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IBC's Applicability for Approximating a Loaded Rectangular Trough

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1 Introduction

Impedance boundary condition (IBC), which has been widely utilized in electromagnetics, is a useful approximation tool for modeling more complex or practical objects in diffraction and scattering analyses. However, as IBC was derived for a material layer on an infinite conducting ground plane [1],[2], it is important to check the validity of IBC for the finite coated objects with some terminations like edges. In Refs.[3] and [4], a scattering problem for a material filled rectangular trough, which is a kind of such finite objects, has been analyzed by using IBCs. Unluckily, their integral equation approaches have not been able to simulate the edge termination effect, so that supplementary padding regions have had to be made an addition in the vicinity of the edges for the accuracy improvement.

In this paper, so as to confirm the genuine applicability of IBC for such configuration, an electromagnetic wave scattering by a loaded rectangular trough on a ground plane is approximately analyzed by using standard impedance boundary condition (SIBC). In accordance with the procedure of the Kobayashi and Nomura's method, the troublesome edge singularity behavior can be analytically included in the formulation [5]-[7]. Therefore, one may easily evaluate the SIBC's applicability to this trough geometry. The validity of the derived approximate solution is discussed by comparing with the rigorous one via various points of view; such as the dependency on the incident angle, the trough's depth and the filled material parameters. Here the time harmonic factor $e^{-i\omega t}$ is assumed and suppressed throughout the context.

2 Formulation

As illustrated in Figure 1 (a), let us consider the electromagnetic scattering problem for the case that H polarized plane wave:

$$\phi^i \,(= H_z^i) = e^{-ik_0(x\cos\theta_0 + y\sin\theta_0)} \tag{1}$$

impinges on the aperture of the rectangular trough, which is filled by a complex material with relative permittivity ϵ_r and relative permeability μ_r. Here k_0 is free space wavenumber. In $y > 0$, the total field $\phi^t (= H_z^t)$ may be considered as $\phi^t = \phi^i + \phi^r + \phi_H$, where ϕ^i is the incident field, ϕ^r is the corresponding specular reflection field on the ground, and ϕ_H is the scattering

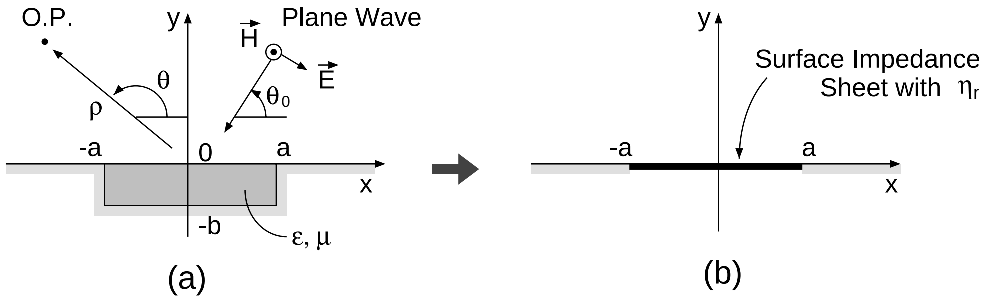

Figure 1: Geometry of the problem

contribution due to the existence of the trough. According to the procedure of Kobayashi and Nomura's method [5]-[7], ϕ_H may be represented as

$$\phi_H = \sqrt{\frac{\pi u}{2}} \sum_{m=0}^{\infty} \int_0^{\infty} \sqrt{\frac{\xi}{\xi^2 - \kappa_0^2}} \{A_m J_{2m}(\xi) J_{-1/2}(\xi u) + B_m J_{2m+1}(\xi) J_{1/2}(\xi u)\} e^{-\sqrt{\xi^2-\kappa_0^2} v} d\xi, \quad (2)$$

where A_m and B_m are the unknown expansion coefficients, and normalization with respect to the half aperture width a $(x = au, y = av, k_0 a = \kappa_0)$ is executed. Each component in the above integral expression can be identified as a class of Weber-Schafheitlin type integrals. Taking into account the discontinuity characteristics of such integrals, the required boundary condition on the ground $(|x| > a, y = 0)$ can be automatically satisfied [7].

In order to check the validity of SIBC, we shall now replace the material loaded trough by an impedance sheet, as illustrated in Fig.1 (b). For determining the unknown coefficients A_m and B_m in Eq.(2), the continuity condition of tangential fields at the trough's aperture $(|x| < a, y = 0)$ have now simplified to an impedance approximate one [1],[2]:

$$E_x^t(= -i\omega\epsilon_0 \cdot \partial\phi^t/\partial y)|_{y=0} = \eta_r Z_0 \cdot H_z^t(= \phi^t)|_{y=0} \quad (3)$$

with

$$\eta_r = -i\frac{N}{\epsilon_r}\tan(N k_0 b), \quad (4)$$

where $Z_0(= \sqrt{\mu_0/\epsilon_0})$ and $N(= \sqrt{\epsilon_r \mu_r})$ are the intrinsic impedance in free space and the complex refractive index, respectively. The simultaneous deterministic equations for the coefficients can be easily obtained as

$$\sum_{m=0}^{\infty} A_m \{GN_C(2m, 2q+1) + GZ(2m, 2q+1)\} = -2\frac{J_{2q+1}(\kappa_0 \cos\theta_0)}{\kappa_0 \cos\theta_0}, \quad (5)$$

$$\sum_{m=0}^{\infty} B_m \{GN_C(2m+1, 2q+2) + GZ(2m+1, 2q+2)\} = i2\frac{J_{2q+2}(\kappa_0 \cos\theta_0)}{\kappa_0 \cos\theta_0}, \quad (6)$$

where $GN_C(\cdot,\cdot)$ is given as

$$GN_C(\alpha,\beta) = i\frac{1}{\eta_r \kappa_0} \cdot \int_0^{\infty} \frac{J_\alpha(\xi) J_\beta(\xi)}{\xi} d\xi = i\frac{1}{\eta_r \kappa_0} \cdot \frac{2\sin\{\frac{\pi}{2}(\alpha-\beta)\}}{\pi(\alpha^2 - \beta^2)} \quad (7)$$

and $GZ(\cdot,\cdot)$, defined as

$$GZ(\alpha,\beta) = \int_0^{\infty} \frac{J_\alpha(\xi) J_\beta(\xi)}{\xi\sqrt{\xi^2 - \kappa_0^2}} d\xi, \quad (8)$$

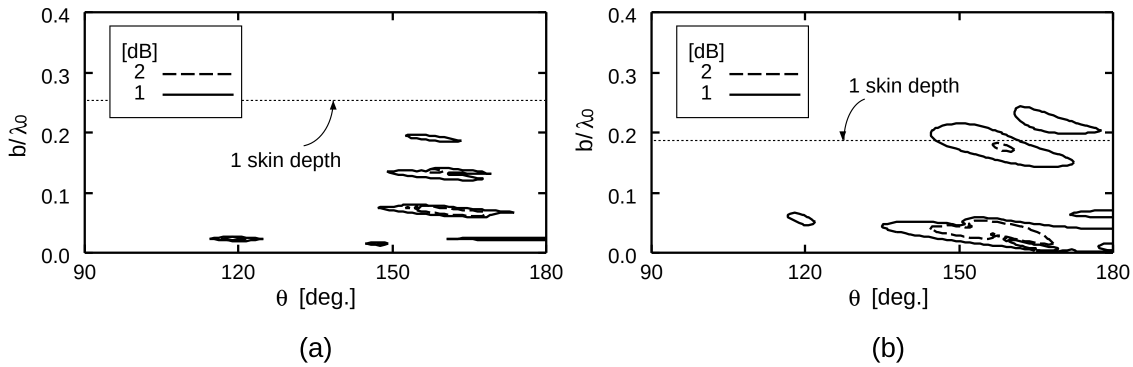

Figure 2: Error distribution of SIBC approximation for the angle-depth variation (monostatic RCS calculation). $2a = 1.0\lambda_0$, $\epsilon_r = 4\mu_r$. (a)$N = 8.0 + i0.6$ (b)$N = 3.0 + i0.85$

can be calculated from a numerically feasible series via the similar procedure in Refs.[5]-[7]. In comparison with the corresponding rigorous formulation in Ref.[7], one can readily solve the above simultaneous equations (5) and (6), since no series expression with slow convergence is included in such equations.

By applying the saddle point method to Eq.(2), the far zone scattering field expression, which is relevant for calculating the corresponding radar cross section (RCS) value, can be easily obtained as

$$\phi_H = \sqrt{\frac{\pi}{2k_0\rho}}e^{i(k_0\rho+\pi/4)}\sum_{m=0}^{\infty}\{A_m J_{2m}(k_0 a\cos\theta) - iB_m J_{2m+1}(k_0 a\cos\theta)\}, \tag{9}$$

where $(\rho,\, \theta)$ is the cylindrical coordinate, as shown in Fig.1 (a).

3 Numerical results and discussions

In general, it is considered that for nearly normal incidence SIBC precisely simulates the electromagnetic wave behavior on an electrically high contrast and/or lossy material layer with a small thickness, by replacing the material's contribution with a surface impedance sheet [1],[2]. It must be therefore checked the SIBC's validity on both oblique incidence and depth dependency. So we shall show first the monostatic RCS error distribution of SIBC approximation for the angle-depth variation in Figure 2. In Fig.2, a comparison is done with the results obtained by the rigorous formulation [7] for checking the accuracy of this approximation. The corresponding skin depth δ, which is defined as [8]

$$\delta = \frac{1}{\Im m\,\{N\}}\cdot\frac{\lambda_0}{2\pi}, \tag{10}$$

is also included in the figure. It seems that the skin depth δ of the material surface may serve for judging the applicability of the approximation, and this observation leads us a condition:

$$b \geq \delta \tag{11}$$

would be imposed. There are some notable errors over 1 dB at the oblique incidences for both the refractive indices. Under the criteria given in Eq.(11), such error can not be observed for

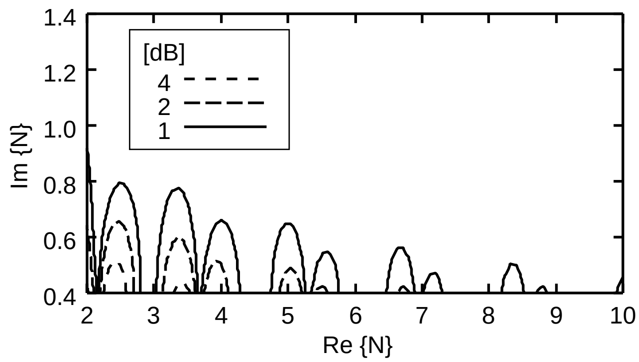

Figure 3: Error distribution of SIBC approximation for the variation of complex refractive index N (monostatic RCS calculation). $\theta_0 = 160^o$, $2a = 1.0\lambda_0$, $b = 0.3\lambda_0$, $\epsilon_r = 4\mu_r$.

$N = 8.0 + i0.5$ as in Fig.2 (a). While, for rather sparse material ($N = 3.0 + i0.85$) in Fig.2 (b), the noticeable error can be still detected even if Eq.(11) is satisfied. This is due to the fact that the wave in such electrically sparse material like $\Re e\{N\} = 3$ does not essentially propagate only perpendicular to the boundary surface, and the reflections at the subsurfaces in the trough may be still appreciable and contribute to the RCS values. Though the detail data is now omitted, we have already checked the accuracy for the dependency on $\Re e\{N\}$ [9] and it has been examined under the satisfaction of the above condition (11) that the error between the present SIBC and exact solutions is decreased to less than 1 dB when $\Re e\{N\} > 6$ with $\Im m\{N\} = 1.0$.

Finally, let us show the error distribution of the present approximation to the complex refractive index N in Figure 3. It is observed from Fig. 3 that a good accurate range less than 1 dB widely exists for the variation of both real and imaginary parts of N.

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