Determination of the number of arrival signals in the atmospheric noise environment

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1. Introduction

A number of high-resolution direction-finding algorithms, such as MUSIC [1] and ESPRIT [2], have been studied recently. These algorithms need the number of arrival signals on the array antenna. The AIC and MDL [3] methods can estimate the number of arrival signals accurately.

The AIC and MDL methods assume that the noise component is uncorrelated between antenna elements. Here, the atmospheric noise, which comes from far field, exists in HF (High Frequency) band. Also, the atmospheric noise power is stronger than the thermal noise power in this frequency band and the atmospheric noise is correlated between antenna elements. Therefore, the accuracy of the AIC and MDL methods degrades in atmospheric noise environment.

It has been reported that the atmospheric noise is transformed to the uncorrelated noise using the whitening method [4]. This whitening method requires the receiving atmospheric noise data which does not include arrival signals. In [4], the receiving atmospheric noise data is analytically calculated assuming the arrival distribution of the atmospheric noise and atmospheric noise power. However, since the arrival distribution of the atmospheric noise changes in a moment, the effect of the whitening method deteriorates.

In this paper, we propose another method to obtain the receiving atmospheric noise data. The receiving atmospheric noise data is calculated at the frequency domain in the proposed method. The effectiveness of the proposed method is shown using the measured data.

2. Conventional whitening method

We assume that HF band signals are incident on an array antenna as shown in Fig. 1. Figure 2 shows the flowchart of the conventional whitening method to determine the number of arrival signals.

A received data vector is defined as $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_K(t)]^T$, where T denotes transpose, $x_m(t)(m=1,2,\dots,K)$ is the received data at mth antenna, and K is the number of antennas. $x_m(t)$ includes arrival signals, atmospheric noise, and thermal noise. A covariance matrix \mathbf{R}_{xx} of the received data $\mathbf{x}(t)$ is calculated by $\mathbf{R}_{xx} = E[\mathbf{x}(t)\mathbf{x}^H(t)]$, where H denotes transpose conjugate. In the conventional AIC and MDL methods, the eigenvalue of the covariance matrix R_{xx} is employed to determine the number of arrival signals. However, the covariance matrix R_{xx} includes the atmospheric noise. It is required that the atmospheric noise can be uncorrelated. To realize uncorrelation of the atmospheric noise, the whitening method [4] is applied to the covariance matrix \mathbf{R}_{xx} .

In the conventional whitening method [4], the received data vector, which includes only the atmospheric noise, is defined as $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_K(t)]^T$, where $y_m(t)(m = 1, 2, \dots, K)$ is the received data at *m*th antenna. The covariance matrix \mathbf{R}_{yy} of the received data $\mathbf{y}(t)$ is also calculated by $\mathbf{R}_{yy} = E[\mathbf{y}(t)\mathbf{y}^H(t)]$. The Cholesky decomposition of the covariance matrix \mathbf{R}_{yy} is carried out by

$$\mathbf{R}_{vv} = \mathbf{C}^H \mathbf{C} . \quad (1)$$

 $\mathbf{R}_{yy} = \mathbf{C}^{H}\mathbf{C}. \quad (1)$ The original covariance matrix \mathbf{R}_{xx} is whitened by

$$\mathbf{R}_{white} = \mathbf{C}^{-H} \mathbf{R}_{xx} \mathbf{C}^{-1} \tag{2}$$

 $\mathbf{R}_{white} = \mathbf{C}^{T} \mathbf{R}_{xx} \mathbf{C}^{T}$ (2) As a result, the atmospheric noise is uncorrelated between antenna elements. The eigenvalue of the whitened covariance matrix \mathbf{R}_{white} is employed in the AIC and MDL methods to determine the number of arrival signals.

In [4], the receiving atmospheric noise data $\mathbf{y}(t)$ is analytically calculated assuming the arrival distribution of the atmospheric noise and the atmospheric noise power. However, since the arrival distribution of the atmospherics noise changes in a moment, the effect of the whitening method deteriorates. Also, it is difficult to obtain the receiving data $\mathbf{x}(t)$ and $\mathbf{y}(t)$ at the same time when both the arrival signals and the atmospheric noise exist. In the next section, we propose the method to obtain the receiving atmospheric noise data $\mathbf{y}(t)$.

3. Proposed method to obtain the atmospheric noise data

Figure 3 shows the proposed flowchart to obtain the atmospheric noise data y(t). In the first step, Fourier transform of the receiving data $x_m(t)(m=1,2,\cdots,K)$, which includes arrival signals, atmospheric noise, and thermal noise, is carried out to frequency domain. In the second step, the frequency bandwidth of the arrival signal is suppressed. The suppressed frequency bandwidth is set to zero. In the third step, these data are transformed to time domain using an inverse Fourier transform. Therefore, the receiving approximate atmospheric noise data $y_m(t)(m=1,2,\cdots,K)$ without the effect of the suppressed frequency bandwidth is obtained. The following step to determine the number of arrival signals is the same as the conventional whitening method. In the proposed method, the receiving atmospheric noise data y(t) can be obtained when both the arrival signals and the atmospheric noise exist.

4. Measurement result

Figure 4 shows the configuration of an array antenna and the measurement condition. The measurement is performed outdoors. The atmospheric noise power is stronger than the thermal noise power in the measurement frequency. A CW signal wave is incident on a four-element circular array antenna. The A/D sampling frequency is 10 kHz. The number of snapshots is 500.

Figure 5 shows the frequency spectrum of the receiving data $x_1(t)$. This corresponds to the first step in Fig. 3. In the conventional AIC and MDL methods, this original receiving data is employed to determine the number of arrival signals. The eigenvalue of the original covariance matrix \mathbf{R}_{xx} is shown in Fig. 6. The second eigenvalue is bigger than the third and fourth eigenvalues. As a result, the AIC and MDL methods both estimate that the number of arrival signals is two. This estimation is incorrect.

The proposed method suppresses the signal bandwidth as shown in Fig. 7, which corresponds to the second step in Fig. 3. Here, the frequency bandwidth of 400 Hz is suppressed. In the third step, $y_m(t)(m=1,2,\cdots,K)$ is calculated using this data. Then the original covariance matrix \mathbf{R}_{xx} is whitened by equation (2). The eigenvalue of the whitened covariance matrix \mathbf{R}_{white} is shown in Fig. 8. The three smallest eigenvalues become almost the same value. Therefore, the AIC and MDL methods both estimate the number of arrival signals correctly.

The atmospheric noise in the suppressed bandwidth in the second step remains in the proposed method. Therefore, the receiving atmospheric noise data y(t) is obtained approximately. Below, the validity of the proposed method is confirmed from a viewpoint of power.

In the suppressed bandwidth in the second step, the power of the atmospheric noise is defined as P_{a1} , and the power of the thermal noise is defined as P_{t1} . On the other hand, in the unsuppressed bandwidth in the second step, the power of the atmospheric noise is defined as P_{a2} , and the power of the thermal noise is defined as P_{t2} .

 P_{a1} , P_{t1} , P_{a2} , and P_{t2} are divided into the correlated noise and the uncorrelated noise. The correlated noise component indicates that the noise is correlated between antenna elements. On the contrary, the uncorrelated noise component indicates that the noise is not correlated between antenna elements. To validate the effectiveness of the proposed method, we assume three cases below. The three cases are summarized in Table 1.

Case 1 is the non-whitening condition. The correlated noise is P_{a1} and P_{a2} . The uncorrelated noise is P_{t1} and P_{t2} . So, the correlated noise power $P_{a1} + P_{a2}$ is stronger than the uncorrelated noise power $P_{t1} + P_{t2}$. The AIC and MDL methods cannot be applied in this case.

Case 2 is the whitening condition. The atmospheric noise P_{a2} is uncorrelated by the whitening method by the proposed method. The correlated noise is P_{a1} . The uncorrelated noise is P_{a2} , P_{t1} and P_{t2} .

If $P_{a1} < P_{a2} + P_{t1} + P_{t2}$ is satisfied, the correlated atmospheric noise P_{a1} is not stronger than the uncorrelated noise power. Therefore, the remaining atmospheric noise P_{a1} can be ignored. The AIC and MDL methods can be applied in this case.

Case 3 is the suppressed bandwidth component. In this case, the correlated noise power P_{a1} is stronger than the uncorrelated noise power P_{t1} . As in Case 1, the AIC and MDL methods cannot be applied.

5. Conclusion

We have proposed a method to obtain the atmospheric noise data which is employed in a whitening method. The effectiveness of the proposed method has been shown using the HF band measurement data. Also, the validity of the proposed method has been proved from the viewpoint of the correlated noise and the uncorrelated noise. The relationship between the suppressed bandwidth in the second step and the accuracy of the determination of the number of arrival signals is a subject for future work.

6. References

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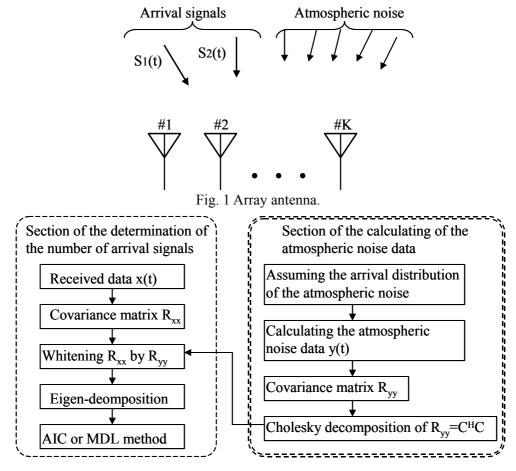


Fig. 2 Flowchart of the conventional whitening method.

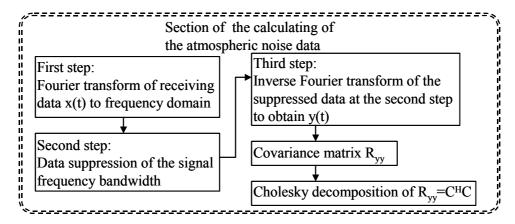


Fig. 3 Proposed method to obtain the atmospheric noise data y(t).

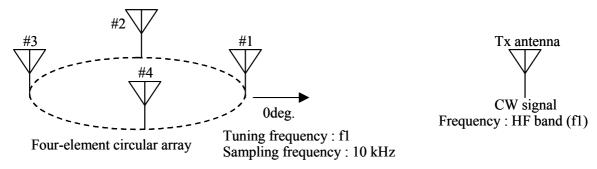


Fig. 4 Measurement array antenna.

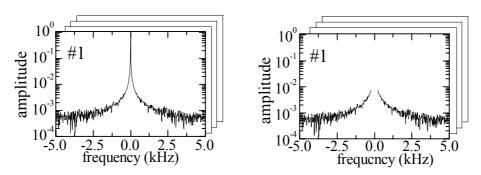


Fig. 5 Frequency spectrum of receiving data. Fig. 7 Frequency spectrum in the second step.

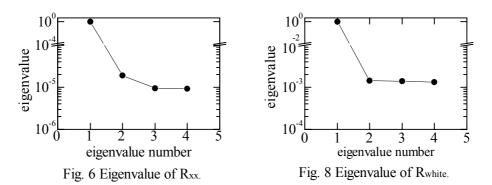


Table 1. Comparison between correlated noise and uncorrelated noise.

	correlated noise		uncorrelated noise
Case 1	Pa1+Pa2	>	Pt1+Pt2
Case 2	Pa1	<	Pa2+Pt1+Pt2
Case 3	Pa1	>	Pt1