PERFOREMANCE EVALUATION OF ANTENNA ARRAYS FOR HIGH-RESOLUTION DOA ESTIMATION IN CHANNEL SOUNDING

M. Landmann, A. Richter and R.S. Thoma
Ilmenau University of Technology, Department of Electrical Engineering and Information Technology, 98684 Ilmenau, Germany, Phone: +49 3677 691160, Fax: +49 3677 691113,
E-mail: markus.landmann@tu-ilmenau.de

Abstract: High-resolution parameter estimation algorithms can significantly enhance path parameter resolution in multidimensional channel sounding. However, variance and reliability of the estimation results are not often clearly defined in practical environments. Especially, Direction-of-Arrival (DoA) resolution of multiple paths can be severely degraded by antenna array imperfections, mutual element coupling, and residual calibration errors. In this paper we describe the fundamental limits on the DoA resolution in terms of the Cramer-Rao-Lower-Bound (CRLB) which is calculated from the calibration data of the antenna array. The CRLB indicates the minimum achievable parameter variance which can not be outperformed by any parameter estimator. We compare the calculated CRLB to the empirical variance taken from measurements using a Stacked Polarimetric Uniform Circular Patch Array (SPUCPA) with 192 output ports. Since the result essentially depends on the constellation of the impinging waves, the most simple "single path scenario" and the "coherent two path scenario" are considered. Since the proposed method relies only on antenna beam pattern measurements, it can easily be applied to real antenna arrays. The results clearly indicate the performance limits to high-resolution DoA estimation.

1 Introduction

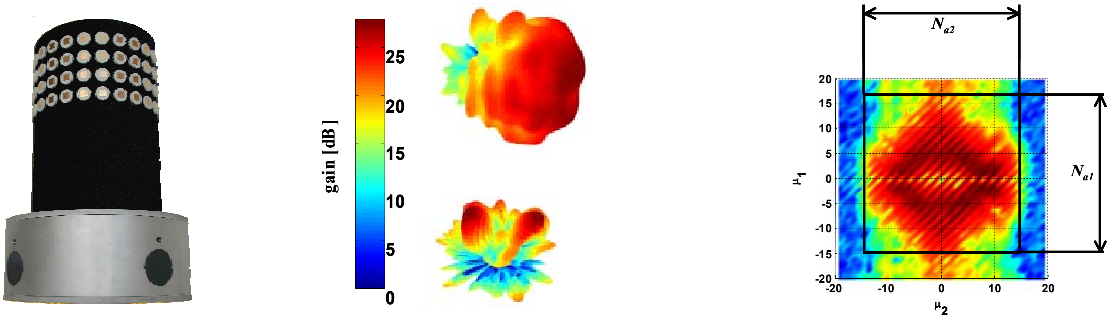
Various types of antenna array architectures have been used in real-time MIMO channel sounding [1]. Depending on their spatial shape these arrays allow estimation in azimuth and/or elevation with more or less complete angular coverage. With a suitable design of the antenna array elements, also the polarimetric channel response can be estimated. Several high-resolution channel parameter estimation algorithms have been proposed for this application, mainly based on subspace [2] or maximum likelihood methods [3] [4] [5] [6] [7]. However, based on an empirical investigation of several arrays (see [8], [2]) we have shown that the resolution performance of these algorithms is severely influenced by the achieved characteristics of the realized antenna array and also depends on the angular and complex polarimetric path weight constellation of the impinging waves. Obviously, design parameters such as the effective array aperture (as seen from a specific DoA), the element distance, and the characteristics of the array elements have a strong influence. This relates also to any kind of mechanical and electrical tolerances, mutual element coupling, and amplitude- and phase mismatch of the antenna elements. From experimental studies it became clear that the fact of closely spaced coherent paths cause severe degradation in variance and resolution. The DoA and path weight variance in terms of the CRLB [9] [10] of an unbiased DoA parameter estimator are derived in [11]. Since it is a fundamental limit on the achievable variance, the CRLB is independent on the intended parameter estimation algorithm. A further advantage of our proposed procedure is that it completely relies on measured complex polarimetric antenna characteristics. Thus any realization degradation is included if only precise calibration measurement data are available. The structure of the data model which is used to calculate the CRLB and also to estimate the channel parameter with a ML based algorithm [6] was described in [11]. Here we demonstrate experimental results. The measurements were taken with a SPUCPA [13] and the sounder [12] in an anechoic chamber. We compare the calculated CRLBs with the empirical variances of the parameters estimated from a series of measurements for a "single path scenario" and "coherent two path scenario".

2 SPUCPA calibration measurement and data model

In this section calibration measurement results are described which are taken from the SPUCPA shown in Fig. 1. This 5.2 GHz cylindrical array consists of 96 dual-polarized dielectric patch antenna elements which are arranged in 4 stacked rings. The spacing between the rings was chosen to 0.5 of the wavelength at the upper frequency in the band thus satisfying the sampling theorem in space. The optimised radius is 1.75 of the wavelength at the upper

ISBN: 4-88552-206-4 C3055©IEICE

frequency. The goal of this optimisation is to minimize the probability of parameter estimation outliers throughout the SAGE iterations (see [8]). The measured beam patterns for vertical and horizontal polarized stimulation and the effective aperture distribution function (EADF) of one element are also depicted in Fig. 1. The EADF results from a 2D-Fouriertransform of complex beam pattern and ends in a 2D-Fourier series expansion of the beam patterns. This allows a very effective handling of the calibration date from a numerical point of view [11]. The calibration measurement was conducted in an anechoic chamber using a channel-sounder [12]. The sounder delivers instantaneous frequency responses at 5.2 GHz within a 120 MHz bandwidth. The beam patterns are calculated from the LOS maxima of the channel impulse responses, thus further reducing the remaining parasitic echoes. This single path LOS scenario is used as reference to construct the calibration data model as described in [11]. The complex beam patterns are recorded in the full 360 degree coverage azimuth domain and 180° in elevation. The transmit antenna was a dual polarized high gain horn with vertical/horizontal polarization matched to the vertical/horizontal polarization vector of the array. High polarization decoupling in the far field of this transmit antenna is of primary importance.



a) SPUCPA b) $\mathbf{B}_{mv}$ & $\mathbf{B}_{mh}$ **c)** EADF $\mathbf{G}_m$

Fig. 1: (a) SPUCPA, (b) normalized magnitude of one beam in azimuth and elevation (vertical & horizontal stimulation), (c) EADF with phase drift correction

The calibration data model is constructed by a discrete Fourier transform of the recorded complex beam pattern Fig. 1 (b) to the EADF Fig. 1 (c):

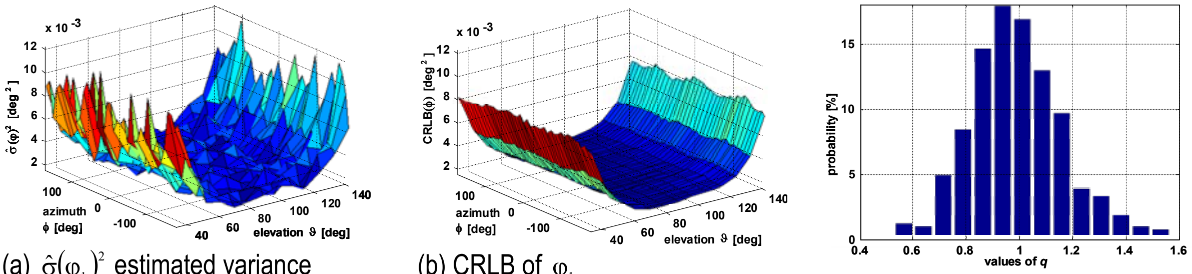
$$\mathbf{G}_m^{[N_{a1}\times N_{a2}]}=\frac{1}{\sqrt{N_1\cdot N_2}}\mathbf{F}_1^{[N_{a1}\times N_1]}\cdot\mathbf{B}_{\mathbf{p}m}\cdot\mathbf{F}_2^{[N_2\times N_{a2}]}\quad .$$

This step, however, not only requires precise rotation of the array around a suitable defined pivot point located in the centre of the array, but also excellent phase stability throughout the whole measurement cycle. Even with synchronized Tx and Rx there can be problems with phase stability as a result of thermal phase drift of cables and filters. Therefore, this phase drift has to be estimated and removed from the calibration data.

3 Comparison of the array evaluation method with measurements

The procedure for determining the CRLB is now applied using the EADF shown in Fig. 1(c). The CRLB represents the variances and co-variances of the estimated parameters dependent on a given signal-to-noise-ratio (SNR). We compare the calculated CRLB to the variance which was empirically achieved by measurements and parameter estimation in well defined reference scenarios. Since we are considering a SPUCPA, the variance of the azimuths, elevations and of the complex polarimetric path weights (separated into real- and imaginary parts) will be considered. Because of space limitations only the results for one of these parameters will be shown.

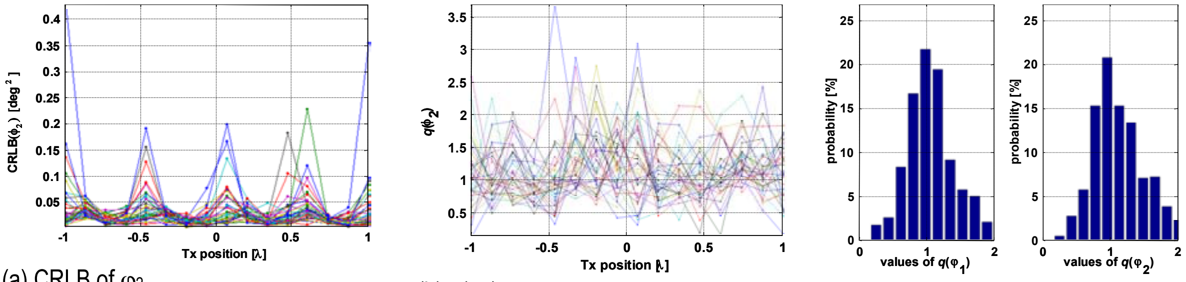
"Single path scenario": At first the achievable accuracy of the investigated SPUCPA will be demonstrated assuming a single source in the far field over the whole azimuth range and elevation range from 30° to 140°. This scenario is described by one LOS path. The measurement consists of 64 stationary observations per azimuth and elevation pair. All parameters of each observation are estimated by using a ML estimator (RIMAX algorithm, [6] [7]). The variances of each parameter from the 64 realizations were calculated. The estimated variance of the azimuth angle is shown (Fig. 2 (a)) vs. the true DoAs in azimuth and elevation. The CRLB-result of the azimuth angle φ_1 (calculated with the estimated SNR (17…18 dB) of the measurement) are depicted in Fig. 2 (b). Because of the limited number of 64 realizations the ratio $q = \frac{\mathrm{CRLB}(\varphi_1)}{\hat{\sigma}(\varphi_1)^2}$ of the CRLB and the estimated variance Fig. 2 (c) has a variance of around 18 %, which in fact matches the theoretically expected value.



(a) $\hat{\sigma}(\varphi_1)^2$ estimated variance (b) CRLB of φ_1 (c) distribution of *q*

Fig. 2: Comparison of the CRLB of φ_1 (b) and estimated variances $\hat{\sigma}(\varphi_1)^2$ from the estimations of φ_1 (b), distribution of the ratio *q* of both (c)

"Coherent two path scenario": Two paths are said to be coherent if their weights have the same frequency and if they have an almost constant phase difference during the available observation window. Because of the superposition of the two coherent wave-fronts, a static spatial pattern results with regions of constructive and destructive interference. This probably leads to an ill-posed parameter estimation problem since it can easily be observed that the resulting degradation depends on the phase difference between the path weights. This scenario has been described in [8], [2]. The two Tx antennas where located in the horizontal plane and at the same distance *l* to the SPUCPA (Rx) and a angular separation in azimuth and elevation of 5°. The distance of one transmit antenna was changed in steps of $\lambda/8$ from $-\lambda$ to λ around the distance *l*. Thus introducing a specific phase difference between both paths. The transmit power of the both sources differed around 3 dB. The results are plotted vs. the relative Tx antenna position. In Fig. 3 (a) the calculated CRLB of the azimuth of source 2 is plotted and in (b) the related ratio $q(\varphi_2)$ of the CRLB and the estimated variance from 32 observations. The different colours in the Fig. 3 (a & b) characterize the different azimuth positions of the Rx array. It is observable that the array behaves not homogeneous for all azimuth angles.



(a) CRLB of φ_2 (b) $q(\varphi_2)$ (c) distributions of $q(\varphi_1)$ & $q(\varphi_2)$

Fig. 3: (a) CRLB(φ_2), (b) ratio of the CRLB(φ_2) and estimated variances $\hat{\sigma}(\varphi_2)^2$ from the 32 estimations of φ_2, (c) distribution of $q(\varphi_1)$ and $q(\varphi_2)$

It becomes obvious that the oscillating behaviour of the CRLB is independent on the azimuth of the Rx array. However it depends on the phase difference of the two waves (see Tx position) which causes an ill posed estimation problem for specific phase constellations.

4 Conclusions

We have shown that the recently introduced DoA resolution performance evaluation method of antenna arrays based on the CRLB [11] very well predicts the attainable estimation variance in specific impinging wave constellations. We have shown that a statistically efficient estimator can reach this CRLB. The comparison has been made with estimated paths in both a single and a multiple path scenario. The proposed performance measure can be used to evaluate antenna arrays on a fair basis since it is not dependent on a specific parameter estimator. It solely requires

calibration measurement data, it can easily applied to any realizable antenna array and reflects all impairments of the physical array. It can also be used to evaluate the array design if simulation data are available.
Furthermore, the proposed method can be used to determine the reachable accuracy and resolution of radio channel parameter estimates calculated from channel measurements.

5 References

[1] Thoma R.S., Hampicke D., Richter A., Sommerkorn, Trautwein U., "MIMO Vector Channel Sounder Measurement for Smart Antenna System Evaluation", European Transactions on Telecommunications ETT, Vol. 12, No. 5, Special Issue on Smart Antennas, pp. 427-438, Sept./Oct. 2001

[2] Haardt M., Thoma R.S., Richter A., "Multidimensional High-Resolution Parameter Estimation with Applications to Channel Sounding. In Yingbo Hua et al. (Ed.): *High-Resolution and Robust Signal Processing*, Marcel Dekker, 2003, p. 253-337

[3] Fleury B.H., Dahlhaus D., Heddergott R., and Tschudin M., "Wideband angle of arrival estimation using the SAGE algorithm," in Proc. IEEE Fourth Int. Symp. spread Spectrum techniques and Applications (ISSSTA'96), Mainz, Germany, pp. 79-85, Sept. 1996.

[4] Pedersen K., Fleury B., Mogensen, P. "High resolution of electromagnetic waves in time-varying radio channels". In: Proceedings from the 8th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'97), Helsinki, Finland, 1-4 Sept., 1997. Piscataway, NJ, IEEE, 1997, 650-654.

[5] Landmann M., Richter A., Thoma R.S., "Estimation Of Multidimensional Polarimetric Channel Model Parameters", COST 273 Temporary Document, TD(02) 132, Lisbon, Portugal, September 2002.

[6] Richter, A., Landmann M., Thoma, R.S., "A Gradient Based Method for Maximum Likelihood Channel Parameter Estimation from Multidimensional Channel Sounding Measurement," XXVIIth URSI General Assembly, Maastricht, NL, pp. CD, August 2002.

[7] Richter, A.; Landmann, M.; Thoma, R., "A Flexible Algorithm for Channel Parameter Estimation from Channel Sounding Measurements," COST 273 Temporary Document, Athens, Greek, No. TD(04)045, pp. 10, January 2004.

[8] Tan C. M., Landmann M., Richter A., Pesik L., Beach M. A., Schneider Ch., Thoma R. S., and Nix A. R., "On the Application of Circular Arrays in Direction Finding Part II: Experimental evaluation on SAGE with different circular arrays" 1st Annual COST 273 Workshop, Espoo, Finland, May 2002

[9] Scharf Lois L., " Statistical Signal Processing: detection, estimation, and time series analysis ", Addison-Wesley, New York, 1991, ISBN 0-201-19038-9

[10] Stoica P., Nehorai T., "Performance Study of Conditional and Unconditional DoA Estimation", IEEE Trans. On ASSP, Vol. 38, No. 10, Oct. 1990

[11] Landmann M., Richter A., Thoma R.S., „DoA Resolution Limits in MIMO Channel Sounding", International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting, Monterey, USA, June 2004,

[12] www.channelsounder.de

[13] www.irk-dresden.de