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RIMAX – A MAXIMUM LIKELIHOOD FRAMEWORK FOR PARAMETER ESTIMATION IN MULTIDIMENSIONAL CHANNEL SOUNDING

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Abstract: For application in multidimensional channel sounding a maximum likelihood parameter estimation framework (RIMAX) is proposed which allows joint high-resolution estimation of both the specular propagation paths parameters and of the dense multipath components. Depending on the available measured dimensions, the algorithm estimates the four coefficients of the polarimetric path weight matrix, the direction of arrival, the direction of departure, the time delay of arrival, and the Doppler-shift of the specular paths. Additional reliability measures are calculated to enhance the robustness of the estimate.

1. Introduction

Advanced wave propagation studies, MIMO (Multiple-Input to Multiple-Output) system performance evaluation, and double directional channel modeling [1] are based on multidimensional channel sounder application [2], [11]. The ultimate goal of subsequent data processing is to estimate the spatial channel structure in the dimensions DoA (direction of arrival at the Rx-site), DoD (direction of departure at the TX-site), TDoA (time delay of arrival), and Doppler-shift. Since resolution and accuracy of classical signal processing algorithms is limited by the available measurement aperture in the space-frequency-time domain, high-resolution parameter estimation algorithms have to be applied to enhance the resolution by fitting an appropriate data model to the measured data. Various algorithms have been proposed including the multidimensional Unitary ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) algorithm [3], and the SAGE (Space Alternating Generalized Expectation maximization) [6], [7] which is essentially a simplified Maximum Likelihood (ML) estimator. Both classes of algorithm are subjected to different model assumptions and underlying conceptual restrictions including applicability to certain antenna array architectures, calculation time in terms of convergence speed and statistical efficiency. So it is well known that ESPRIT is an unbiased DoA/DoD estimator only if the antenna arrays used for the measurements show a so called shift invariant structure (uniform linear and planar arrays ULA URA, and circular uniform beam arrays (CUBA)) [3]. The SAGE approach can be applied to a larger variety of antenna array architectures. The drawback is its slow convergence rate if two "closely spaced" propagation paths exist in the coherent multi-path propagation scenario [4].

In the sequel the data model for parameter estimation is described which also includes the dense multipath components and the influence of the measurement devices. A gradient based multidimensional ML channel parameter estimation framework is developed, and finally we present an example.

2. The Data Model

The definition of the data model is crucial for parameter estimation. It has to represent the reality of wave propagation and the influence of the measurement device. A proper choice can dramatically reduce the algorithmic complexity and enhance the reliability and resolution of the results. We prefer a data model comprising two components which can be handled separately throughout the estimation procedure. The first part is deterministic and results from specular-like reflection. The second part represents distributed diffuse scattering which typical occurs in a complicated, multipath rich environment. It is observed as dense multipath components that cannot be resolved by the measurement device. As an example: a sounder having a measurement bandwidth of 120 MHz [2], [11] gives us excellent possibilities to resolve a number of specular components. Even tough, the spatial resolution is only about 2.5 m which corresponds to 43 wave lengths at 5.2 GHz. Hence, in a "microscopic" sense we can expect quite a big number of superimposed diffused components in any delay-bin. This part is therefore adequately modeled by a complex circular normal distribution. Its contribution varies depending on

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the complexity of the propagation environment. It can be almost negligible in macrocell LOS scenarios and can even dominate in complicated propagation environments such as factory halls. In the discrete angular-delay Doppler domain the specular part is described by a superposition of *K R*-dimensional Dirac deltas weighted by a 2x2 complex polarimetric path weight matrix with its components $\gamma_{xy,k}$, where the indices x,y indicate horizontal and vertical polarization at Tx and Rx resp. The *R* dimensions are the DoD φ_T, ϑ_T (azimuth and elevation), TDoA τ, Doppler-shift α, and DoA φ_R, ϑ_R:

$$\mathbf{H}(\alpha,\tau,\varphi_R,\vartheta_R,\varphi_T,\vartheta_T)=\sum_{k=1}^{K}\begin{bmatrix}\gamma_{HH,k} & \gamma_{VH,k}\\ \gamma_{HV,k} & \gamma_{VV,k}\end{bmatrix}\delta(\alpha-\alpha_k)\,\delta(\tau-\tau_k)\,\delta(\varphi_R-\varphi_{R_k})\,\delta(\vartheta_R-\vartheta_{R_k})\,\delta(\varphi_T-\varphi_{T_k})\,\delta(\vartheta_T-\vartheta_{T_k}) \quad (1)$$

The observable channel response $\mathbf{s}(\boldsymbol{\theta}_k)$ in the multidimensional aperture domain is defined by the limited observation time, finite bandwidth, and finite (effective) antenna apertures. $\boldsymbol{\theta}_k$ is the condensed propagation path parameter vector containing 14 real-valued unknowns. We arrange the sampled channel response in vectors as $\mathbf{a}(\boldsymbol{\mu}_k)=\mathbf{a}(\mu_k^{(R)})\otimes\mathbf{a}(\mu_k^{(R-1)})\otimes\ldots\otimes\mathbf{a}(\mu_k^{(1)})$, whereby the $\mathbf{a}(\mu_k^{(i)})$ are complex exponentials resulting from Fourier transform of (1) and the $\mu_k^{(i)}$ are normalized path parameters [9]:

$$\mathbf{s}(\boldsymbol{\theta}_k)=\gamma_{HH,k}\cdot\mathbf{G}_{HH}\cdot\mathbf{a}(\boldsymbol{\mu}_k)+\gamma_{HV,k}\cdot\mathbf{G}_{HV}\cdot\mathbf{a}(\boldsymbol{\mu}_k)+\gamma_{VH,k}\cdot\mathbf{G}_{VH}\cdot\mathbf{a}(\boldsymbol{\mu}_k)+\gamma_{VV,k}\cdot\mathbf{G}_{VV}\cdot\mathbf{a}(\boldsymbol{\mu}_k) \quad (2)$$

The linear projector matrices $\mathbf{G}_{xy}$ describe the measurement systems response which is composed by the Kronecker product of the frequency, Doppler and spatial responses, respectively. Whereas the frequency response is represented by a diagonal matrix and the Doppler response is simply an identity matrix, the spatial response is described by the effective aperture distribution function (EDAF) of the antenna arrays. The EADF has been found to be a very powerful method to describe the antenna array behavior for calibration purposes [10]. It is calculated by a 2D-Fourier transform and subsequent data compression from the precisely measured complex polarimetric beam patterns.

Resulting from many observations of measured channel responses an exponential decaying data model was defined to represent the dense multipath components in the delay (correlation) domain $\psi(\tau)$ with its corresponding frequency response $\Psi(f)$ [8]. The parameter vector $\boldsymbol{\theta}_{dds}$ is composed of the parameters β_d, τ_d, α_1 which are the normalized coherence bandwidth, base delay and maximum power respectively. Note that due to the limited observation bandwidth, a distortion of this response will be observed in the delay domain:

$$\psi_x(\tau)=\mathrm{E}\left\{|x(\tau)|^2\right\}=\begin{Bmatrix}0 & \tau<\tau_d\\ \alpha_1\cdot\frac{1}{2} & \tau=\tau_d\\ \alpha_1\cdot\mathrm{e}^{-B_d(\tau-\tau_d)} & \tau>\tau_d\end{Bmatrix}\quad \circ\!\!-\!\!\bullet \quad \psi_x(f)=\frac{\alpha_1}{\beta_d+\mathrm{j}2\pi f}\cdot\mathbf{e}^{-\mathrm{j}2\pi f\tau_d} \quad (3)$$

3. Maximum Likelihood Parameter Estimation

With the stationary measurement noise **n** and the dense multipath and specular components **d** and **s** resp. the total observed signal vector **x** is modeled as follows:

$$\mathbf{x}=\mathbf{n}+\mathbf{d}(\boldsymbol{\theta}_{dds})+\sum_{k=1}^{K}\mathbf{s}(\boldsymbol{\theta}_k)=\mathbf{n}+\mathbf{d}(\boldsymbol{\theta}_{dds})+\mathbf{s}(\boldsymbol{\theta}_{sp}) \quad (4)$$

having a conditional probability density of:

$$pdf(\mathbf{x}|\boldsymbol{\theta}_{sp},\boldsymbol{\theta}_{dds})=\frac{1}{\boldsymbol{\pi}^M\det(\mathbf{R}(\boldsymbol{\theta}_{dds}))}\mathbf{e}^{-(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))^H\mathbf{R}(\boldsymbol{\theta}_{dds})^{-1}\cdot(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))}. \quad (5)$$

The related log-likelihood function is:

$$L(\mathbf{x};\boldsymbol{\theta}_{sp},\boldsymbol{\theta}_{dds})=-M\cdot\ln(\pi)-\ln(\det(\mathbf{R}(\boldsymbol{\theta}_{dds})))-(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp}))^H\cdot\mathbf{R}(\boldsymbol{\theta}_{dds})^{-1}\cdot(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}_{sp})) \quad (6)$$

Because of the Gaussian nature of the probability density, the maximization of (6) in essence is a nonlinear least squares problem. Since an exhaustive search in the multidimensional parameter space is not feasible, we are proposing an iterative search framework. This procedure proceeds snapshot by snapshot and takes advantage as much as possible from typical channel behavior which is known a-priori from propagation physics and from experimental experience. So the estimated parameter set of every snapshot is taken as the initial estimate for the next one. This is of specific importance for the $\boldsymbol{\theta}_{dds}$ vector since the statistic parameters of the dense multipath changes only slowly as long as the "average environment" does not change completely. The *K* parameter vectors $\boldsymbol{\theta}_k$, on the other hand, change much faster since they directly comprise geometric parameters. Moreover, existing paths can temporarily or definitely disappear and new paths can suddenly show up. So we have not only to track existing paths but as well have to search for new paths. This also changes the model order *K*, which is adaptively controlled throughout the sequence of snapshots. A considerable simplification of the search procedure is possible according to the expectation maximization (EM) principle if the parameters are independent in its influence. The strongest simplification leads to the space alternating generalized expectation maximization algorithm (SAGE) [6], [7]. Since the parameter sets $\boldsymbol{\theta}_{sp}$, and $\boldsymbol{\theta}_{dds}$ are obviously independent, we can use alternating search procedures to maximize (6) with respect to $\boldsymbol{\theta}_{sp}$, and $\boldsymbol{\theta}_{dds}$. Which means that we successively remove the estimated deterministic specular paths from the observed data. For estimation of the parameters $\boldsymbol{\theta}_{dds}$, a Gauss-Newton algorithm is applied. This gives us also a parametric representation of the covariance matrix $\mathbf{R}(\boldsymbol{\theta}_{dds})$. The knowledge of $\mathbf{R}(\boldsymbol{\theta}_{dds})$ is necessary for the estimation of specular parameters $\boldsymbol{\theta}_{sp}$ since it provides appropriate weighting of the observed data in the nonlinear weighted least squares problem:

$$\hat{\boldsymbol{\theta}}_{sp} = \arg\min_{\boldsymbol{\theta}_{sp}} \left(\mathbf{x} - \mathbf{s}\left(\hat{\boldsymbol{\theta}}_{sp}\right)\right)^{H} \cdot \mathbf{R}\left(\hat{\boldsymbol{\theta}}_{dds}\right)^{-1} \cdot \left(\mathbf{x} - \mathbf{s}\left(\boldsymbol{\theta}_{sp}\right)\right). \quad (7)$$

The global search for new paths (which has to be carried not only at the beginning of the sequence but continuously step by step) is also carried out by a SAGE-like procedure. Rather than a random assumption for unknown parameters we use some kind of non-coherent combining of observations to reduce the parameter dimension. To explain the strategy, let us discuss an example. Suppose the channel impulse response has been measured using a 10 element ULA at one link end. At first we treat the 10 individual channel impulse responses as independent realizations of the same process and maximize the log-likelihood function with respect to the time delay. In the next step we keep the estimated time delay fixed and maximize for DoA. Although this reduces the maximization problem to two concatenated one dimensional problems, assumption on the unknown DoA is required in the first step. This kind of noncoherent handling of data dimensions related to unknown parameters (e.g. DoA) gives us a higher probability to detect the relevant parameters which is the time delay in our example. As opposed to this, any arbitrary assumption of the DoA in the example would imply coherent combining which potentially disregards paths impinging from other angles by beamforming.

The problem of local search is completely different. We have found that in case of closely spaced coherent paths the coordinate-wise search strategy of SAGE has serious disadvantages because of its slow convergence rate which is not only time-consuming but may also end in erroneous estimates when using a quantized parameter data base [4]. On the other hand, it is well known that the ML function is, under mild restrictions, quadratic at its maximum (in the local “attractor area”). Therefore a conjugate gradient search promises much better convergence performance when the parameters are coupled in its influence to the minimization of (7) [5]. From the variety of available procedures for nonlinear optimization we have decided to use Levenberg-Marquardt because of its robustness. To calculate the optimum step size and direction for parameter change these algorithms require the gradient, the Jacobian and the Hessian of the log-likelihood function at the actual point in the parameter space. Fortunately enough, with the algebraic data model defined by (2) the derivatives are easily available. This is especially true for the EADF model of the antenna arrays. The approximation of the Hessian as its is used in the Gauss-Newton / Levenberg-Marquardt algorithm is essentially an estimate of the Fisher information matrix (FIM). This provides us with information on the variance and on the interdependency of the parameter estimates. The variance estimate helps to evaluate the reliability of the parameters and is used to accept or drop estimated paths. The two major sources of excessive variance are line splitting and noise enhancement.

The example in Fig. 1 shows the estimation results in the delay domain (power delay profile, pdp). The specular path weight magnitudes are indicated by blue dots. The reconstruction of the pdp within the measurement bandwidth is given by the blue curve. The green curve is the difference between the reconstructed and the measured pdp, thus it is an instantaneous realization of the dense multipath (dds). The expectation of the same part (which is estimated from the data) is given by the red triangular curve. The vertical red lines indicate the relative variance of the specular path weight estimates as it is calculated from the FIM. Most reliable paths are indicated by a variance contribution that directly follows the “dds” slope. Noise enhancement is indicated by red points above the dds slope (see, eg at 2800 ns). The outliers around 3050 and 3350 ns are caused by line splitting, which is characterized by two very closely spaced, excessively strong paths with opposite signs. There is clear evidence of a wrong estimate by the relative variance >1. As a consequence, one of those paths has to be omitted which will further lead to accurate estimate of the remaining path.

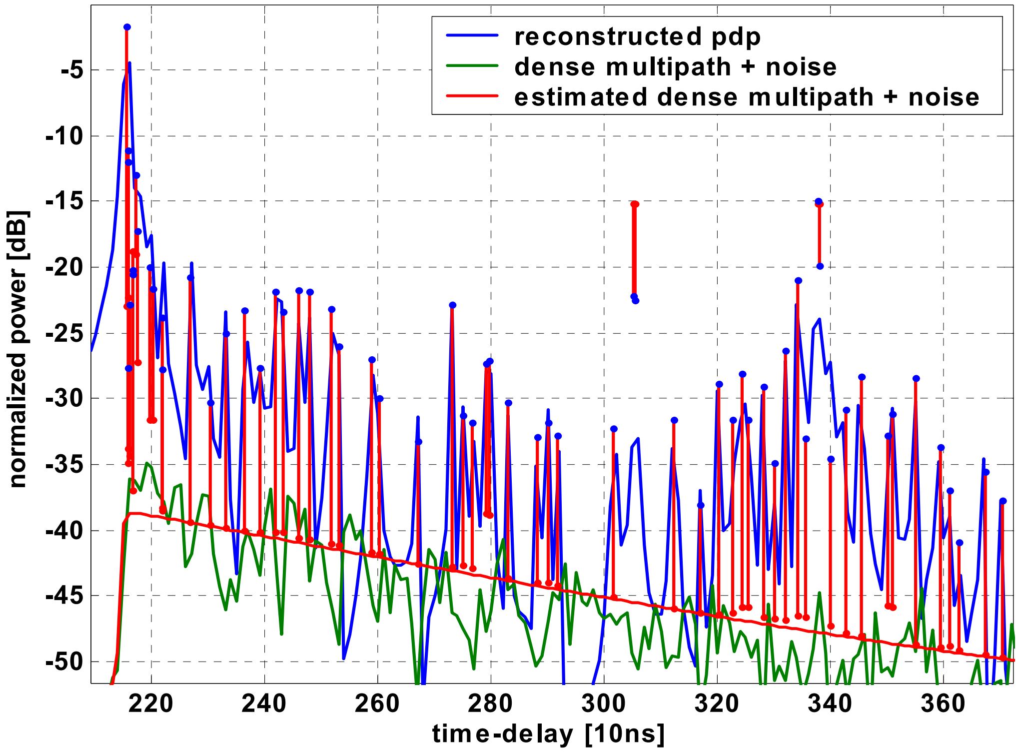

Fig. 1: Measurement example and parameter estimates

4. Conclusions

- We proposed a ML channel parameter estimation framework (RIMAX) for multidimensional channel sounding which comprises estimation of the multidimensional specular paths parameters and of the delay distribution of dense multi-path contributions which clearly outperforms existing procedures in terms of applicability, flexibility, robustness and convergence. The estimator provides additional reliability information on the estimated parameters and uses a novel general antenna array model, the so called EADF function. Further enhancements may include enhanced path tracking and directional respectively spatial modeling of dense multipath components.

5. References

[1] M. Steinbauer, A.F. Molisch, E. Bonek., “The Double-Directional Radio Channel”, IEEE Antennas and Propagation Magazine, vol. 45, pp. 51-63, No.4, Aug. 2001

[2] R.S. Thoma, D. Hampicke, A. Richter, G. Sommerkorn, U. Trautwein, “MIMO Vector Channel Sounder Measurement for Smart Antenna System Evaluation”, European Transactions on Telecommunications ETT, Vol. 12, No. 5, Special Issue on Smart Antennas, pp. 427-438, Sept./Oct. 2001

[3] M. Haardt., R.S. Thoma, A. Richter, “Multidimensional High-Resolution Parameter Estimation with Applications to Channel Sounding. In Yingbo Hua et al. (Ed.): *High-Resolution and Robust Signal Processing*, Marcel Dekker, 2003, p. 253-337

[4] A. Richter, M. Landmann, R.S. Thoma, “Maximum Likelihood Channel Parameter Estimation from Multidimensional Channel Sounding Measurements,” Proc. IEEE VTC2003-Spring, Jeju, Korea, April 2003

[5] A. Richter, M. Landmann, R.S. Thoma, “A Gradient Based Method for Maximum Likelihood Channel Parameter Estimation from Multidimensional Channel Sounding Measurement,” XXVIIth URSI General Assembly, Maastricht, NL, pp. CD, August 2002.

[6] J.A. Fessler, A.O. Hero, “Space Alternating Generalized Expectation Maximization Algorithm,” IEEE Trans. on Signal Processing, Vol. 42, pp. 2664-2677, October 1994.

[7] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. Ingeman Pedersen, “Channel Parameter Estimation in Mobile Radio Environment Using the SAGE Algorithm,” IEEE Journal on Selected Areas in Communications, Vol. 17, No. 3, pp. 434-450, March 1999.

[8] A. Richter, R. S. Thoma,” Parametric Modeling and Estimation of Distributed Diffuse Scattering Components of Radio Channels”, COST273, TD(03)198, Sept. 24-26, 2003, Prague, http://www.lx.it.pt/cost273/

[9] A.Richter, M. Landmann, R.S. Thoma,” RIMAX-A Flexible Algorithm for Channel Parameter Estimation from Channel Sounding Measurements”, COST273, TD(04)045, Jan. 26-28, 2004, Athens, http://www.lx.it.pt/cost273/

[10] M. Landmann, A. Richter, R.S. Thoma „DoA Resolution Limits in MIMO Channel Sounding”, International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting, Monterey, USA, June 2004

[11] www.channelsounder.de