : \text{E - wave} \quad (4a)$$

$$\frac{\partial \phi_x^{\text{sc}}}{\partial n} = -\frac{\partial \phi_x^{\text{in}}}{\partial n} \quad : \text{H - wave} \quad (4b)$$

A boundary element method is applied to Eq.(3), the following equation is obtained^{[1], [2]}.

$$\phi_{xi}^{\text{sc}} = \int_{\Gamma} \left(\phi \frac{\partial \phi_x^{\text{sc}}}{\partial n} - \frac{\partial \phi}{\partial n} \phi_x^{\text{sc}} \right) d\Gamma \quad (5)$$

By dividing boundary Γ with N elements and using interpolation functions, the scattered field ϕ_z^{sc} with respect to nodal point i on the boundary Γ is given from Eq.(5). $\phi_z^{\text{sc}}, \partial \phi_z^{\text{sc}} / \partial n$, in Eq.(5) are given by the matrix representation, Eq.(6).

$$H \phi_x^{\text{sc}} = G \frac{\partial \phi_x^{\text{sc}}}{\partial n} \quad (6a)$$

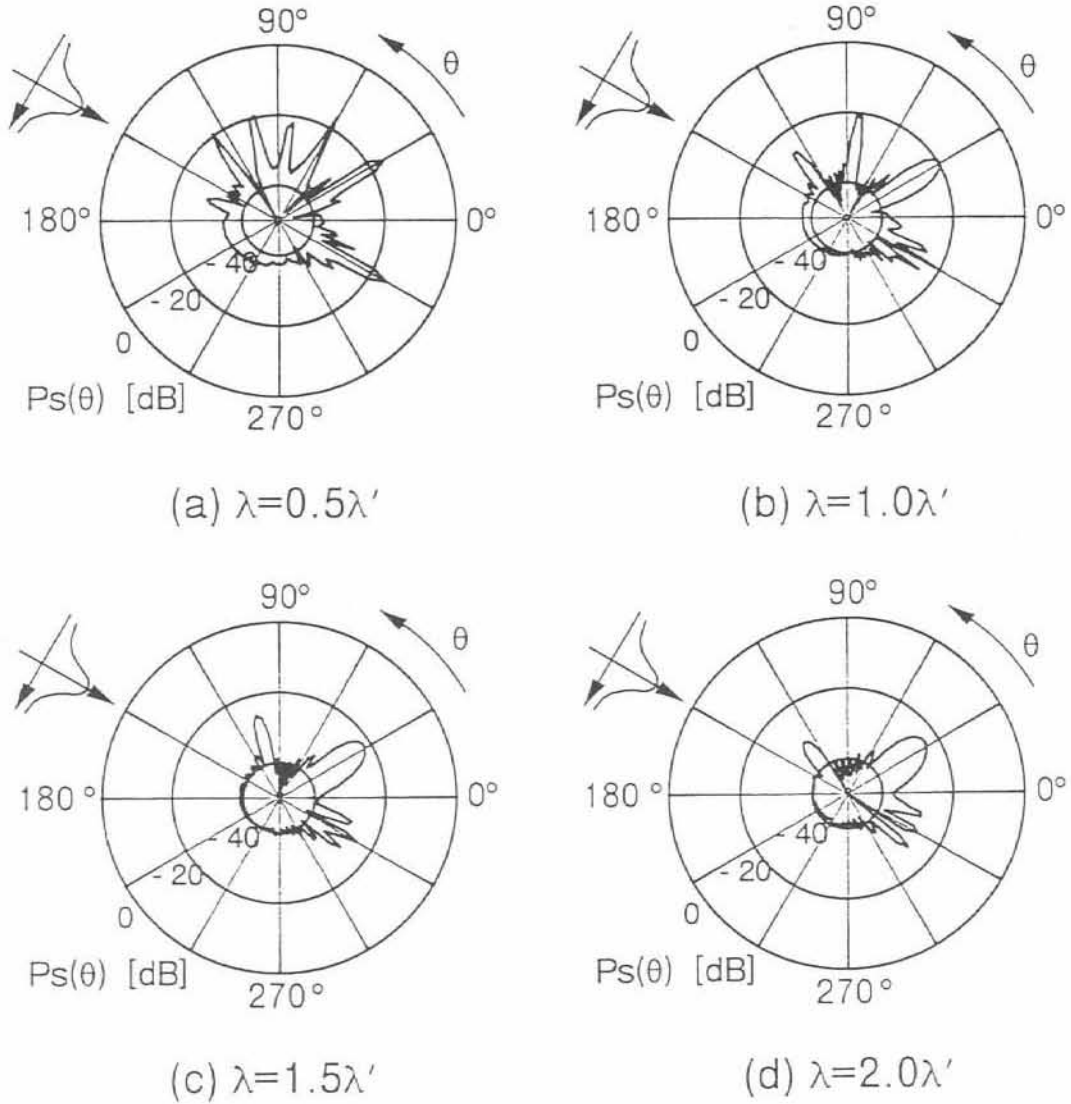
$$\phi_{xi}^{\text{sc}} = \sum_{j=1}^N G_{ij} \frac{\partial \phi_{xj}^{\text{sc}}}{\partial n} - \sum_{j=1}^N H_{ij} \phi_{xj}^{\text{sc}} \quad (6b)$$

Matrices H and G represent square matrices of $((N+1), (N+1))$. The magnitude of scattering is defined by Eq.(7).

$$P_s(\theta) = 20 \cdot \log_{10} \left(\frac{|\phi_x^{\text{sc}}(\rho, \theta)|}{\max |\phi_x^{\text{in}}(\rho, \theta)|} \right) \quad (7)$$

3. Numerical Results

In this analysis parameters are considered as actual model, and used wavelength of characteristics X-ray of Cr, atomic radius and lattice parameter as parameter a and $d_y (=d_z)$. Figure 2 shows comparison of scattered field patterns by different incident wavelength for E-wave. It is found that the number of the diffraction directions is changed by the wavelength. The amplitude of the near scattered field for E-wave is shown in Fig.3.

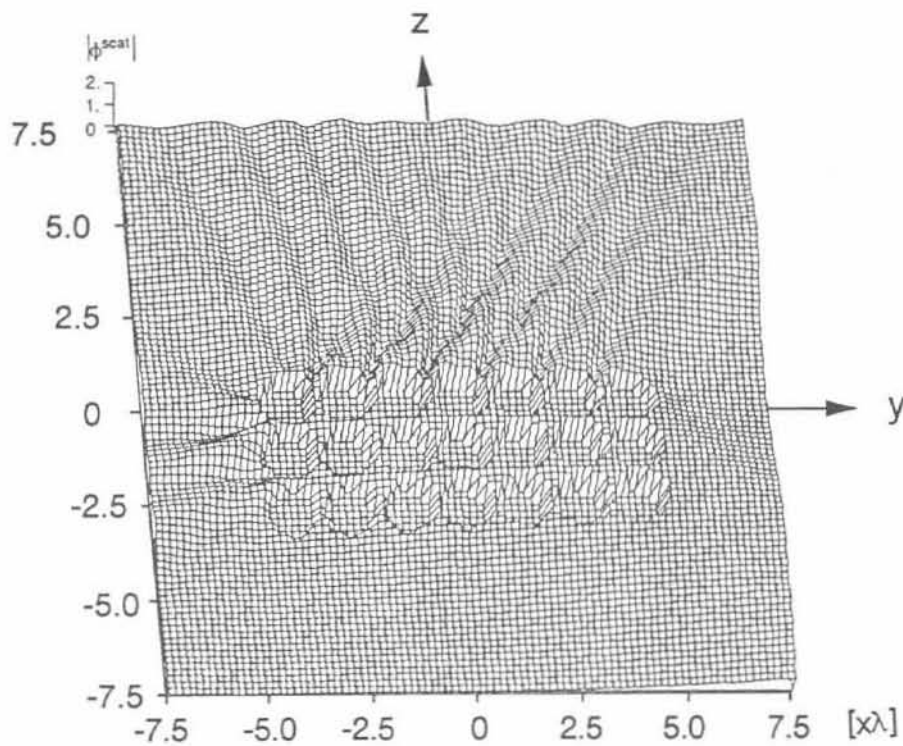


λ' : standard wave length

$$a=0.607\lambda', d_y=1.37\lambda', \theta^n=60^\circ, w_0=4.240\lambda',$$

$$(y_0, z_0)=(-4.113\lambda', 4.113\lambda'/\tan\theta^n), n_y=7, n_z=1.$$

Fig.2 Comparison of scattered field patterns by different incident wavelength for E-wave, (Mo : $a=1.39A, d_y=3.1399A$. Cr : $\lambda'=2.29A$.)



$$E_0=1, a=0.607\lambda, d_y=d_z=1.37\lambda, \theta^{in}=60^\circ, w_0=3.964\lambda, (y_0, z_0)=(0,0), n_y=7, n_z=3.$$

Fig.3 Amplitude of the scattered electromagnetic field for E - Wave
(Mo : $a=1.39\text{\AA}, d_y=d_z=3.1399\text{\AA}$. Cr : $\lambda=2.29\text{\AA}$.)

4. Conclusion.

The scattering characteristics of beam waves by the parallel multi cylinders are analyzed using the boundary element method, which can easily be applied to various cylindrical array forms under different conditions of the incident beam. The properties of far and near fields, that are ruled by parameters (radius of cylinders, distance between cylinders, the number of cylinders and others), have been clarified.

In this paper, scattering objects are perfectly conductor, however in fact, the each object has distribution of refractive index. Therefore, it is expected that the finite element method is applied at the inner area of scattering objects.

References.

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