

# ON IMPROVING CYCLIC ESPRIT FOR SIGNAL-SELECTIVE DOA ESTIMATION

Yoshiyuki INAGAKI      Nobuyoshi KIKUMA      Kunio SAKAKIBARA

Hiroshi HIRAYAMA

Department of Computer Science and Engineering

Nagoya Institute of Technology

Gokiso-cho, Showa-ku, Nagoya 466-8555, Japan

E-mail: yoshiina@luna.elcom.nitech.ac.jp

## 1 Introduction

The mobile radio propagation is much complicated because of interference and multipath waves deteriorating seriously the quality of radio communications. In order to understand the mobile radio propagation structures and also consider the signal recovering techniques in the mobile environments, it is most effective to estimate the DOAs (directions of arrival) of the individual incoming waves. Furthermore, utilizing the estimated DOA information in the adaptive array, we can form easily optimum beam patterns of antenna arrays[1].

Currently, MUSIC and ESPRIT algorithms are much popular because of their high angular resolution[1][2]. They require that the number of antennas should be more than the number of incident waves, and therefore they have to use the large number of antennas in complicated radio environments including both desired signals and interference.

In this paper, we focus on Cyclic ESPRIT which is one of the signal-selective DOA estimators using the cyclostationarity[3][4]. Although ESPRIT exploits the signal subspace of correlation matrix, we try to use the noise subspace to improve the Cyclic ESPRIT because the correlation matrix produced in Cyclic ESPRIT is generally not Hermitian. Via computer simulation, the performance of the proposed Cyclic ESPRIT will be clarified in comparison with the conventional one.

## 2 Principle of Signal-Selective DOA Estimation

### 2.1 Cyclostationarity and Cyclic ESPRIT

Most of digital signals used mobile communications have cyclostationarity which is the property that the cyclic auto-correlation function of signal  $s(t)$  defined by

$$R_{ss}^{\alpha}(\tau) \triangleq \langle s(t + \tau/2) \{s(t - \tau/2)e^{j2\pi\alpha t}\}^* \rangle \quad (1)$$

is not identically zero for some cycle frequency  $\alpha$  and some lag parameter  $\tau$ [3][4]. Here  $\langle \cdot \rangle$  denotes the finite time average operator. The cycle frequencies are normally determined by carrier frequency or symbol rate, depending on modulation schemes.

Consider a uniform linear array (ULA) having  $K$  antenna elements which receives  $L$  desired signals  $s_i(t) (i = 1, \dots, L)$  that have a particular cycle frequency  $\alpha$ . Those signals are supposed to be uncorrelated with each other and incident from different directions:  $\theta_i (i = 1, \dots, L)$ . Although an arbitrary number of interfering signals and arbitrary noise are included in the received signals, it is assumed that they do not have cyclic correlation at frequency shift  $\alpha$ . Then, vector notation of the array received signals is given by

$$\mathbf{x}(t) = \mathbf{A}s(t) + \mathbf{n}(t) \quad (2)$$

$$\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_L)] \quad (3)$$

$$\mathbf{s}(t) = [s_1(t), \dots, s_L(t)]^T \quad (4)$$

where  $\mathbf{a}(\theta_i)$  is an array response vector to the direction  $\theta_i$ , and  $\mathbf{n}(t)$  is a vector consisting of the interfering signals and noise. In this paper,  $\mathbf{a}(\theta_i)$  is called direction vector.

The cyclic auto-correlation matrix of  $\mathbf{x}(t)$  is given by

$$\mathbf{R}_{xx}^\alpha(\tau) \triangleq \langle \mathbf{x}(t + \tau/2)\mathbf{x}^H(t - \tau/2)e^{-j2\pi\alpha t} \rangle \quad (5)$$

$$= \mathbf{A}\mathbf{R}_{ss}^\alpha(\tau)\mathbf{A}^H \quad (6)$$

$$\mathbf{R}_{ss}^\alpha(\tau) \triangleq \langle \mathbf{s}(t)\mathbf{s}^H(t)e^{-j2\pi\alpha t} \rangle \quad (7)$$

Applying TLS-ESPRIT algorithm[1][2] to the matrix  $\mathbf{R}_{xx}^\alpha(\tau)$ , we can obtain the DOA estimates of  $L$  desired signals. This is called Cyclic ESPRIT, and it will be compared with the proposed method explained below.

## 2.2 Cyclic ESPRIT Using Noise Subspace

Since the auto-correlation matrix  $\mathbf{R}_{xx}^\alpha(\tau)$  in Cyclic ESPRIT is generally not Hermitian, there might be some degradation in DOA estimates. In spite of being able to rely on SVD (singularvalue decomposition) to get a signal subspace matrix from  $\mathbf{R}_{xx}^\alpha(\tau)$ , we try to reconstruct the signal subspace matrix from noise subspace of  $\mathbf{R}_{xx}^\alpha(\tau)$ . Obviously, rank of  $\mathbf{R}_{xx}^\alpha(\tau)$  is  $L$  because both  $\mathbf{R}_{ss}^\alpha(\tau)$  and  $\mathbf{A}$  have full rank equal to  $L$ . Let  $\mathbf{E}_N$  be a noise subspace matrix composed of eigenvectors corresponding to  $K - L$  zero eigenvalues of  $\mathbf{R}_{xx}^\alpha(\tau)$ . Then, we have following relationship.

$$\mathbf{R}_{xx}^\alpha(\tau)\mathbf{E}_N = \mathbf{0} \quad \text{or} \quad \mathbf{E}_N^H\mathbf{a}(\theta_i) = 0 \quad (i = 1, \dots, L) \quad (8)$$

Therefore, Hermitian matrix  $\mathbf{E}_N\mathbf{E}_N^H$  has  $L$ -dimensional null space which is orthogonal to  $\mathbf{E}_N$ . In other words, the  $L$ -dimensional null space that is denoted by  $\mathbf{E}_S$  spans the same space as the direction vectors  $\mathbf{a}(\theta_i)(i = 1, \dots, L)$ . This leads consequently to

$$\mathbf{E}_S = \mathbf{A}\mathbf{T} \quad (9)$$

where  $\mathbf{T}$  is an  $L$ -dimensional non-singular matrix. Equation (9) means that  $\mathbf{E}_S$  can be used as the signal subspace matrix for TLS-ESPRIT.

## 3 Computer Simulation

In this section, computer simulation is carried out to examine the performance of proposed Cyclic ESPRIT. The simulation conditions we used are described in Table 1, and the radio environment is given in Table 2. There are two multipath waves of the desired signal. The estimation accuracy is quantified by root-mean-square error (RMSE) of 100 independent estimation trials.

Figure 1 shows convergence characteristics of RMSE of DOA estimates of 1st desired signal when the interference is not incident. For RMSE to be less than  $0.5^\circ$ , the conventional method requires 12 snapshots, while the proposed one needs only 6 snapshots.

Next, we investigate the effect of input SNR on RMSE of DOA estimate of 1st desired signal. The results in the case of no interference are plotted in Fig. 2. Those when the interference is incident are depicted in Fig. 3. In both figures, the number of snapshots is 100. From Fig. 2, we can see that the proposed method has almost same estimation accuracy as the conventional method, although the former is slightly better than the latter. On the other hand, when the interference is incident on the array, the difference between the two methods is remarkable as seen from Fig. 3. Since the number of snapshots is 100, the influence of interference remains in the cyclic auto-correlation matrix and so both methods have floor in RMSE for high SNR. However, the proposed method reveals significantly better performance than the conventional one.

## 4 Conclusion

In this paper, we have improved Cyclic ESPRIT that features carrying out signal-selective DOA estimation. The cyclic auto-correlation matrix exploited in Cyclic ESPRIT is generally not Hermitian, and so the direct use of signal subspace eigenvectors in TLS-ESPRIT procedure yields degradation of DOA estimation. The proposed method reconstructs the signal subspace from noise subspace. Simulation results have shown that the proposed method outperforms the conventional one particularly when the interference is incident. In addition, it is noted that the proposed one requires less number of snapshots than the conventional one to attain the same estimation accuracy.

## References

- [1] N.Kikuma: *Adaptive Antenna Technologies (in Japanese)* , Ohmsha, Inc., 2003.
- [2] S.U.Pillai: *Array Signal Processing*, Springer-Verlag New York Inc., 1989.
- [3] W.A.Gardner: “Simplification of MUSIC and ESPRIT by Exploitation of Cyclostationarity,” Proceedings of the IEEE, Vol.76, No.7, pp.845–847, July 1988.
- [4] S.V.Schell, R.A.Calabreta, W.A.Gardner, and B.G.Agee: “Cyclic MUSIC Algorithms for Signal-Selective Direction Estimation,” Proceedings of IEEE ICASSP, pp.2278–2281, May 1989.

Table 1: Simulation conditions.

Carrier frequency	2.0[GHz]
Number of elements	4
Element spacing	Half wavelength
Antenna element	Isotropic antenna
Input SNR	20[dB]

Table 2: Radio environment.

	Desired signals		Interference
	1st wave	2nd wave	
Modulation scheme	QPSK	QPSK	QPSK
DOA	20°	-40°	-60°
Power [dB]	0	-3	-3
Symbol rate [Msps]	2.0	2.0	1.5

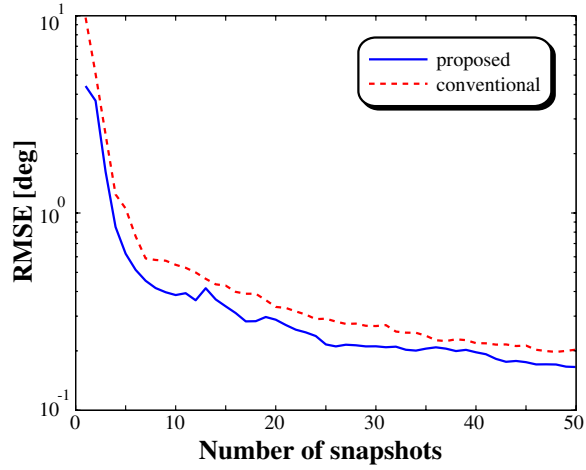


Figure 1: Convergence characteristics of RMSE of DOA estimates of 1st desired signal in the case of no interference.

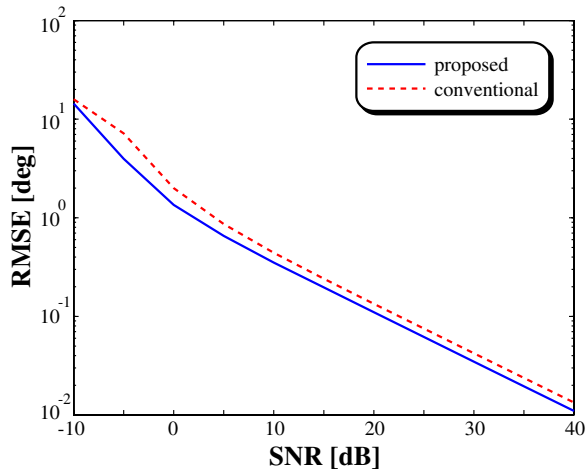


Figure 2: RMSE of DOA estimates of 1st desired signal versus input SNR in the case of no interference.

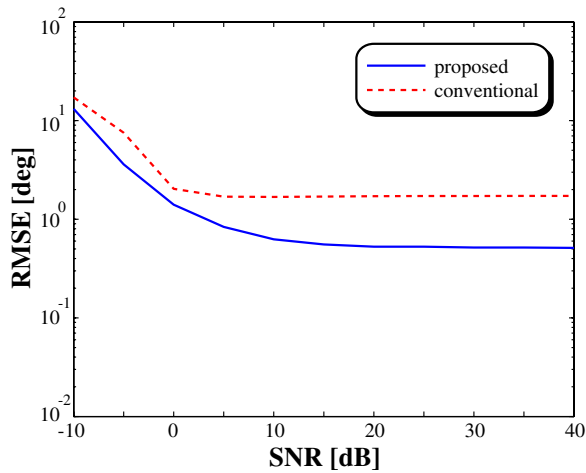


Figure 3: RMSE of DOA estimates of 1st desired signal versus input SNR in the case of interference incident.