1A4-3

Proceedings of ISAP'04, Sendai, JAPAN

PERFORMANCE ANALYSIS OF RECURSIVE UNITARY MUSIC ALGORITHM FOR ITERATIVE DOA ESTIMATION

Ayib Rosdi ZAINUN Nobuyoshi KIKUMA Kunio SAKAKIBARA
Hiroshi HIRAYAMA
Department of Computer Science and Engineering, Nagoya Institute of Technology
Gokiso-cho, Showa-ku, Nagoya 466-8555, Japan
E-mail: ayib@luna.elcom.nitech.ac.jp

1 Introduction

DOA estimation algorithms have been studied and developed by many researchers[1][2]. As one of the main categories of the signal parameter estimation techniques, MUSIC algorithm is well received by researchers[1][2]. MUSIC requires the eigenvectors in noise subspace of correlation matrix and therefore we have to execute EVD (eigen-value decomposition) of correlation matrix or SVD (singular-value decomposition) of input data matrix. However, the EVD or SVD generally brings us heavy computational load and as a result it would be difficult to do real-time processing in DOA estimation.

In this paper, we proposes the recursive Unitary MUSIC to improve the computation efficiency toward real-time DOA estimation[3]. The basic algorithm of this is Unitary MUSIC which deals with real-valued matrices and in addition BiSVD subspace tracking method[4] is employed to carry out iterative DOA estimation without heavy EVD or SVD. Through computer simulation, we will compare the recursive Unitary MUSIC (proposed algorithm) with the recursive Standard MUSIC (complex-valued version), and discuss initial values of the proposed algorithm.

2 Principle of DOA Estimation

2.1 Receiving system and signal model

We develop the signal model for DOA estimation by MUSIC algorithm. Receiving system is a K-element equispaced linear array. Assuming that there is no mutual coupling between the antennas. We have L incoming signals to the array, the directions of which are $\theta_l (l = 1, 2, \cdots, L)$ respectively. Then, snapshot vector of the array at time instant t can be expressed as follows.

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad (1)$$

$$A = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_L)] \quad (2)$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \quad (3)$$

Note that $s_i(t)$ is the ith incoming signal and $\boldsymbol{n}(t)$ is an internal noise vector. $\boldsymbol{a}(\theta_i)$ is the array response vector (mode vector) of the ith signal and A is the matrix of array response (mode matrix). From the above input data expression, it is possible to estimate DOA by recursive Unitary MUSIC algorithm[3].

2.2 Unitary operation to input data

First, to perform iterative estimation at every time instant $t (t = 1, 2, \cdots)$, the data matrix is defined as follows.

$$\boldsymbol{X}(t) = [\alpha^{1/2}\boldsymbol{X}(t-1) \ \ (1-\alpha)^{1/2}\boldsymbol{x}(t)] \quad (4)$$

where $\boldsymbol{X}(t-1)$ is the old data matrix, $\boldsymbol{x}(t)$ is a current snapshot vector, and α is a forgetting factor $(0 < \alpha < 1)$. The snapshot vector of $\boldsymbol{x}(t)$ is transformed into another vector $\boldsymbol{y}(t)$ with a unitary matrix, and it is written as

$$\boldsymbol{y}(t) = \boldsymbol{Q}_K^H \boldsymbol{x}(t) \quad (5)$$

where $\boldsymbol{Q}_K$ is the unitary matrix and is defined as follows[1][2].

$$Q_K = Q_{2M} = \frac{1}{\sqrt{2}}\begin{bmatrix} I_M & jI_M \\ \Pi_M & -j\Pi_M \end{bmatrix} \text{ when } K \text{ is even } (K = 2M) \tag{6}$$

$$Q_K = Q_{2M+1} = \frac{1}{\sqrt{2}}\begin{bmatrix} I_M & \mathbf{0} & jI_M \\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T \\ \Pi_M & \mathbf{0} & -j\Pi_M \end{bmatrix} \text{ when } K \text{ is odd } (K = 2M+1) \tag{7}$$

Here, I_M is the identity matrix of the dimension M, and Π_M is the square matrix of dimension M, which is defined as

$$\Pi_M = \begin{bmatrix} \mathbf{0} & & 1 \\ & 1 & \\ 1 & & \mathbf{0} \end{bmatrix} \tag{8}$$

For Unitary MUSIC the data matrix is modified as follows[3].

$$X(t) = [\alpha^{1/2}X(t-1)\ (1-\alpha)^{1/2}y_c(t)] \tag{9}$$

$$y_c(t) = [\mathrm{Re}[y(t)]\ \mathrm{Im}[y(t)]] \tag{10}$$

It is noted that the data matrix of eq.(9) is real-valued.

2.3 BiSVD subspace tracking method

In the proposed algorithm, we use BiSVD (Bi-iteration Singular-Value Decomposition) subspace tracking method[4] to estimate the signal subspace. BiSVD is based on decomposition of singular values of $X(t)$. It determines recursively the signal subspace matrix $Q_A(t)$ which consists of eigenvectors belonging to signal subspace of $X(t)X^T(t)$. With ref.[4], the BiSVD to $X(t)$ can be expressed as follows.

$$B(t) \triangleq X^T(t)Q_A(t-1) \tag{11}$$

$$B(t) = Q_B(t)R_B(t) \quad \text{(QR decomposition)} \tag{12}$$

$$A(t) \triangleq X(t)Q_B(t) \tag{13}$$

$$A(t) = Q_A(t)R_A(t) \quad \text{(QR decomposition)} \tag{14}$$

where $X(t)$ is supposed to be $(K \times N)$ matrix. Through eqs.(11) to (14) matrices $B(t)$ and $A(t)$ are decomposed into $\{Q_B(t) \in R^{N\times L}, R_B(t) \in R^{L\times L}\}$ and $\{Q_A(t) \in R^{K\times L}, R_A(t) \in R^{L\times L}\}$ respectively. By BiSVD subspace tracking method, $Q_A(t), R_A(t), R_B(t)$ are recursively determined. For the initial value of $Q_A(t)$, we consider the following two methods[3].

(a) Initial value A

$$Q_A(0) = \begin{bmatrix} I_L \\ \mathbf{0} \end{bmatrix} \times p \in R^{K\times L} \quad R_A(0) = \mathbf{0}_{L\times L} \quad R_B(0) = I_L$$

where p determines the magnitude of initial value which is in the range $0 < p \leq 1$.

(b) Initial value B

$Q_A(0)$ is derived from the SVD of $X(t)$ with first several snapshots. $R_A(0)$, $R_B(0)$ are in turn determined from $Q_A(0)$ through BiSVD.

2.4 Recursive Unitary MUSIC spectrum

From unitary operation of eq.(5), $a(\theta)$ can be transformed into a real-valued vector which is given by

$$d(\theta) = Q_K^H a(\theta) \tag{15}$$

This is because the components of $a(\theta)$ have property of so called conjugate centrosymmetry with respect to the center component[2][4]. This property is important to create angular spectrum with Unitary MUSIC. From the property, Unitary MUSIC algorithm can compute the angular spectrum by using

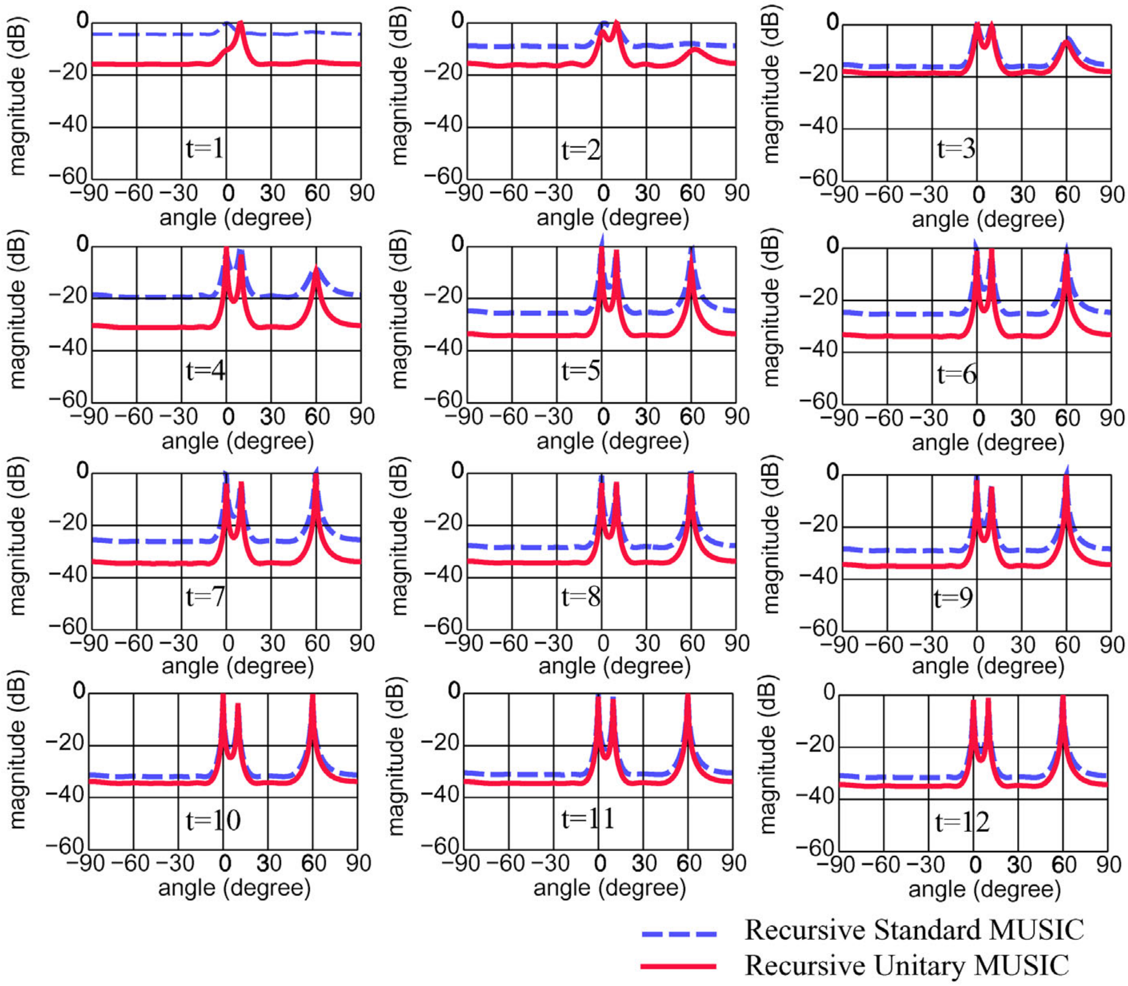

Figure 1: MUSIC spectrum of recursive MUSIC with initial value A

noise space eigenvectors[1] of the real-valued correlation matrix. Then, the angular spectrum of recursive Unitary MUSIC algorithm at time instant t can be written as follows[2].

$$P_{MU}(\theta; t) = \frac{\boldsymbol{d}^T(\theta)\boldsymbol{d}(\theta)}{\boldsymbol{d}^T(\theta)\left\{\boldsymbol{I}_L - \boldsymbol{Q}_A(t)\boldsymbol{Q}_A^T(t)\right\}\boldsymbol{d}(\theta)} \tag{16}$$

3 Computer Simulation

Table 1 shows the simulation conditions used in our simulation. In the simulation, we estimate 3 incoming waves ($L = 3$), which are uncorrelated with each other. We also examine the performance of recursive Standard MUSIC algorithm for comparison to our proposed algorithm.

Table 1: Simulation conditions.

Type of array	equispaced linear array
Antenna element	isotropic
Element spacing	0.5λ (λ : wavelength)
Number of elements	10
Number of incoming waves	1
SNR	20dB
Forgetting factor: α	0.9

Figures 1 and 2 show the change of spectrum from $t = 1$ to $t = 12$. In Fig. 1, the recursive Unitary

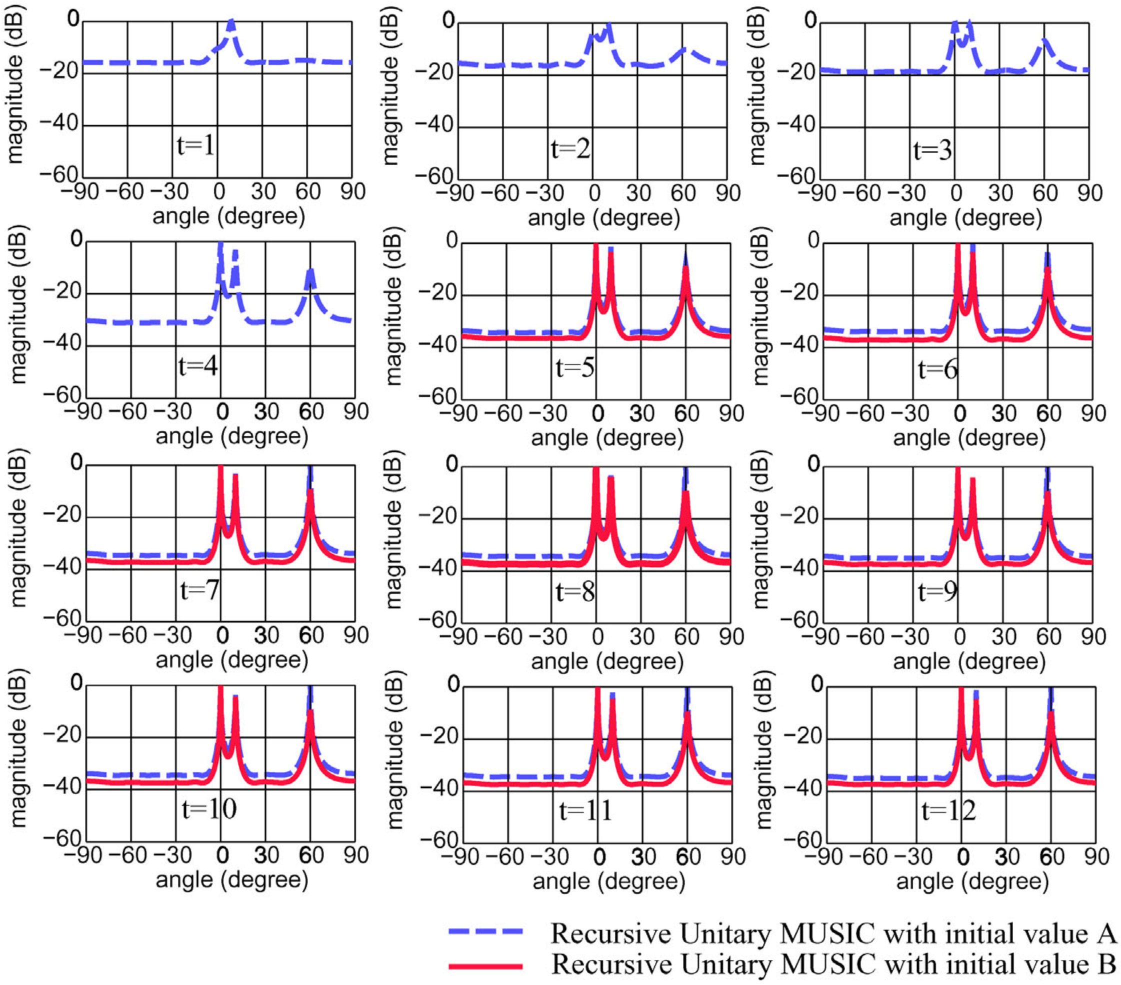

Figure 2: MUSIC spectrum of recursive Unitary MUSIC with initial values A and B

MUSIC and recursive Standard MUSIC are compared in the case of initial value A. On the other hand, in Fig. 2, the initial values A and B are compared in the recursive Unitary MUSIC. Five snapshots are used in the calculation of initial value B. From the results, it is found that the estimation accuracy of unitary type is better than standard type. Also, the initial value B provides more stable spectrum than the initial value A immediately after $t = 5$.

4 Conclusion

In this paper, we have carried out performance analysis of DOA estimation with an equispaced linear array using recursive Unitary MUSIC algorithm. From the simulation results, we can see that recursive Unitary MUSIC with the initial value determined by SVD of the first several snapshots can provide accurate and stable DOA estimation as well as high computation efficiency.

References

[1] Nobuyoshi Kikuma: *Adaptive Signal Processing with Array Antenna (in Japanese)* , Science and Technology Publishing Company, Inc. 1998.

[2] Nobuyoshi Kikuma : *Adaptive Antenna Technologies (in Japanese)*, Ohmsha, 2003.

[3] T.Sasaki, N.Kikuma and N.Inagaki:"On Improving the Initial Value Problem of the Recursive Unitary ESPRIT for Iterative DOA Estimation", Technical Report AP2002-96, Oct. 2002.

[4] Peter Strobach : "Bi-iteration SVD subspace tracking algorithms and applications," IEEE Trans. Signal Processing, vol.45, no.5, pp.1222-1240, May 1997.