Proceedings of ISAP'04, Sendai, JAPAN

An Application of Extended Array Mode Vector to ISI-SAGE

Kriangsak Sivasondhivat and Jun-ichi TAKADA
Tokyo Institute of Technology 2–12–1, O-okayama, Meguro-ku, Tokyo 152–8552
E–mail:{joe,takada}@ap.ide.titech.ac.jp

1.Introduction

Recently ISI-SAGE (Initialization and Searching Improved-Space Alternating Generalized Expectation Maximization) has been proposed to jointly estimate channel parameters in mobile environments [1]. Since its introduction, the application of ISI-SAGE has been widely reported by many researchers, especially in the channel sounding activity.

However, as will be shown in the subsequent section, the direction of arrival (DOA) estimation of each wave in ISI-SAGE is founded on beamforming principles, hence coherent waves generated from a slightly distributed source cannot be separated if the angular spread (AS) of that source seen by an antenna array is smaller than the 3dB beamwidth of the antenna array. In this case, the signal model of the ISI-SAGE depending on a single plane wave model is not reasonable and should be reconsidered. As a consequence of model error, the accuracy of estimating the nominal direction of such a source is degraded. More importantly, the residual power from waves estimated and cancelled earlier, but not perfectly due to model errors, cause multiple sidelobes, which could be detected as different waves in some future iterations of the estimation.

In this paper, we propose an application of the extended array mode vector (AMV) based on the first-order Taylor series expansion to the ISI-SAGE algorithm with the aim of improving the accuracy of estimating a nominal DOA and alleviating the problem of detecting spurious waves when ASs are less than the resolution of the array antenna. Moreover, since the first-order approximation becomes invalid as the AS becomes wider, we also extend the AMV by the second-order approximation and apply it to the ISI-SAGE.

The paper is organized as follows. In Section 2, the preliminary data model is formulated and followed by a brief overview of the ISI-SAGE. Next, the first-order approximation of the AMV is explained in Section 3. In Section 4, by using second-order approximation, the more accurate ISI-SAGE is enlightened, and simulation results are carried out to verify the performance of the ISI-SAGE based on our proposed extended AMV in Section 5.

2.Data Model and ISI-SAGE Algorithm

A. Data Model

From here onwards, we assume the received signal can be mathematically expressed as the superposition of K waves from different K clusters. The kth cluster is possibly further decomposed as the sum of many plane waves from local scatterers around the center of the kth cluster. Furthermore, for simplicity, all K waves are presumably undistinguishable by sampling in other parameter dimensions, except in DOA in the azimuth direction. Thus, our problem can be considered to be equivalent to the problem of estimating DOAs of narrowband signals by a uniform linear array (ULA). Note that the received signal is assumed to be collected in the observation time less than the coherence time of the channel, so that the channel is approximately invariant. With these assumptions, a measurement of the M-ULA output $\mathbf{x}(t)$, which is corrupted by additive noise, can be written as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{v}_k s_k(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{v}_k$ is the spatial signature (SS) of the kth signal, whose size is $M \times 1$, and $s_k(t)$ denotes the baseband signal from the kth cluster. Given the nominal DOA $\theta_{0,k}$ and the angle deviation

ISBN: 4-88552-206-4 C3055©IEICE

from the nominal DOA of the dth scattered multipath $\tilde{\theta}_{d,k}$, the kth SS from D_k scatterings takes the form

$$\mathbf{v}_k = \sum_{d=1}^{D_k} \beta_{d,k} \mathbf{a}\left(\theta_{0,k} + \tilde{\theta}_{d,k}\right), \tag{2}$$

where the complex gain of each scattered wave is denoted by $\beta_{d,k}$. $\mathbf{a}(\theta)$ is the AMV of ULA in direction θ. It has been known that the form in $\mathbf{v}_k$ cannot be represented by any single DOA in the form of the AMV $\mathbf{a}(\theta)$.

B. ISI-SAGE Algorithm

The framework of ISI-SAGE hinges on expectation maximization (EM) consisting of the expectation step and maximization step. In the expectation step, the complete data for the kth wave is estimated by using estimated parameters from the maximization step maximizing loglikelihood function of the measured data. According to our assumptions and the Successive Interference Cancellation (SIC) method, in which the parameters of waves are estimated successively, the maximization step for the DOA θ_k and the complex gain of the kth cluster γ_k is carried out as

$$\hat{\theta}_k = \arg\max_{\theta} |E[\mathbf{a}^H(\theta)\,\hat{\mathbf{x}}_k]|, \tag{3}$$

$$\hat{\gamma}_k = \frac{E[\mathbf{a}^H(\theta)\,\hat{\mathbf{x}}_k]}{M}, \tag{4}$$

with $\hat{\mathbf{x}}_k$ denoting the estimated signal for the kth wave by canceling previously estimated waves depending on the estimated DOAs. As will be shown later, the model mismatch due to the use of $\mathbf{a}(\theta)$ causes the error in estimating nominal DOA and results in spurious waves.

3.The Extension of ISI-SAGE by the First-Order Approximation

The first-order Taylor series expansion has been applied to the signal model of MUSIC for estimating a wave with AS caused by local scatterings near a mobile [2, 3]. Assuming that the AS of each kth wave $\Delta\theta_k$ is small, the first order approximation of the Taylor expansion around $\theta_{0,k}$ can be used, and each $\mathbf{a}\left(\theta_{0,k} + \tilde{\theta}_{d,k}\right)$ can be decomposed as $\mathbf{a}\left(\theta_{0,k} + \tilde{\theta}_{d,k}\right) \cong \mathbf{a}(\theta_{0,k}) + \tilde{\theta}_{d,k}\mathbf{d}(\theta_{0,k})$, where $\mathbf{d}(\theta_{0,k}) = \frac{\partial \mathbf{a}(\theta)}{\partial \theta}\Big|_{\theta=\theta_{0,k}}$. Substituting this into Eq. (2), then

$$\mathbf{v}_k = \gamma_k \left\{\mathbf{a}(\theta_{0,k}) + \xi_k \mathbf{d}(\theta_{0,k})\right\}, \tag{5}$$

where, $\gamma_k = \sum_{d=1}^{D_k} \beta_{d,k}, \xi_k = \xi_{\mathrm{r}k} + \jmath\xi_{\mathrm{i}k} = \frac{\sum_{d=1}^{D_k} \beta_{d,k}\tilde{\theta}_{d,k}}{\gamma_k}$. In Rayleigh fading channel, we assume that $\beta_{d,k}$'s are independent and identically distributed (iid) with the mean $E[\beta_{d,k}]$ is zero. In addition, if $\beta_{d,k}$'s are assumed to be independent of directions, and D_k is large enough so that the central limit theorem (CLT) is satisfied, γ_k is reasonably to be approximated as a complex Gaussian random variable. Finally, $\mathbf{v}_k$ can be further simplified as

$$\mathbf{v}_k \cong \gamma_k \mathbf{a}\left((\theta_{0,k} + \xi_{\mathrm{r}k}) + \jmath\xi_{\mathrm{i}k}\right). \tag{6}$$

Here, we extend the maximization step in Eq. (3) to a more general AMV by using $\mathbf{a}\left((\theta + \xi_{\mathrm{r}}) + \jmath\xi_{\mathrm{i}}\right)$ in the above form of $\mathbf{v}$ instead of $\mathbf{a}(\theta)$. To reduce the number of parameters to be estimated, $\theta + \xi_r$ can be jointly estimated as one parameter ζ, i.e. $\mathbf{a}\left((\theta + \xi_{\mathrm{r}}) + \jmath\xi_{\mathrm{i}}\right) = \mathbf{a}(\zeta + \jmath\xi_{\mathrm{i}})$. Altogether, comparing with the standard ISI-SAGE, the number of parameters to be estimated increases by only one parameter, that is $\xi_{\mathrm{i}k}$. As will be shown via simulations, while the first-order approximation improves the performance of ISI-SAGE to some extent, it is unsuccessful in representing the signal model when AS becomes wider, but still by far narrower than the 3dB beamwidth.

4. More Accurate ISI-SAGE Based on the Second-Order Approximation

In this section, the second-order analysis is carried out. For the case that the contribution of the second-order Taylor expansion cannot be neglected, $\mathbf{a}\left(\theta_{0,k}+\tilde{\theta}_{d,k}\right)$ of the kth cluster should be modeled as

$$\mathbf{a}\left(\theta_{0,k}+\tilde{\theta}_{d,k}\right) \cong \mathbf{a}\left(\theta_{0,k}\right)+\tilde{\theta}_{d,k}\mathbf{d}\left(\theta_{0,k}\right)+\frac{\tilde{\theta}_{d,k}^{2}}{2}\mathbf{f}\left(\theta_{0,k}\right). \tag{7}$$

Then,

$$\mathbf{v}_k=\gamma_k\left\{\mathbf{a}\left(\theta_{0,k}\right)+\xi_k\mathbf{d}\left(\theta_{0,k}\right)+\eta_k\mathbf{f}\left(\theta_{0,k}\right)\right\}, \tag{8}$$

where $\mathbf{f}\left(\theta_{0,k}\right)=\frac{\partial^2\mathbf{a}(\theta)}{\partial\theta^2}\Big|_{\theta=\theta_{0,k}}$, and $\eta_k=\eta_{\mathrm{r}k}+\jmath\eta_{\mathrm{i}k}=\frac{\sum_{d=1}^{D_k}\beta_{d,k}\tilde{\theta}_{d,k}^{2}}{2\gamma_k}$. We define

$$\mathbf{g}\left(\theta,\xi,\eta\right)=\left\{\mathbf{a}\left(\theta\right)+\xi\mathbf{d}\left(\theta\right)+\eta\mathbf{f}\left(\theta\right)\right\}. \tag{9}$$

The cost-function of the maximization step for this case of the AMV can be expressed in terms of $\mathbf{g}\left(\theta,\xi,\eta\right)$. It can be seen that the number of parameters including real and imaginary parts of ξ_k and η_k for the second-order approximation case is 5, which is a rather large number and may increase computation time. However, considering the need for accurate characterization of the channel, the second-order approximation approach is certainly worthy of consideration.

5.Simulations

A. Nominal DOA Estimation

To simulate the performance of AMVs mentioned above, we consider the case where one wave with $D_1=30$, which distribute uniformly over the interval $[-\Delta\theta_1/2,\Delta\theta_1/2]$ impinges on ULA, whose M and the antenna spacing are 6 elements and 0.487 wavelength, respectively. The nominal DOA is fixed at 0°, and the average signal to noise ratio (SNR) is 20 dB. For this condition, the 3dB bandwidth of the array antenna is 20.62 degrees. In Fig. 1, the mean error of DOA estimates from 2000 simulations of each AMV are plotted in terms of $\Delta\theta_1$, which is varied from 1 to 10 degrees. From the figure, considering the results from the ISI-SAGE using the first-order approximation, the accuracy of the nominal DOA estimation is somewhat improved. This is possibly due to the effect of the simplification in Eq. (6). In the case of the second-order approximation, a great improvement can be easily observed. It means that the AMV in the presence of AS can be well approximated by extending the AMV to the second-order of the Taylor series.

B. Problem of Detecting Spurious Waves

Next, we assume that a second wave with $\Delta\theta_2=0°$ is present in the direction of $-35°$ in addition to the first wave as mentioned above, except that the SNR is 40 dB. For the sake of simplicity, the power of the second wave is set to be 20 dB below that of the first wave, so that the spurious waves could be detected by the algorithms before the second wave. To evaluate the performance on reducing the occurrence of spurious waves, we observe the DOA estimates of the second wave by using 20 simulations. If the estimated DOA is between $-40°$ and $-30°$, it is considered to be of the second wave, and no spurious wave is detected. Figures 2, 3 and 4 show the DOA estimates of the second wave using the standard, the first and second-order approximated AMVs in the ISI-SAGE, respectively. Two cases of $\Delta\theta_1=1°$ and $\Delta\theta_1=6°$ are investigated in each figure. From the figures, the percentages of correction estimation based on the criterion mentioned earlier are summarized in the Table 1. It can be seen that with the first and the second-order approximations of AMV, the ISI-SAGE can efficiently suppress the presence of spurious waves as compared with the standard ISI-SAGE. Moreover, the ISI-SAGE using the second-order approximation can successfully work even when the AS is wide, in which the first-order approximation cannot give satisfactory results.

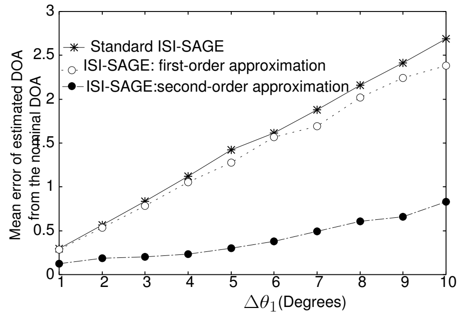

Figure 1: Mean error of estimated DOA from the nominal DOA.

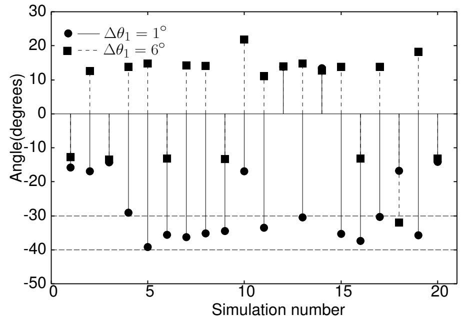

Figure 2: Estimated DOA of the second wave by using the standard ISI-SAGE.

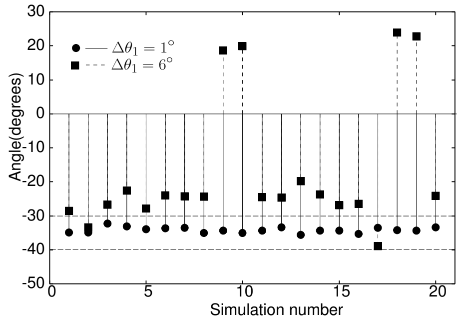

Figure 3: Estimated DOA of the second wave by using the ISI-SAGE using the first-order approximation.

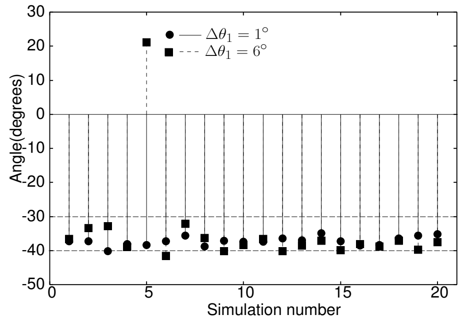

Figure 4: Estimated DOA of the second wave by using the ISI-SAGE using the second-order approximation.

6.Conclusions

In this paper, we proposed the extended AMVs based on the first and the second-order approximations for the ISI-SAGE algorithm. According to numerical simulations, the results clearly demonstrate that our proposed approach can be effectively used for DOA estimation in real mobile scenarios in which a wave with AS arrives at an array antenna.

Table 1: Percentages of correction estimation

	$\Delta_1 = 1°$	$\Delta_1 = 6°$
Standard ISI-SAGE	55 %	5 %
ISI-SAGE using the first-order approximation	100 %	10 %
ISI-SAGE using the second-order approximation	100 %	90 %

References

[1] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. I. Pedersen, Proc. 2002 Spring IEEE Veh. Tech. Conf. (VTC), May 2002.

[2] J. S. Jeong, K. Sakaguchi, K. Araki, and J. Takada, IEICE Trans. Communs. Vol. E84-B, No. 7, pp. 1781-1789, July 2001.

[3] D. Asztély and B. Ottersten, IEEE Trans. Signal Processing. Vol. 47, No. 12, pp. 3220-3234, Dec. 1999.