# SPECTRAL-DOMAIN GALERKIN APPROACH FOR SCATTERING BY A HOMOGENEOUS ANISOTROPIC OBJECT

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### 1. ABSTRACT

A new prescription is given to the electromagnetic scattering by a homogeneous and anisotropic dielectric object. The scatterer is located in free space. A volume integral equation is used to formulate the boundary value problem. Via the plane wave angular spectrum expansion of electromagnetic field inside a homogeneous scatterer, entire domain basis functions are chosen to be solutions to the wave equation inside the scatterer. A Galerkin method in the Fourier transform spectral domain, which gives numerically stable solutions, is applied to convert the volume integral equation to a linear system of equations. The Fredholm integral equation is further regularized by taking into account the singularity of dyadic Green's function in source region.

### 2. INTRODUCTION

Recently, there has been a growing interest in the electromagnetic scattering of three dimensional object. The conjugate gradient method and the fast Fourier transform(CGFFT)[1,2] have widely used to the scattering by a dielectric body with arbitrary inhomogeneity and anisotropy. The Fredholm integral equation method(FIM)[3-6] have successfully applied to the scattering by a homogeneous object. However, for a homogeneous scatterer, CGFFT does not explore the homogeneity of a scatterer and FIM does not utilize the convolution properties of the volume integral equation. These two methods call for the present approach which fully utilizes both the homogeneity of scatterer and the convolution characteristic of the volume integral equation.

The approach addressed in this paper is strongly related to the method widely used in the analysis of guidance[7], resonance[8], radiation[9], and scattering[10-12] of rectangular conducting plates. Of greatest importance to this work is the paper series by Kaklamani and Uzunoglu[11.12]. Also, recent work based on the plane wave angular spectrum expansions[3-6,13-16] is important to this paper. In this paper, a new approach consisting of regularized fredholm integral equation technique in conjunction with the Galerkin method in the Fourier transform spectral domain is presented to treat the electromagnetic scattering by a homogeneous anisotropic object. Instead of the algebra manipulation, the main emphasis is given to the idea. Throughout this paper, time factor is  $\exp(j\omega t)$ .

### 3. FORMULATION

Consider scattered fields generated by a homogeneous scatterer illuminated by a given incident wave  $E_{inc}(r)$ . The scatterer is located in free space with constant permittivity  $\varepsilon_0$ , magnetic permeability  $\mu_0$ , and wavenumber  $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ . Assume a homogeneous and anisotropic object with constant tensor permittivity  $\varepsilon_0 \underline{\varepsilon}$  and permeability  $\mu_0 \underline{I}$ , where  $\underline{I}$  is the unit dyad. The source induced in this homogeneous dielectric scatterer is polarization current  $\underline{J}$ 

$$J(r) = j\omega\varepsilon_0[\underline{\varepsilon} - \underline{I}] \bullet E(r) \tag{1}$$

The total electric field can be described by the following volume integral equation

$$E(r) = E_{inc}(r) + k_0^2(\underline{\epsilon} - \underline{I}) \bullet \int_{\underline{I}} \underline{G}_0(r, r') \bullet E(r') dr'$$
 (2)

where

$$\underline{G_0}(\boldsymbol{r}, \boldsymbol{r}') = (\underline{I} + \frac{1}{k_0^2} \nabla \nabla) \frac{\exp[-jk_0|\boldsymbol{r} - \boldsymbol{r}'|]}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|}$$
(3)

is the dyadic Green's function in free space.

In order to proceed with the solution of (2), let the observation point r in (2) is restricted inside the homogeneous anisotropic scatterer, an integral equation is obtained for the unknown electric field E(r) inside the scatterer volume V. Following the method used in the references[3-6,13-16], we can express the electric field inside the homogeneous and anisotropic scatterer as

$$E(r) = \sum_{n=1}^{2} \int_{0}^{2\pi} \int_{0}^{\pi} d\phi_{k} \sin\theta_{k} d\theta_{k} C_{n}(\theta_{k}, \phi_{k}) E_{n}(\theta_{k}, \phi_{k}) \exp[-jk_{n}(\theta_{k}, \phi_{k}) \cdot r]$$
(4)

$$E_n(\theta_k, \phi_k) = E_{nx}(\theta_k, \phi_k)\hat{x} + E_{ny}(\theta_k, \phi_k)\hat{y} + E_{nz}(\theta_k, \phi_k)\hat{z}$$
(5)

$$\mathbf{k}_n(\theta_k, \phi_k) = k_n(\theta_k, \phi_k) \hat{k}(\theta_k, \phi_k) \tag{6}$$

$$\hat{k}(\theta_k, \phi_k) = \cos \phi_k \sin \theta_k \hat{x} + \sin \phi_k \sin \theta_k \hat{y} + \cos \theta_k \hat{z} \tag{7}$$

where  $C_n(\theta_k, \phi_k).k_n(\theta_k, \phi_k)$  and  $E_n(\theta_k, \phi_k)$  are the undetermined amplitude, wave number and wavevector direction of the nth eigenwave in an unbounded homogeneous anisotropic medium[13-16], respectively. Furthermore,  $\hat{x}, \hat{y}$ , and  $\hat{z}$  are unit vectors in a rectangular coordinate system. (4) can be discreted as[16]

$$E(r) = \sum_{n=1}^{2} \sum_{s=0}^{S} \sum_{t=0}^{T} C_{nst} E_n(\theta_{kt}, \phi_{ks}) \exp[-jk_n(\theta_{kt}, \phi_{ks}) \cdot r]$$
 (8)

$$C_{nst} = \frac{2\pi^2}{ST} W_s W_t k_n(\theta_{kt}, \phi_{ks}) C_n(\theta_{kt}, \phi_{ks}) \sin \theta_{kt}$$
(9)

$$\theta_{kl} = \frac{t\pi}{T} \tag{10}$$

$$\phi_{ks} = \frac{s2\pi}{S} \tag{11}$$

where T and S, and  $W_t$  and  $W_s$  are the node numbers, and weight coefficients of numerical quadratures with respect to  $\theta_k$  and  $\phi_k$ , respectively. Moreover, the multi-index nst can be denoted by a single index m and therefore (8) can be rewritten as

$$E(r) = \sum C_m E_m(r) \tag{12}$$

Taking into account the Fourier expansion of scalar Green's function in a spherical coordinate system

$$\frac{\exp[-jk_0|\mathbf{r} - \mathbf{r}'|]}{4\pi|\mathbf{r} - \mathbf{r}'|} = \frac{1}{(2\pi)^3} \int_0^\infty \int_0^{2\pi} \int_0^{\pi} k^2 dk d\phi_k \sin\theta_k d\theta_k \frac{\exp[-jk\hat{k} \cdot (\mathbf{r} - \mathbf{r}')]}{k^2 - k_0^2}$$
(13)

:multiplying both side of (2) by the following vector  $F_{m'}^*(r)$  (\* denotes the complex conjugate)

$$F_{m'}(r) = (\underline{\varepsilon} - \underline{I}) \bullet E_{m'}(r)$$

$$(m' = n's't', n' = 1, 2; s' = 0, \dots, S; t' = 0, \dots, T)$$
(14)

where the explicit expression of  $E_{m'}(r)$  is given in (12); using the Parseval theorem in the spectral domain approach[7-12], we finally obtain

$$[Z_{mm'}]_{M \times M}[I_m]_{M \times 1} = [V_{m'}]_{M \times 1}$$
(15)

where

$$V_{m'} = \int_{V} F_{m'}^{*}(r) \bullet E_{inc}(r) dr$$
 (16)

$$I_m = C_m \tag{17}$$

$$Z_{mm'} = \int_0^{2\pi} \int_0^\pi d\phi_k \sin\theta_k d\theta_k \tilde{\boldsymbol{F}}_{m'}^*(\theta_k,\phi_k)$$

$$\frac{\omega \varepsilon_0 k_0}{8\pi^2} \bullet [\underline{I} - \widehat{k}\widehat{k}] \bullet \check{F}_m(\theta_k, \phi_k)$$

$$+\frac{1}{3}\int_{V}F_{m'}^{*}(r)\bullet F_{m}(r)dr \tag{18}$$

$$\hat{\boldsymbol{F}}_{m}(\theta_{k},\phi_{k}) = \int_{V} \boldsymbol{F}_{m}(\boldsymbol{r}) \exp[jk_{0}\hat{k}\cdot\boldsymbol{r}]d\boldsymbol{r}$$
(19)

Note that the singularity of dyadic Green's function in the source region and its counterpart in the Fourier transform spectral domain[17] has been taken into consideration. By the way, it is possible to remove the singularity of dyadic Green's function by introducing the polarization charge source  $\rho$  via the continuity equation of current. However, previous numerical experiences show that it is just the supersingularity of dyadic Green's function that leads to better numerical stability of the final linear system of equations. It is also interesting that for a rectangular dielectric parallelepiped[11], a finite circular cylinder and a sphere, the Fourier transform (19) can be analytically calculated. In addition, for an arbitrary volume V, all the volume integrals can be reduced to surface integrals by virtue of the second Green's identity[18].

This section would not be complete without mentioning the counterpart of (4) and (5) for isotropic homogeneous object. Because the well known complete set of vector wave functions are derived via the pilot vector r [17], which loses many useful properties of (5), and meanwhile the set of vector wave functions derived from pilot vector  $\hat{z}$  is not complete [18], the following expressions are suggested:

$$E(r) = \int_{0}^{2\pi} \int_{0}^{\pi} d\phi_{k} \sin \theta_{k} d\theta_{k}$$

$$[C_{x}(\theta_{k}, \phi_{k})\hat{x} + C_{y}(\theta_{k}, \phi_{k})\hat{y} + C_{z}(\theta_{k}, \phi_{k})\hat{z}] \exp[-jk_{n}(\theta_{k}, \phi_{k}) \cdot r]$$

$$C_{x}(\theta_{k}, \phi_{k}) \cos \phi_{k} \sin \theta_{k} + C_{y}(\theta_{k}, \phi_{k}) \sin \phi_{k} \sin \theta_{k} + C_{z}(\theta_{k}, \phi_{k}) \cos \theta_{k} = 0$$

$$(21)$$

where (20) is based on that each rectangular component of electric field in a isotropic medium satisfies the wave equation and (21) is due to the fact that the divergence of electric field must equal to zero. So we identify

$$k_1 = k_2 = k_0 \sqrt{\varepsilon} \tag{22}$$

$$E_1(\theta_k, \phi_k) = \hat{x} - \frac{\cos \phi_k \sin \theta_k}{\cos \theta_k} \hat{z}$$
 (23)

$$E_2(\theta_k, \phi_k) = \hat{y} - \frac{\sin \phi_k \sin \theta_k}{\cos \theta_k} \hat{z}$$
 (24)

Let T in (8) be a odd number, no numerical trouble is involved even if  $\theta_k = \pi/2$ . Notice that our choice of basis functions for isotropic media is better than that given in [11] since the undetermined coefficient  $C_z(\theta_k, \phi_k)$  of [11] not only increases the matrix size but also results in the possibility of spurious solutions[18].

Finally, we point out that if a homogeneous scatterer is of complex shape, we may use the following multiple center expansion to represent the field inside the homogeneous scatterer[18]

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{l=1}^{N} \sum_{n=1}^{2} \int_{0}^{2\pi} \int_{0}^{\pi} d\phi_{k} \sin \theta_{k} d\theta_{k} C_{n}^{l}(\theta_{k}, \phi_{k}) \boldsymbol{E}_{n}(\theta_{k}, \phi_{k}) \exp[-j\boldsymbol{k}_{n}(\theta_{k}, \phi_{k}) \cdot (\boldsymbol{r} - \boldsymbol{r}_{l})]$$
(25)

### 4. CONCLUDING REMARKS

The proposed approach makes a full use of the homogeneity of a scatterer and the convolution characteristic of the volume integral equation. Probably, this work can be viewed as a minor extension of spectral domain approach for strips/slots[7] and plates/apertures[8-10] to the three dimensional geometries. The essential difference between this paper and [11,12] is that we utilize the Fourier expansion of scalar Green's function in a spherical coordinate system while [11,12] use the Fourier expansion of scalar Green's function in a rectangular coordinate system. It is evident that the latter is a travail extension of previous work[7-10], but the former is not. The great advantage of our approach is that the fourier integrals are with respect to finite regions. Therefore, instead of the fast Fourier transform, we will use the novel numerical quadrature applied in [15] to evaluate all the closed surface integrals appearing in this article. This procedure totally avoids the so called aliasing problem[1,2] arising from the application of fast Fourier transform. It is noticed that this paper is based on the same approximation of the field inside a homogeneous and anisotropic medium as [3-6]. So our method should be applicable to the geometries suitable for the method of [3-6]. The method proposed here can also be applied to the elastic wave scattering[14]. The numerical implementation of present approach is in consideration and will be reported later.

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