WHEELERS SIZE LIMITATION AS APPLIED TO SMALL MOBILE PHONE ANTENNAS

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1. Summary
Modern mobile phones are from user viewpoint desired to be small and attractively designed. There is a general observation that many modern small phones tend to have less efficiency than older and bigger ones and especially network operators are regarding possibly reduced efficiency as a threat to the coverage and the cell planning. Already in the early fifties Wheeler [1], Chu [2] and others formulated limitations for antenna bandwidth as proportional to the volume of the radiating structure expressed in cubic wavelengths. Below this two observations are discussed and compared with application to mobile phones. The antenna function in general terms is also discussed indicating what properties are especially important to obtain good bandwidth and efficiency.

2. Wheelers limitation of antenna bandwidth versus volume

The size limitation stated by Wheeler and Chu can also be expressed as a Q-value as given by Harrington [3]. The Q-value for the lossfree case is closely related to the bandwidth. In all these three references the radiating structure (“antenna”) is thought to be enclosed by a sphere which is as small as possible still enclosing the radiating structure and all other conductive parts involved. The fields outside the sphere can quite generally be expressed as a sum of spherical wave functions (see for instance ref [4] by Hansen). The spherical fields will, as opposed to plane fields, store energy but only at small distances from the origin. As in any circuit the stored energy as compared to the transported power (during one period) give a measure of the Q-value (or rather $2\pi Q$) and thus the bandwidth. This connection to the waves outside of the antenna structure is important as it gives a bandwidth limitation independent to what can be done within the radiating structure. In ref [5] (Ch 6 written by Wheeler) the term “power factor” is used for the inverted Q and for the loss-less case of an antenna enclosed within a sphere the power factor $4\pi^2V/(3\lambda^3)\approx 13V/\lambda^3$ is deduced. V is the volume of the sphere which is the smallest sphere still containing the radiating structure. In the real case (with losses) the term efficiency η can be used and if the inverted loaded (i.e. incl. losses) Q is said to represent the fractional bandwidth ($\Delta f/f$) then the volume limitation can be formulated as:

$$(\Delta f/f)\eta < 13V/\lambda^3 \qquad \text{(eq 1)}$$

It follows from the deduction that that both bandwidth and efficiency can be smaller, i.e. the formula is a bit optimistic. The formula is mainly an estimation as it is hard to apply exactly among other because the calculation of the volume V is not obvious for a non-spherical body. However it gives a good understanding of influence of the size of any small antenna. Two examples of very small antennas are a ceramic GPS-patch and a battery powered medium-wave receiver. For the GPS patch at 1575 MHz (size 18x18x4 mm) the quotient is about 0.006 indicating 0.6% bandwidth or a bit more with regard to the losses. For the AM receiver (needing 1% bandwidth) the formula indicates 1% bandwidth and the efficiency 10^{-7}. The low efficiency is acceptable due to the extremely high atmospheric noise below 1 MHz enabling signal recovery with moderate quality loss.

3. Application of Wheelers limitation to a mobile phone

With application to a mobile phone the formula above gives the result that is the radiating structure (or rather the sphere around it) should have a volume of around 0.2 liter corresponding to a diameter of the sphere of 75 mm. This indicates that the whole phone structure is active in the radiating process.

For a conventional stubby antenna it is rather obvious that the phone plus antenna can be seen as an asymmetrical dipole. Due to the big thickness to length ratio this dipole is wideband and the total length is not too far from $\lambda/2$ at 800-1000 MHz. As will be discussed subsequently this can be applied to other small types of antennas too. The current along the phone body is the dominating source for the radiating function and all small antennas can be seen as a feeder structure. It is a common observation from work with mobile phones that all phones at the low frequency bands (<1000 MHz) have very similar antenna pattern all like an electrical dipole along the axis of the phone.

4. One classification of mobile phone antennas

In general terms antennas for mobile phones can be divided after two criteria: If they are fixed or have a variable geometry and if the antenna for its radiating function is independent of the phone body or not. Examples from these 4 basic combinations are:

- Fixed independent antenna: Sleeve $\lambda/2$-antenna atop of the phone sometimes used at 1900 MHz band. A GPS antenna belongs to the same class.
- Fixed antenna radiating through the phone body: All stubby antennas as well as built in antennas. A stubby antenna typically includes the required tuning inductance.
- Extendable independent antenna: A $\lambda/2$ whip. The feeding impedance is high giving a low current through the phone body.
- Extendable antenna using the phone body for radiating. An extended $\lambda/4$ whip it the typical case where the whip is the upper half of the radiating dipole and the phone body the lower.

Generally it is desirable to have an antenna function independent of the phone body but that also will make the assembly bigger. The most common case, at least in Europe, is to gain size by using the phone body as a radiator. The negative influence on the function of the antenna by the user is generally accepted but by suitable design it can be minimized. The discussion below will be concentrated to the antennas radiating mainly through the phone body which means short external antennas or built in antennas. This discussion fully excludes independent antennas such as extendible $\lambda/2$-whips.

It follows from typical measures of modern phones that the phones within this selected group at least at the low frequency bands (somewhere within 800-1000 MHz) will radiate similar to a $\lambda/2$ dipole more or less regardless of the antenna element. The term “antenna element” rather than “antenna” will be used for the physical units intended to create the radiation. On the higher frequency bands the situation will be different and more complicated antenna pattern can be seen. However the idea with this paper is to study the effect of the small antennas as expressed in wavelengths.

5. Radiation resistance/conductance

The important function of any antenna used as a transmitting device is to create a far field and the radiated power is conventionally expressed as R_sI^2 or g_sV^2 where radiation resistance R_s or radiation conductance g_s is used depending on the type of feeding. Depending on the type of circuit “resistance” or “conductance” may be the most suitable measure but it is only when the circuit is tuned (pure real impedance) as the resistance and conductance are the inverse of each other. Resistivity of conductors or lossy dielectric materials will in practical cases give losses and change the real part of the input resistance accordingly.

6. Radiation resistance/conductance as applied to mobile phones

As the phone, at least at the low bands (800-1000), is fairly small as expressed in wavelengths the phone body is by no means any kind of “groundplane” for the antenna element independent of its type. This can be illustrated by a typical helical stubby antenna which on the phone may have 20 ohms radiation resistance while it when mounted on a big groundplane may have 2-4 ohms radiation resistance. The latter figure is more in accordance with the figure given in ref [5] chapter 5 where the radiation resistance of a short monopole is given as $10(kL)^2$ where L is the length and k the wave number ($=2\pi/\lambda$). For 900 MHz ($k\approx19$) and L=24-33 mm this corresponds to 2-4 ohms.

This can be further understood if the antenna located on the phone is regarded as a receiving antenna rather than a transmitting one. For some kind of patch or PIFA antenna the short circuit current at the antenna connection is a suitable measure and especially for this application the short will make the antenna more or less as a part of the phone body. “Phone body” refers to the conductive and connected parts of the phone which can be the PCB with screens, possibly metallic frame or enclosure etc. The current distribution over the phone body when it is exited by an external field while the antenna is shortcircuited thus can be expected to be fairly independent of the style of antenna as long as it is fairly conform with the phone body. When the receiving efficiency is described as a short-circuit current the corresponding radiation properties are described as a radiation conductance as a consequence of the reciprocity theorem. The short circuit current through the antenna element will corresponds to the displacement current hitting the antenna element when the phone is surrounded by the incoming field. Heuristically the radiation conductance can be described as a coupling factor α describing how big part of the displacement current hitting the phone that hits the antenna element. By applying the law of reciprocity it can be shown that the product of radiation conductance g_s and corresponding gain G_s can be deduced from that coupling factor α as:

$$g_s G_s = \frac{p Z_0}{l^2}\left[\frac{2La}{p Z_T}\right]^2 \qquad \text{(eq 2)}$$

The effective length of the phone is L and Z_T is the radiation resistance (ideally 73 ohms) of the phone if it was cut in the middle (i.e. corresponding to a standard λ/2 dipole possibly with the slightly different length L). Z_0 is 377 ohms. α can be calculated from the distribution of the displacement current and with regard to the different density of displacement current over the phone α should approximately be proportional to the surface of the antenna element divided by the surface of half the phone body. If the antenna element is a patch or PIFA element the conductance g_s above refers to a connection where the voltage is maximum (at the open end).

7. Reactance/suceptance of a small resonant structure
In the typical case on a modern phone without a whip the antenna element is a small structure far below λ/4 etc at the low frequency band (i.e. below 1000 MHz). It is usually tuned to give a real input impedance but the tuning element may sometimes be located outside of the antenna element itself. The typical stubby antenna (without whip or with a possible whip retracted) is from radiation viewpoint a very short whip which consequently has a capacitive impedance. By making the short whip of a helical or meandering structure inductance is added to tune the impedance to real. This location of the tuning impedance has two advantages. First the inductance will be physically big as compared to a corresponding component giving lower losses for the same inductance. Secondly the helical structure is a kind of transmission line with a low wave velocity and a piece having a quarter wave length (with low velocity) is thus feasible. The radiation resistance is basically depending on the physical length only but will anyway be about twice as high for this slow wave structure (having a sinusoidal current distribution) as compared to a straight whip of the same length (having a current linearly decreasing towards the end). As an average the current is “moved upwards” which is desirable.

An illustrative antenna element for a built in antenna element is the PIFA-element. In its basic version it is a quarter-wave tongue (or patch) mounted on and in one end connected to a conductive plane. One of its virtues is that the resonance gives a real input impedance which can be chosen within a wide range to 50 ohm or any other impedance. A connection close to the shorted end will give a very low impedance while a connection close to the open end will give a very high impedance (such as >1000 ohms).

For PIFA elements of the same shape but different heights the radiation conductance will be the same but the stored energy will increase if the PIFA element is made thinner with correspondingly increased capacitance. Seen as a circuit element the PIFA looks like a parallel resonant circuit (or a shorted quarter wave stub) and if the total capacitance is C then the Q-value of the resonant structure can be estimated as $\omega C/g_s$ where g_s is the radiation conductance as given in eq 2. It can be shown that the volume under the PIFA element is the critical parameter for the bandwidth. A bigger and lower PIFA can be used with (at constant volume) similar results. The extra stored energy due to the shape will be compensated for by a bigger α due to the bigger area.

In most practical cases the total Q is determined by the circuit solution (i.e. by the energy stored in the PIFA structure) but in case this structure is very favorable the energy stored in the fields outside of the phone as given by eq 1 can limit the bandwidth. In both cases the conductance is the most important quantity to keep the Q down.

8. Conclusions

Volume is the critical quantity both for the phone and for the built-in antenna element. The volumes limits the product of fractional bandwidth and efficiency and thus at very small antenna structures efficiency may have to be sacrified to get enough bandwidth.

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[3] R F Harrington. Time harmonic electromagnetical fields. Ch 6-4. Mc Graw Hill 1961
[4] J E Hansen. Spherical near-field antenna measurements
[5] R C Johnson. Antenna engineering handbook. Ch 6. Mc Graw Hill 1993.