T-MATRIX ANALYSIS OF ELECTROMAGNETIC WAVE DIFFRACTION FROM A METALLIC FOURIER GRATING WITH COMPLEX PERMITTIVITY

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1 Introduction

The diffraction problem of electromagnetic wave from a sinusoidal grating is one of the basic problem in light wave technique and radio wave engineering. Chuang and Kong[1] described an extensive literature survey of this important boundary value problem. Recently, the Fourier grating has been developed by using holographic exposure process, and analysis of that grating with metallic loss has already been carried out by using the boundary element method and mode matching method[2]. We are interested in the diffraction problem from a metallic Fourier grating whose profiles are represented by a superposiotion of sinusoidal waves. When diffraction characteristic of the Fourier grating are analyzed in short wavelength region of light, lossy metallic Fourier gratings with arbitrary complex permittivity must be considered[4].

In this paper, extended boundary condition method was applied to analyze the two media boundary value problem of Fourier gratings given by Chuang and Kong[1]. This method is so called T-matrix method in the formulation of the problem and also it is regarded as an adjoint method to the mode matching method. We obtained elements of T-matrix for Fourier grating with basic sinusoidal wave and the second harmonic wave newly, while an analysis of Chuang only basic sinusoidal wave is treated. Numerical results of the diffraction efffeciency and the energy flow near the Fourier grating are presented.

2 T-matrix formulation

Let us consider the electromagnetic wave diffraction by periodic varying suface(e.g. Fourier grating) illuminated by a plane wave. We assume a two-dimensional problem where the surface does not vary in the y direction and the plane of incidence is the $x - z$ plane for incident angle θ_i as shown in Fig.1. The surface is given by

$$z(x) = -h\left\{\cos\left(\frac{2\pi x}{P}\right) + \gamma\cos\left(\frac{4\pi x}{P} + \delta\right)\right\} \tag{1}$$

where P is the period of grating, h is the amplitude of the grating, $h\gamma$ and δ denote the amplitude and phase of the second harmonic wave, respectively.

In the region 1 and 2, the two-dimentional wave equation of electric and magnetic fields in the rectanglar coordinate is derived form Maxwell's equations as follows:

$$\frac{\partial^2\psi_\nu}{\partial x^2} + \frac{\partial^2\psi_\nu}{\partial z^2} + k_\nu^2\psi_\nu = 0, \quad \psi_\nu = \begin{cases} E_{y\nu}\text{: TE wave} \\ H_{y\nu}\text{: TM wave} \end{cases} \tag{2}$$

where $k_\nu = \omega\sqrt{\epsilon_\nu\mu_\nu}$ (the suffix ν denotes $\nu = 1$ for region 1, $\nu = 2$ for region 2), ω is the angular frequency, ϵ_ν and $\mu_\nu$are the permittivity and the permeability in each region respectively. Periodic time dependency $\exp(j\omega t)$ is supressed throughout.

From Huygens' principle and Green's theorem, it follows that

$$\left.\begin{array}{ll} z > z(x) & \psi_1(\bar{r}) \\ z < z(x) & 0 \end{array}\right\} = \psi_i(\bar{r}) - \int d\overline{\sigma'} \cdot \left\{ G_1\left(\bar{r}, \overline{r'}\right) \nabla' \psi_1\left(\overline{r'}\right) - \psi_1\left(\overline{r'}\right) \nabla' G_1\left(\bar{r}, \overline{r'}\right) \right\},$$

$$\left.\begin{array}{ll} z > z(x) & 0 \\ z < z(x) & \psi_2(\bar{r}) \end{array}\right\} = \int d\overline{\sigma'} \cdot \left\{ G_2\left(\bar{r}, \overline{r'}\right) \nabla' \psi_2\left(\overline{r'}\right) - \psi_2\left(\overline{r'}\right) \nabla' G_2\left(\bar{r}, \overline{r'}\right) \right\} \tag{3}$$

where $\bar{r} = x\hat{x} + z\hat{z}, \overline{r'} = x'\hat{x} + z'\hat{z}, \nabla' = \left(\frac{\partial}{\partial x'}\hat{x} + \frac{\partial}{\partial z'}\hat{z}\right), G_\nu(\bar{r}, \overline{r'}) = -\frac{j}{4} H_0^{(2)}(k_\nu|\bar{r} - \overline{r'}|), (\nu = 1, 2),$ and $d\overline{\sigma'}$ is also the normal directionvector defined by $d\overline{\sigma'} = dx'\left(-\frac{dz(x')}{dx'}\hat{x} + \hat{z}\right)$ which $\hat{x}$ and $\hat{z}$ are the unit vector in x and z direction, respectively.

We start with introducting a basis of Bloch function[1]

$$\varphi_\nu\left(\pm\overline{k_{\nu m}^{\pm}} \cdot \bar{r}\right) \equiv \frac{\exp\left(\pm j\overline{k_{\nu m}^{\pm}} \cdot \bar{r}\right)}{\sqrt{\beta_{\nu m}}}, \quad \nu = 1, 2, \quad m = 0, \pm 1, \pm 2, \cdots \tag{4}$$

with

$$\overline{k_{\nu m}^{\pm}} = k_\nu(\alpha_{\nu m}\hat{x} \pm \beta_{\nu m}\hat{z}), \quad k_\nu \alpha_{\nu m} = k_\nu \sin\theta_{\nu m} = k_1 \sin\theta_i + m\frac{2\pi}{P},$$

$$\beta_{\nu m} = \cos\theta_{\nu m} = \begin{cases} \sqrt{1 - \alpha_{\nu m}^2}, & \alpha_{\nu m}^2 \le 1, \\ -j\sqrt{\alpha_{\nu m}^2 - 1}, & \alpha_{\nu m}^2 \ge 1. \end{cases}$$

The function $\varphi_\nu\left(-\overline{k_{\nu m}^{+}} \cdot \bar{r}\right)$ corresponds to the up-going reflected wave, whereas the $\varphi_\nu\left(-\overline{k_{\nu m}^{-}} \cdot \bar{r}\right)$ represent the down-coming wave. Futher $\varphi_\nu\left(\overline{k_{\nu m}^{\pm}} \cdot \bar{r}\right)$ arises in expanding the Green functions.

A field expansion may be expanded to get

$$\left.\begin{array}{l} \psi_1(\bar{r}) \\ 0 \end{array}\right\} = \psi_i(\bar{r}) + \begin{cases} \sum_m b_m \varphi_1\left(-\overline{k_{1m}^{+}} \cdot \bar{r}\right), & z \ge \max z(x), \\ -\sum_m a_m \varphi_1\left(-\overline{k_{1m}^{-}} \cdot \bar{r}\right), & z \le \min z(x), \end{cases}$$

$$\left.\begin{array}{l} 0 \\ \psi_2(\bar{r}) \end{array}\right\} = \begin{cases} -\sum_m B_m \varphi_2\left(-\overline{k_{2m}^{+}} \cdot \bar{r}\right), & z \ge \max z(x), \\ \sum_m A_m \varphi_2\left(-\overline{k_{2m}^{-}} \cdot \bar{r}\right), & z \le \min z(x). \end{cases} \tag{5}$$

The surface electromagnetic field depends on x only, the unkonown field in terms Fourier series expansions is used. Applying the boundary conditions for the tangential fields on the periodic surface, we obtain the following matrix equation:

$$\begin{bmatrix} b_m \\ A_m \end{bmatrix} = [T_{mn}] \begin{bmatrix} B_m \\ a_m \end{bmatrix} \tag{6}$$

where

$$[T_{mn}] = \begin{bmatrix} -c_2\frac{k_2}{k_1}Q_{Dmn}^{+}(k_1) & -Q_{Nnm}^{+}(k_1) \\ Q_{Dmn}^{-}(k_2) & Q_{Nnm}^{-}(k_2) \end{bmatrix} \begin{bmatrix} -Q_{Dmn}^{-}(k_2) & -Q_{Nnm}^{-}(k_2) \\ c_2\frac{k_2}{k_1}Q_{Dmn}^{+}(k_1) & Q_{Nnm}^{+}(k_1) \end{bmatrix}^{-1}$$

In the above expression, $c_2 = 1$ for the TE case, $c_2 = \epsilon_1/\epsilon_2$ for the TM case, B_m must be zero for the radiation condition in region 2. The incident plane wave is expressed

$$\psi_i(\bar{r}) = \frac{\exp\left(-j\overline{k_{10}^{-}} \cdot \bar{r}\right)}{\sqrt{\beta_{10}}} = \frac{\exp(-jk_1 x \sin\theta_i + jk_1 z \cos\theta_i)}{\sqrt{\cos\theta_i}} \tag{7}$$

where θ_i being the angle of incidence with respect to the z axis. Therefore it is determined

$$a_m = \begin{cases} 1 & (m=0), \\ 0 & (m \neq 0). \end{cases} \tag{8}$$

In the eq.(6), the element of T-matrix expressed in terms of Bessel functions are obtained as

$$\begin{aligned}
Q^{\pm}_{Dmn}(k_\nu) &= \frac{-1}{\sqrt{\beta_{\nu m}}}\sum_l \exp\left\{\mp jl\left(\frac{\pi}{2}+\delta\right)\right\}(\mp j)^{|n-m\pm 2l|}J_{|n-m\pm 2l|}(k_\nu h\beta_{\nu m})J_l(k_\nu h\gamma\beta_{\nu m}). \\
Q^{\pm}_{Nmn}(k_\nu) &= \frac{-1}{\sqrt{\beta_{\nu m}}}\sum_l \exp\left\{\mp jl\left(\frac{\pi}{2}+\delta\right)\right\}J_l(k_\nu h\gamma\beta_{\nu m}) \\
&\left[\left\{\frac{k_\nu P\beta^2_{\nu m} - 2\pi\alpha_{\nu m}(n-m\pm 2l)}{\pm k_\nu P\beta_{\nu m}}\right\}J_1 + \left(\frac{2\pi jh\gamma\alpha_{\nu m}}{P}\right)\left(e^{j\delta}J_2^- - e^{-j\delta}J_2^+\right)\right]
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
J_1 &= (\mp j)^{|n-m\pm 2l|}J_{|n-m\pm 2l|}(k_\nu h\beta_{\nu m}), \quad J_2^- = (\mp j)^{|n-m-2\pm 2l|}J_{|n-m-2\pm 2l|}(k_\nu h\beta_{\nu m}). \\
J_2^+ &= (\mp j)^{|n-m+2\pm 2l|}J_{|n-m+2\pm 2l|}(k_\nu h\beta_{\nu m}).
\end{aligned}$$

The result of a sinusoidal wave grating is obtained by setting $\gamma = 0$ in this section.

3 Numerical calculations

In Fig.2, it is numerically simulated that the energy error for the trancted number N(=M) is calculated in the case that Fourier grating is made of complete dielectric body. Thus result is a similar in the case that is perfect conductor. We found that the truncted number N depends on the groove deep of the Fourier grating h.

Numerical result of the diffraction effeciency by using this method agrees well with the mode matching method[4] while metallic loss of a grating medium is presented as shown in Fig.3. When the resonance absorption for TM polarization is caused by metallic loss, the energy flow near the Fourier grating with allow of the Poynting vector is illustrated in the Fig.4 for $\theta_i = 6.8°$, $\gamma = 0.3$ and other parameters which are indicated in Fig.3.

4 Conclusion

Extended boundary condition method was applied to analysis of two medium boundary value problems of the Fourier grating. We obtained the element of T-matrix for the Fourier grating that basic sinusoidal wave and the second harmonic wave are considered. Numerical results showed that this method is useful for analysis of the Fourier grating with metallic loss.

References

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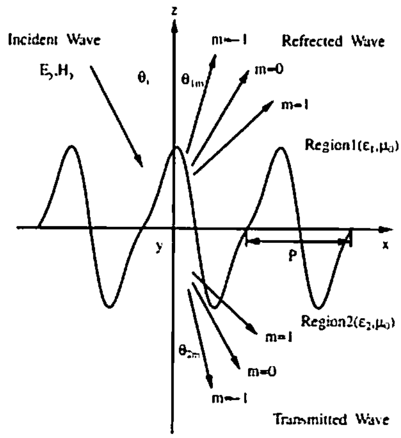

Fig.1 Geometry of the ploblem.

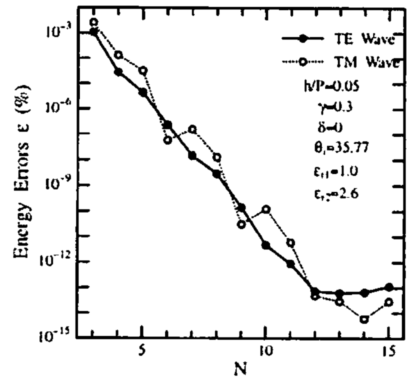

Fig.2 Energy errors ϵ as a function of N.

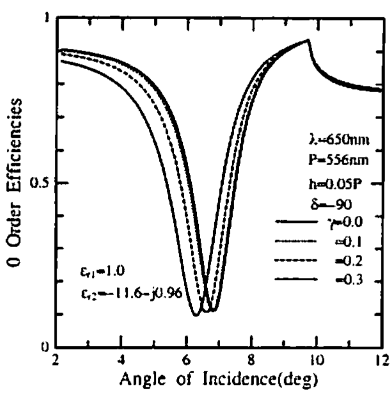

Fig.3 0 order diffraction effiencies as a function of the incident angle θ_i (TM wave).

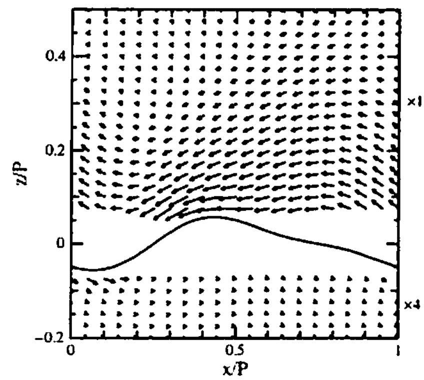

Fig.4 Energy flow of the total field near the surface (TM wave).