ELECTROMAGNETIC SCATTERING BY THE MODIFIED LUNEBERG LENS

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Abstract:
A new solution for the modified Luneberg lens with arbitrary parameter is presented. For which the focusing effect is examined and also useful in developing the RCS of the Luneberg lens.

1. Introduction

The electromagnetic scattering by the conventional Luneberg lens (δ =1, b=2) has been widely studied by using a homogeneous multilayered approximate method[1] and is also exactly obtained[2][3].

For the purpose of studying the focussing effect of the Luneberg lens, the new solution of the modified Luneberg lens ($\varepsilon_r = b - \delta (r/a)^2$) is obtained and numerical example of the electric field distribution along the propagation distance is also presented. From which the optimum variation of the arbitrary parameter b are also obtained.

2. Analysis

As shown in Fig. 1, the plane wave propagating along the z axis is considered and the specific permitivity $\varepsilon_s(r)$ is assumed to be defined by

$$\varepsilon_s(r) = b - \delta\left(\frac{r}{a}\right)^2 \tag{1}$$

For the special case b=2, δ =1 (the Luneberg lens) many workers[1-3] are widely treated.

In Fig.1, the incident electric field E_i and the scattered electric field E_s for the free space are represented by using vector mode functions respectively

$$\mathbf{E}^i = E^0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left(\mathbf{M}^i_{o1n} + j\mathbf{N}^i_{e1n}\right) \tag{2}$$

$$\mathbf{E}^s = E^0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left(a_n^s \mathbf{M}_{o1n}^s + j b_n^s \mathbf{N}_{e1n}^s \right) \tag{3}$$

where M, N are the radial function given by spherical Bessel function for the incident wave and spherical Hankel function of the second kind for the scattered wave respectively.

For the modified Luneberg lens the transmitted wave is also represented as

$$\mathbf{E}^t = E^0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left(a_n^s \mathbf{M}_{o1n}^t + j b_n^t \mathbf{N}_{e1n}^t \right) \tag{4}$$

where the radial functions involved in $\mathbf{M}_{o1n}^t$, $\mathbf{N}_{e1n}^t$ are defined by

$$S_n(\rho) = \rho^{n+1} \exp\left(-\frac{\sqrt{\delta}\rho^2}{2\rho_a} \right) {}_1F_1(\alpha, \gamma, \varsigma) \tag{5}$$

$$T_n(\rho) = (b\rho_a^2 - \delta\rho^2)\rho^{n+1} \exp\left(-\frac{\sqrt{\delta}\rho^2}{2\rho_a} \right) U_n(\rho) \tag{6}$$

where ρ=kr, ρ_a=ka.

In the above expression, ${}_1F_1(\alpha, \gamma, \varsigma)$ is the Kummer function and a new function $U_n(\rho)$ is represented in the following series solution as

$$U_n(\rho) = \sum_{m=0}^{\infty} A_m \varsigma^m \tag{7}$$

where

$$A_0 = 1,\ A_1 = \frac{\alpha_1}{\gamma},\ A_2 = \frac{1}{2}\frac{\alpha_1(\alpha_1+1)}{\gamma(\gamma+1)} - \frac{\alpha_2+\alpha_3}{(\gamma+1)a_2}$$

$$A_3 = \frac{1}{3!}\frac{\alpha_1(\alpha_1+1)(\alpha_1+2)}{\gamma(\gamma+1)(\gamma+2)} - \frac{1}{3(\gamma+2)}\left\{ \frac{(\alpha_1+2)(\alpha_2+\alpha_3)}{(\gamma+1)a_2} + \frac{2\alpha_1(\alpha_2+\alpha_3)}{\gamma a_2} + \frac{2(\alpha_2+\alpha_3)}{a_2^2} \right\} \tag{8}$$

$$a_2^2(m+1)(m+\gamma)A_{m+1} - a_2\left\{ a_2(m+\alpha_1) + 2m(\gamma+m-1) \right\} A_m + \left\{ 2a_2(\alpha_1+\alpha_2+\alpha_3+m-1) + (m-1)(\gamma+m-2) \right\} A_{m-1} - (\alpha_1 + 2\alpha_2 + m - 2)A_{m-2} = 0$$

The unknown coefficient a_n^s, b_n^s and b_n^t are determined from the boundary conditions on r=a, thus the all electromagnetic fields are obtained.

3. Numerical results

In Fig.2 the amplitude of the total electric field $E_x/E_0 = (E_x^i + E_x^s)/E_0$ is plotted versus the propagation distance of z for the various parameters of b and ka=14.5. In which the electric field amplitude for the parameter of b=2.0 and b=2.5 becomes maximum at z < a (inner sphere).

Thus the focal point where the peak amplitude becomes maximum occurs at the inner of the sphere. This decreases the performance of a lens. In order to obtain the maximum field amplitude at the surface of the sphere (z=a), the different parameter of b must be determined. Fig.3 shows the optimum variation in the various parameters of b versus ka for b- δ =1 corresponding the impedance matching condition of the inner (ε_r (r=0)=b) and outer (ε_r (r=a)=1) sphere. This new data is useful in developing the RCS of the Luneberg lens.

References:

[1] Mikulski, J. J. and Murphy, E. L., "The computation of electromagnetic scattering from concentric spherical structures," IEEE Trans. Antennas & Propag., AP-11, pp.169-177, 1963

[2] Tai, C. T., "The electromagnetic theory of the spherical Luneberg lens," Appl. sci. Res., 7, pp.113-130, 1958

[3] Rozenfeld, P., "The electromagnetic theory of three dimensional inhomogeneous lenses," IEEE Trans. Antennas & Propag., AP-24, pp.365-370, 1976

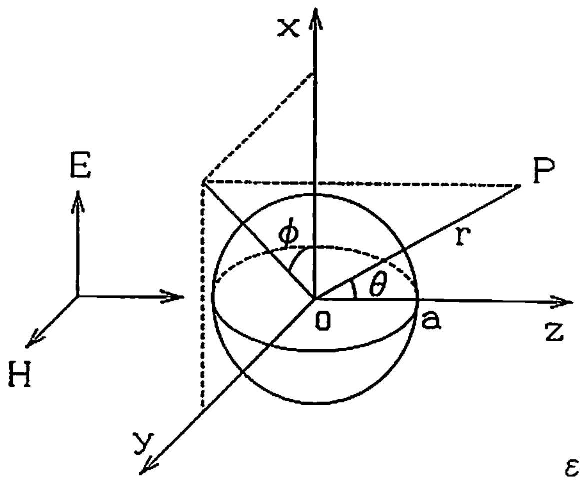

Fig.1(a) Geometry of the problem

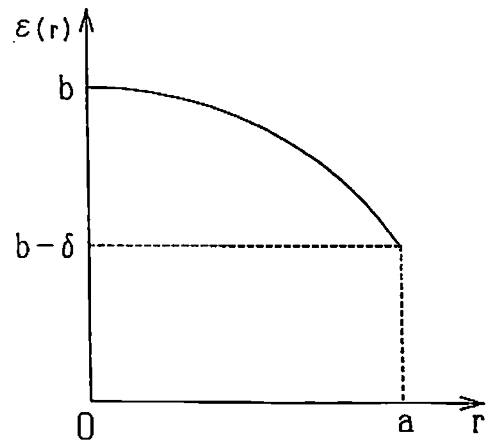

Fig.1(b) Dielectric constant variation in r

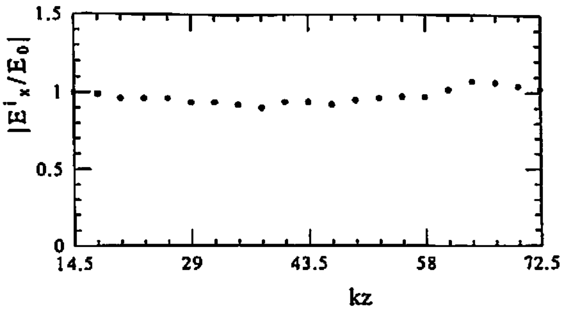

Fig. 2(a). Amplitude of the incident field along the z axis (free space).

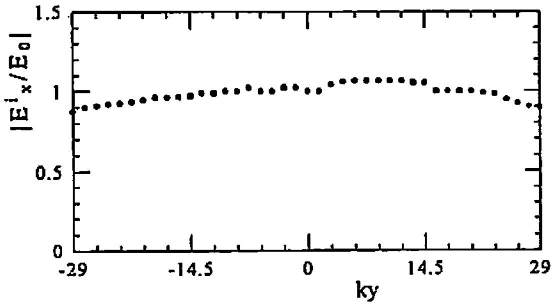

Fig. 2(b) Amplitude of the incident field along the y axis (free space).

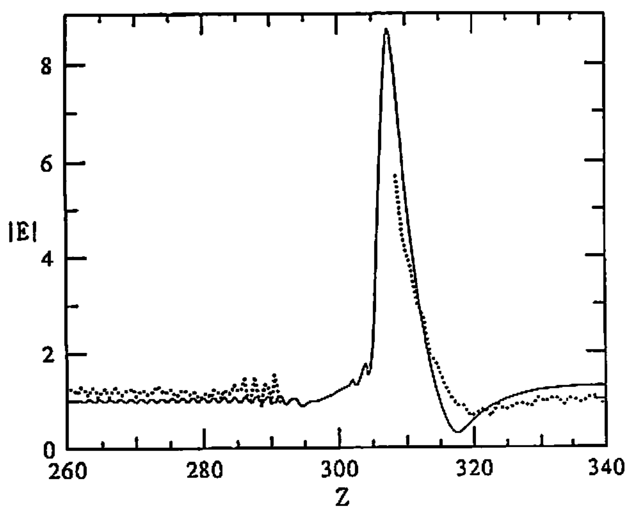

Fig. 3(a) Comparison between theoretical value(solid line) and experimental value(small circle) for the total electric field along the z axis.

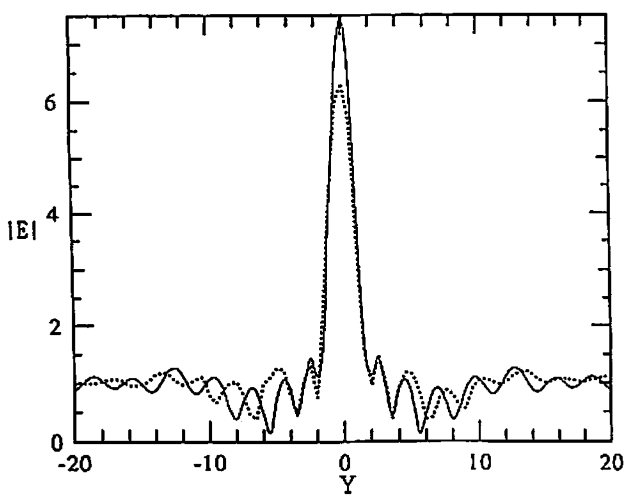

Fig. 3(b) Comparison between theoretical value(solid line) and experimental value(small circle) for the total electric field along the y axis.