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Equivalent SNR Expression for Location Error of Elements on MUSIC Algorithm

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Abstract There are various methods of direction-of-arrival(DOA) estimation. In these methods, however, accuracy of DOA estimation is decreased because of distortion of steering vector due to location error of elements. In this paper, using computer simulations with MUSIC algorithm as example, results that steering vector distortion due to location error of elements causes DOA estimation error are described. In addition, the relation between the location error of elements and input SNR is discussed. Furthermore, it is shown that the location errors are expressed as equivalent SNR by using probability of incorrect estimation as a parameter.

Keyword DOA Estimation, MUSIC Algorithm, Location Error, Equivalent SNR

1. Introduction

There are various methods of direction-of-arrival(DOA) estimation such as beamformer, Capon, LP(Linear Prediction), MUSIC (MUltiple SIgnal Classification) [1] and ESPRIT (Estimation of Signal Parameters via Rotational Invariarance Techniques). On these methods, in order to estimate the arrival direction, it is necessary to use an array antenna. In these techniques, the location of antenna elements should be known, however arrangement of an element is not exact in fact. Because an antenna surely has a location error which occurred in the process of antenna manufacture. Therefore, estimation error of the arrival direction is increased[2][3]. In this paper, we evaluate the influence of the location error of an antenna element on the estimation accuracy of the arrival direction by using the MUSIC Algorithm as one example. The location errors of antenna element are expressed as equivalent SNR, by regarding the probability of incorrect estimation of the arrival direction as a parameter.

The analysis model used in this paper is shown in Chapter 2. In Chapter 3, we evaluate how the location error of the antenna elements affects the estimative accuracy of the arrival direction. Chapter 4 shows that a location error and input SNR can has one-to-one correspondence from the results of simulations.

2. Analysis model and Evaluation method

Fig. 1 shows the linear array, equally spaced d apart, for N elements. Except for the 1st element, the elements from the 2nd to the Nth are shifted from an exact location. The Nth element error shifted from the original location is set at Δd_n, which is normal random value according to average μ and standard deviation σ. The original element interval d is set to 0.5λ. On this condition, the simulation of the arrival direction has been conducted using the MUSIC algorithm. When the estimated arrival direction differs from the true arrival direction, the

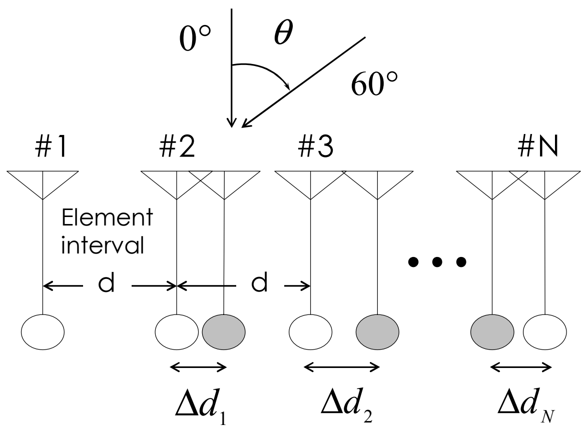

Fig.1 array antenna as an analysis model estimation is fail, in short, incorrect estimation. Here, if $\Delta\theta$ which is the difference between the estimated arrival direction estimated and the true arrival direction, is larger than $\Delta\theta_P$ which is the permitted estimation error, the estimation is judged to be failure. The same trials are done repeatedly, and then we evaluate the probability of the failure. We call the probability of incorrect estimation as "PIE", here after. We supposed that the number of the arrival wave is two in the simulation. The arrival direction of the 1st wave and 2nd wave is 0 degree and 60 degrees respectively. The average μ is 0%.

3. Validity of trials

First, for the purpose of determining the suitable number of trial times, we evaluate the PIE with changing the number of trials. Here, the simulation was repeated 10 times in the same condition, and we evaluate the reliability of PIE by check the fluctuation of PIE. Since it is clear that the suitable number of trials is affected by the number of the elements N, we changed N from 3 to 5. Fig. 2 shows the result for N= 5. In Fig. 2, a horizontal axis is the number of trials and a vertical axis denotes the probability of incorrect estimation, PIE. From the figure, if number of trials is small, the values of the PIE differ greatly for every simulation. In the case the number of trials is 8000 times, the difference among PIE is about ±1% of range. Thus, we can say that the suitable number of trials is more than 8000 times. In order to obtain the result of a simulation in the fewer amount of calculation, we will adopt 8000 times as a number of trials, hereafter.

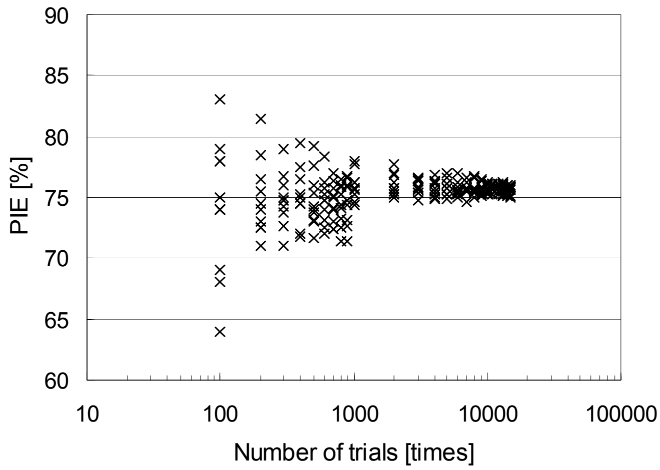

Fig. 2 Effect of trial times

4. Influence of location error

It examined how the PIE would vary with variation of the location error of antenna elements. The number of elements was fixed to N=3. Fig.3 and Fig.4 show the PIE respectively as input SNR was changed from 0 to 40dB. In Fig.3 and Fig.4, a horizontal axis is input SNR and a vertical axis is the PIE value. The parameter is the permitted estimation error.

In Fig.3, the average value of the location error σ is 0%, which means there is no location error. The PIE becomes lower as input SNR becomes larger. The arrival direction is estimated correctly. On the other hand, the PIE becomes high as input SNR becomes smaller. In Fig.4 the average value of the location error σ is 3%. Due to the location error of each element, a floor of PIE appears in the case that input SNR is large, and the estimation accuracy of the arrival direction cannot be improved even if it the input SNR is increased.

Next, the number of the element was increased, and we examined the PIE for σ=3% and $\Delta\theta_P$=1.5 degrees. The result is shown in Fig.5. A horizontal axis is input SNR and a vertical axis is the value of PIE. The parameter is the number of elements. As the number of elements is increased, a floor of PIE appears incuse of large SNR as the same as above. From these results, it is indicated that the estimation error cannot be eliminated completely, even we can decrease the PIE by increasing SNR or number of elements

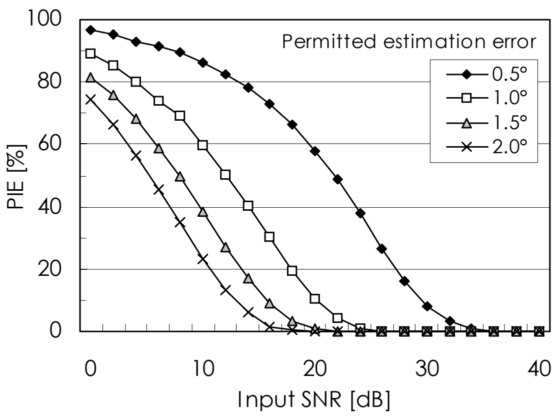

Fig.3 Input SNR and PIE (N=3,σ=0%)
(PIE : probability of incorrect estimation)

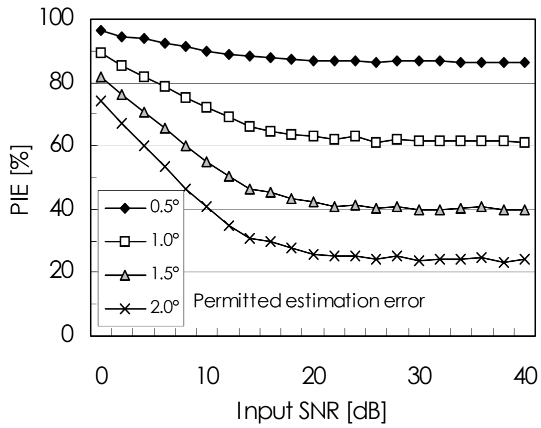

Fig.4 Input SNR and PIE (N=3,σ=3%)

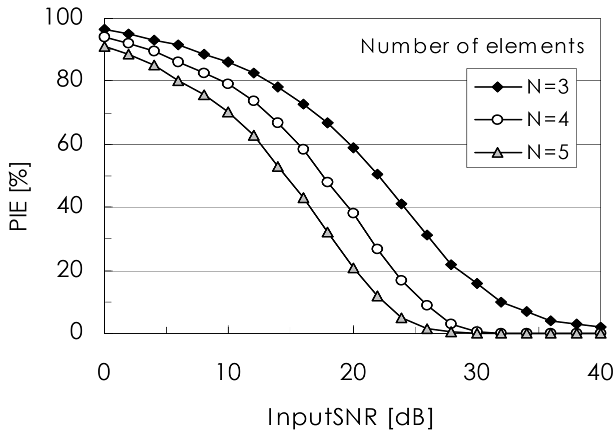

Fig.5 input SNR and PIE (σ=3%, $\Delta\theta_P = 1.5°$)

5. Equivalent SNR expression of location error

We consider the relation between an element location error and input SNR, here. First of all, we picked up the value of PIE at SNR=40dB in Fig.4, and find the corresponding value of SNR in Fig.3. For example, the value of PIE in Fig.4 at SNR=40dB and $\Delta\theta_p$=1.5 degrees is 40%. Then, we find value of SNR correspond to 40% of PIE in Fig.3. It is approximately 10dB. This means that σ=3% of element location error is equivalent to thermal noise at SNR=10dB.

Furthermore, we evaluated the relation between location error σ and input SNR with

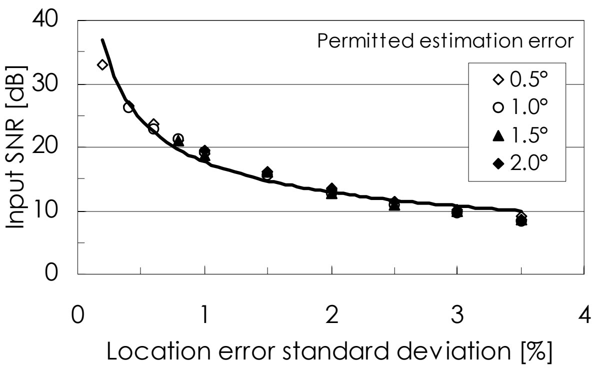

Fig.6 relation between location error σ and input SNR (N=3) parameter of permitted estimation error. A result is shown in Fig.6. A horizontal axis is the location error σ and a vertical axis is input SNR.

From Fig.6, we can find that the relation between location error and input SNR can be expressed with one line. We can see that the relation is not dependent on the permitted estimation error. In other words, concerned with the estimated error of the arrival direction in the MUSIC method, the location error of elements is equivalent to degradation of input SNR. Thus, we defined the value "Equivalent SNR" as a parameter which indicates the effect of location error of antenna elements to the input SNR. For example, it supposed from Fig.6 that Equivalent SNR when 1.5% of location error is 15dB.

Furthermore, the number of elements is changed from 3 to 5. The result is shown in Fig.7. A horizontal axis is the location error σ, and a vertical axis is input SNR. A parameter is the number of elements. From the figure, it confirmed that input SNR and a location error have the one-to-one correspondence and the relation is not dependent on the number of elements. For example, we can say that when we cannot keep the input in SNR only 15dB, the location error σ is permissible to about 1.5%.

From the results it can be said that not only the improvements of an antenna gain and input SNR by the reducing noise but also element location accuracy are very important.

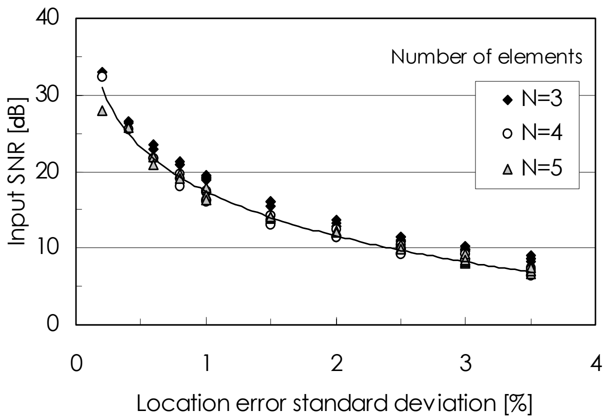

Fig.7 relation between location error σ and input SNR

6. Conclusion

The effect of location error of antenna elements on estimation accuracy of MUSIC Algorithm was evaluated. It was shown through computer simulation that the relation between the location error of elements and input SNR can be expressed using one curve, and it is not depend on permitted estimation error. Therefore, the effect of the location error of the elements can be expressed as equivalent SNR. Furthermore, it has been shown that the equivalent SNR does not be affected by number of elements.

For future works, it should be investigate whether the location error can be expressed as Equivalent SNR also in case of other array arrangement or other estimation methods.

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