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ASYMPTOTIC EQUIVALENCY BETWEEN ADAPTIVE ARRAYS AND SYNTHESIZED ARRAYS

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INTRODUCTION

Array pattern synthesis has been investigated since Schelknoff's theory [1]. They are shaped beams[2] and optimization of array characteristics [3]. On the other hand, the concept of adaptive arrays for interference rejection [4],[5],[6] has been established and some works have been published based on the least-mean-square-error method[7], maximum signal-to-noise ratio method[8] and constrained optimization method[9].

So far, these two techniques are treated independently, and this paper points out their relationship and its application.

ANALYSIS

We have already reported the concept and the detailed analysis of a directionally constrained adaptive array[10]. This system acts to minimize the output power subject to a constraint which specifies the array system response in a given direction. On the other hand, synthesized array with maximum directive gain under specified nulls has been reported by Drane and McIlvenna[11]. To investigate the relationship between them, the method by Zahm[12] will be applied to the problem of [11].

For the analytical simplicity, the array system under consideration is assumed to be linear with K-element. The spacing between adjacent elements is a half-wavelength of an operating frequency.

The directive pattern of the array under consideration is expressed as;

$$y(\theta) = \sum_{i=1}^{K} w_i \exp\{-j\psi_i(\theta)\}, \qquad (1)$$

with

$$\psi_k(\theta) = \pi(i - \frac{K+1}{2}) \cos\theta, \qquad (2)$$

where w_k (i=1, 2,..., K) are the "weights" or complex excitation coefficients at each element. θ is the angle measured from the array baseline. For the ease of treatment, vector notations are introduced as follows:

$$y(\theta) = Z^* W, \qquad (3)$$

with

$$Z^* = [\exp\{-j\psi_1(\theta)\}, \exp\{-j\psi_2(\theta)\}, \ldots, \exp\{-j\psi_K(\theta)\}], \qquad (4)$$

$$W^* = [w_1{}^\dagger, w_2{}^\dagger, \ldots, w_K{}^\dagger], \qquad (5)$$

where superscripts * and † denote the complex conjugate transpose and the complex conjugate, respectively. The power pattern is, therefore, given as:

$$|y(\theta)|^2 = W^* Z Z^* W. \qquad (6)$$

And, the directive gain G in the specific direction θ_s is expressed from the definition as follows.

$$G = \frac{W^* Z_s Z_s{}^* W}{W^* W}, \qquad (7)$$

with

$$Z_s{}^* = [\exp\{-j\psi_1(\theta_s)\}, \exp\{-j\psi_2(\theta_s)\}, \ldots, \exp\{-j\psi_K(\theta_s)\}]. \qquad (8)$$

Following the concept of [11], null constraint toward the given direction θ_b can be expressed as:

$$W^* Z_b = Z_b^* W = 0 , \quad (9)$$

with

$$Z_b^* = [\exp\{-j\psi_1(\theta_b)\},\ \exp\{-j\psi_2(\theta_b)\},\ \ldots,\ \exp\{-j\psi_K(\theta_b)\}] . \quad (10)$$

By using Zahm's method, the optimal weight vector W_{DM} can be derived as a following manner.

$$W_{DM} = \frac{1}{K-1} (Z_s - A_{sb} Z_b) , \quad (11)$$

with

$$A_{sb} = \frac{\sin[\frac{K\pi}{2} (\cos\theta_s - \cos\theta_b)]}{K \sin[\frac{\pi}{2} (\cos\theta_s - \cos\theta_b)]} . \quad (12)$$

On the other hand, the optimal weight vector of directionally constrained adaptive array W_{DC} is given by the next formula[10], where θ_s and θ_b as the directions of the desired signal (which is assumed to coincide with the constraint direction) and the interference, respectively.

$$W_{DC} = \frac{1}{K} Z_s + \frac{P_b A_{sb}}{P_r + KP_b(1 - A_{sb}^2)} \times (A_{sb} Z_s - Z_b) , \quad (13)$$

where P_r and P_b are the average power of the internal noise and of the interference, respectively. Now, for the case with negligible internal noise compared with the interference, we have,

$$\lim_{P_r/P_b \to 0} W_{DC} = \frac{1}{K(1 - A_{sb}^2)} (Z_s - A_{sb} Z_b) . \quad (14)$$

By comparing (11) with (14), the difference lies only in the magnitude of the vectors. From this fact, it can be concluded that the Drane and Mc-Ilvenna's array with a single null constraint and the directionally constrained adaptive array for single interfering source are similar to each other if the internal noise power is negligible compared with the interference power. Although the analysis has been performed only for single-null case, it will be easily expanded to multi-null case. In addition, following [13] and [14], some adaptive arrays are also asymptotic to Drane and Mc-Ilvenna's array.

APPLICATION

In the preceding section, asymptotic feature of DC adaptive array and DM array has been proved. The analytical result implies that some deviation will arise due to the internal noise. This reminds us of the fact the internal noise may become large at the output of an array in the case of super-gain antenna. In order to evaluate this effect, the concept Q-factor is sometimes introduced, but in DM array, it cannot be expressed conveniently.

On the contrary, an adaptive array follows the guideline to minimize the noise component at the output. In other words, it acts on the balance of the desired signal versus total noise, which in turn consists of the interference and the internal noise. Therefore, application to the pattern synthesis problem may well enable us to include the effect of internal noise without introducing Q-factor or others. The output SNR for DM array and for DC adaptive array are shown as follows, respectively.

$$S/N_{DM} = (1 - A_{sb}^2) \frac{P_s}{\frac{P_r}{K}} , \quad (15)$$

$$S/N_{DC} = P_s \Big/ \left\{ \frac{P_r^2 P_b A_{sb}^2}{\{P_r + KP_b(1-A_{sb}^2)\}^2} + \frac{P_r\{P_r^2 + KP_b(1-A_{sb}^2)(2P_r + KP_b)\}}{K\{P_r + KP_b(1-A_{sb}^2)\}^2} \right\} \quad (16)$$

As shown in the previous part, with $P_b >> P_r$, (16) approaches to (15). For the comparison between these two systems, we take the example of a four-element array with spacing of a half-wavelength, assume θ_s=90° and P_s/P_r to be 20dB and calculate the output signal-to-noise ratios with parameters of θ_b and P_b/P_r, the results of which are shown in Fig.1. Apparently, in DM array, large increase of the internal noise power at the array output causes remarkable deterioration in the output signal-to-noise ratio. This feature results from steering the null too close to the direction of desired signal without consideration on the internal noise. On the other hand, the system response of the adaptive array toward the interference direction is not always perfect null but depends on the interference power relative to the internal noise so that maximum output signal-to-noise ratio can be obtained. This null depth depends also on the angular distance between the desired signal and interference in terms of the beamwidth. Fig.2 shows the part of our interest of an example of the synthesized patterns for these cases where θ_s and θ_b are taken to be 90° and 89°, respectively. Arrarently, the array system response toward the interference in DC adaptive array deviates further from null as the ratio of the interference power to the internal noise power decreases. On the other hand, DM array always gives perfect null regardless of the change of external or internal noises. Fig.3 shows the output SNR as a function of P_b/P_r, for the case with the same example with Fig.2. (Fig.3 is the section of Fig.2 at the angle of 89°.) With small P_b/P_r, the output SNR of DC adaptive array is much higher than that of DM array. As P_b/P_r increases, the former becomes lower, but it is only at P_b/P_r =∞ where we have the same output SNR for DM array and DC adaptive array.

Summing up, the adaptive technique approach to array pattern synthesis always gives the higher output signal-to-noise ratio than DM array. In addition, for the reference of the array design, beam efficiency indices of the synthesized array are shown in Table 1. The beam efficiency index is defined as follows in the present case.

$$\frac{1}{K} \frac{W^* Z_s Z_s^* W}{W^* W} . \qquad (17)$$

CONCLUSION

In this paper, the authors have given an analytical proof of the asymptotic feature of the adaptive array under directional constraint to an array of maximum directive gain subject to null constraint proposed by Drane and McIlvenna. The proportional relationship with other types of adaptive arrays was proved previously, so that these systems are also asymptotic to Drane and McIlvenna's array. It is also presented that our method can solve the system proposed by Drane and McIlvenna.

From the results of analysis, application of adaptive technique to array pattern synthesis is proposed to improve the output signal-to-noise ratio. Some example show that the array synthesized by the adaptive technique can give much higher output signal-to-noise ratios. When the interference power is high enough compared with the internal noise power, e.g. more than 20dB, the effect of internal noise can be neglected, but if it is low, adaptive technique approach must be adopted to provide the satisfactory performance.

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	P_b/P_r	Beam Efficiency
DC Adaptive Array	10dB	18.11%
	20dB	1.03%
	30dB	0.43%
	40dB	0.38%
DM Array		0.38%

Table 1 Comparison of the beam efficiency indices between synthesized arrays by DM method and DC adaptive method. (θ_s=90°, θ_b=89°, P_s/P_r=20dB, K=4)

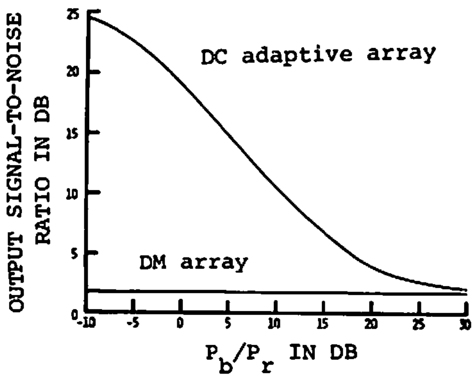

Fig.3 Variation of the output signal-to-noise ratio in terms of the interference power for DC adaptive array. Output signal-to-noise ratio of DM array is also shown. (θ_s=90°, θ_b=89°, P_s/P_r= 20dB, K=4)

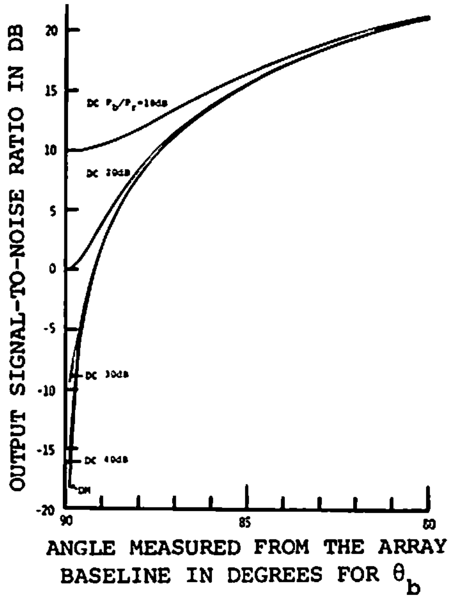

Fig.1 Comparison of the output signal-to-noise ratio between Drane and McIlvenna's array and adaptive array under directional constraint. (θ_s=90°, P_s/P_r=20dB)

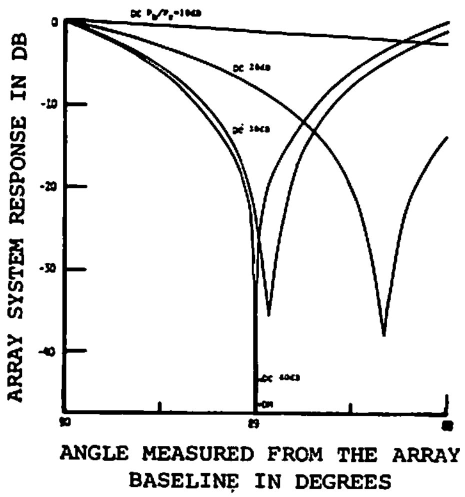

Fig.2 Synthesized pattern by DC adaptive method and DM method near interference direction.