PROCEEDINGS OF ISAP '96, CHIBA, JAPAN

Electromagnetic scattering of a Gaussian beam wave from finite periodic slots in a parallel-plate waveguide

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Abstract

Electromagnetic scattering from the finite periodic slots in a parallel-plate waveguide is investigated. The off-Bragg as well as Bragg blazing phenomena observed in the geometry is discussed.

I. Introduction

Recently experimental result that periodic strip grating on a grounded dielectric behaves like a corrugated structure was reported[1]. There have been lots of studies[2,3] on the scattering such as blazing phenomena of the rectangular groove(corrugated) grating so far. But this is not the case with the periodic strip grating on a grounded dielectric. Hence the further study on this geometry is needed.

The main objective of this work is to consider an analysis method for the electromagnetic scattering of a Gaussian beam wave(finite source) by finite periodic slots(finite reflection grating) in a parallel-plate waveguide(PPW) filled with a homogeneous dielectric and to present some theoretical results which are shown to be simulating well the scattering behaviour of the rectangular groove geometry, and to discuss characteristics of blazing observed in the geometry.

II. Formulation

Consider a Gaussian beam E_i(E-polarization case ; electric field parallel to the y axis) incident upon the slotted region(period d, slot width 2a, guide height b, total slot number 2L+1) at an incidence angle θ_i , as shown in Fig.1.

If the width of the incident beam is many wavelengths(i.e., $\bar{g}\,\lambda_o k_{xo} < 1$, here λ_o ; free space wavelength), the approximate expression for the incident Gaussian beam wave[3] is given by, under the time convention exp($j\omega t$),

$$E_i(x,z) = e^{jk_{xo}x} e^{-jk_{zo}z} e^{-(\bar{g}k_{xo})^2(z-z_o+xk_{zo}/k_{xo})^2} , \qquad (1)$$

in which $k_{zo} = k_o \sin\theta_i$, $k_{xo} = k_o \cos\theta_i$ and $\bar{g} = g/k_o = 1/(\beta R k_o)$. Here $k_o = \omega\sqrt{\mu_o \varepsilon_o}$ and β is the Gaussian beamwidth. Similarly that for the reflected Gaussian beam wave is expressed as $E_r(x,z) = -E_i(-x,z)$. Following the previous method in [4,5], we obtain the integro-differential equation whose unknowns are the z-component magnetic currents M_ℓ over the 2L+1 slots(ℓ =-L, ⋯,-1,0,1, ⋯,L) as follows :

$$\sum_{\ell=-L}^{L} \int_{\ell d-a}^{\ell d+a} \frac{1}{j2} \left\{ \left(k_o^2 + \frac{\partial^2}{\partial z^2} \right) H_o^{(2)}(k_o|z-z'|) + \frac{2}{b} \sum_{m=1}^{\infty} \frac{(m\pi/b)^2}{\alpha_m} e^{-j\alpha_m|z-z'|} \right\} M_\ell(z')\,dz'$$
$$= -\frac{\partial}{\partial x}\left[E_i(x,z) + E_r(x,z)\right]\Big|_{x=0^+} , \text{ over each slot,} \qquad (2)$$

where $\alpha_m = \sqrt{k^2 - (m\pi/b)^2}$ and $k = \sqrt{\mu_o \varepsilon_o \varepsilon_r}$.

Next, the piecewise sinusoidal Galerkin method[5] is employed to reduce the equation to a following system of algebraic linear equation :

$$[Y_{pr}^{qs}][V_{pr}] = [I^{qs}] \ , 1 \leq p, q \leq 2L+1 \ , \ 1 \leq s, r \leq N-1 \ , \tag{3}$$

where the super(sub)script set (q,s)((p,r)) means the s(r)-th piecewise sinusoid over q(p)-th slot and N is the segment number over each slot. The detailed expressions for elements of the admittance and excitation matrices are obtained analytically by the method in [5]. Once the coefficients of r-th piecewise sinusoid over the p-th slot, V_{pr}, are known, the far zone scattered electric field $E_s(\rho, \phi)$ due to the equivalent surface magnetic current is expressed as

$$E_s(\rho, \phi) = \sqrt{\frac{2}{\pi k_o \rho}} \frac{\cos(k_o h \sin\phi) - \cos k_o h}{\sin k_o h \cos\phi} e^{-j(k_o\rho - \pi/4)} \sum_{p=1}^{2L+1} \sum_{r=1}^{N-1} V_{pr} e^{jk_o z_{pr} \sin\phi}, \tag{4}$$

where z_{pr} is the location of the junction of r-th piecewise sinusoid over p-th slot and h is the segment length($=2a/N = z_{pr} - z_{pr-1}$). The far-zone reflected electric field $E_r(\rho, \phi)$ is obtained with the use of the saddle point method as follows :

$$E_r(\rho, \phi) = -\sqrt{\frac{k_o}{2\rho}} \frac{\cos\phi}{\bar{g}\, k_{xo}} e^{-j(k_o\rho - \pi/4)} e^{-k_o^2(\sin\phi - \sin\theta_i)^2/(2\bar{g}\, k_{xo})^2 - jz_o(k_{xo} - k_o \sin\phi)}. \tag{5}$$

By use of (4) and (5), the scattering behaviour of the Gaussian wave by the the geometry under consideration is investigated.

III. Numerical results and discussion

Firstly, in order to examine the similarity between scattering behaviours of the geometry of Fig.1 and the corrugated structure, we have calculated the angular behaviours of the relative power $\sigma(= |E_s(\phi) + E_r(\phi)|^2 / |E_s(\phi) + E_r(\phi)|^2_{max})$ for the normally incident Gaussian beam case and compared the results with those of the prior work[6] for the rectangular grooves on a perfect conductor under the condition that the groove width and height are set equal to the slot width and waveguide height in the geometry of Fig.1, respectively. The results for two geometries are shown in Fig.2. It is interesting to note that both geometries show almost the same scattering patterns. It is felt that this good agreement backs up the previous experimental result[1] that the geometry in Fig.1 can behave like the rectangular groove geometry. Note that $b = 0.25\lambda_o$ (ε_r=1) has been chosen for the results in Fig.2. Under this condition, in the PPW region for the single slot case, the lowest TE_{10} is cut off and so any incident wave power from outside cannot penetrate into the inside region of the PPW and, in the PPW region too under the strip for the multislot case, the lowest mode is cut off and so the guiding structure cannot be formed between two adjacent periodic cells, which are also the case with the rectangular groove geometry. In this respect, some similarity between two geometries can be expected from the viewpoint of electromagnetic scattering.

Recently theoretical results on the Bragg blazing of the infinite geometry of Fig.1 were reported[7]. Using the results we have chosen the parameters of the finite reflection grating as $d = 1.084\lambda_o$, $2a = 0.6135\lambda_o$, $b = 0.1944\lambda_o$, $\varepsilon_r = 2.57$. This choice corresponds to the case A Bragg blazing in Fig.2 in [7]. In this case too, as mentioned above, the guiding structure cannot be formed between two adjacent periodic cells. The angular behaviour of σ for the case that the total slot number $2L+1=51$ and incident angle of the Gaussian beam($g=0.1/\lambda_o$) $\theta_i=27.45^o$ is drawn in Fig.3. Comparison of this result with that[3] for the finite rectangular groove grating suggests that the geometry of Fig.1 will give the Bragg blazing characteristics which are comparable to or better than that given by metallic corrugations even with the smaller size.

There is other kind of Bragg blazing(for example, corresponding to the case C blazing in Fig.2 in [7]), in which case guided(leaky) wave is supportable by the infinite geometry of Fig.1. However, because the Bragg blazing of this type is relatively more sensitive to errors in angle of incidence and grating dimensions than that of the type in Fig.3 as pointed out in [7], the Bragg blazing of the type in Fig.3 is more useful for the applications such as polarizers and nonreflecting planes(which minimize specular reflection), for example, used to reduce the instrument landing system interference.

It is worthy of note that also off-Bragg blazing phenomenon, which is a kind of resonance type(P-type) anomaly[7], can be observed in the geometry(Fig.1), considering that the E-polarized off-Bragg blazing in the rectangular groove geometry has not been found up to now[2]. We have calculated the angular behaviour of the normalized power σ for the case that $d=1.133\lambda_0$, $2a=0.6413\lambda_0$, $b=0.5185\lambda_o$, $\epsilon_r=2.57$, total slot number $2L+1=61$, and the incident angle of the Gaussian beam($g=0.1/\lambda_0$) $\theta_i=37.6^\circ$. The results are drawn in Fig.4. This off-Bragg blazing is thought to be useful for such applications as multiplexers, demultiplexers, and frequency scanners at optical and millimeter wavelengths, where the inconvenience of Bragg blazing severely restricts the use of very high efficiency diffraction gratings.

IV. Conclusion

We have considered an analysis method for the electromagnetic scattering problem when the Gaussian beam wave is incident upon the finite periodic slots in a PPW. Some similarity between scattering behaviours of the geometry and the rectangular groove geometry has been examined. The characteristics of off-Bragg as well as Bragg blazing phenomena observed in the geometry of this work have been discussed.

References

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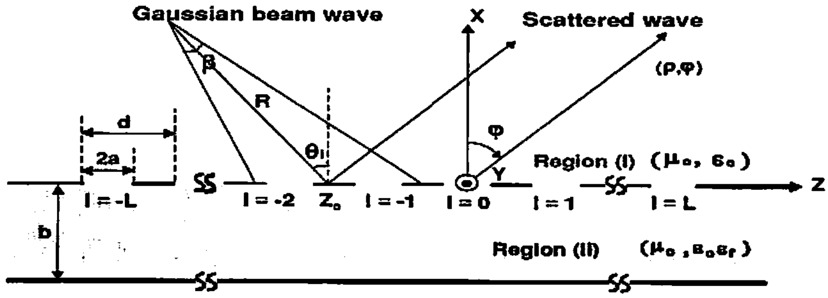
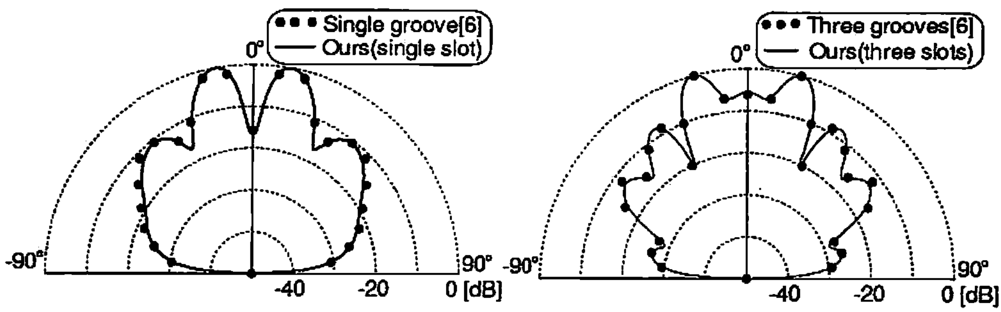


Fig.1. Geometry of the problem and notations.



a. Single slot(L=0).

b. Three slots(L=1).

Fig.2. Angular behaviours of the relative power σ for $d=3.75\lambda_o$, $2a=2.5\lambda_o$, $b=0.25\lambda_o$, $\varepsilon_r=1$, $\theta_i=0°$, $g=0.4/\lambda_o$, and $z_o=0$.

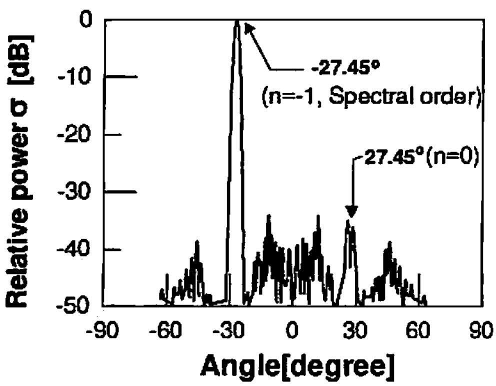


Fig.3. Angular behaviour of σ (Bragg blazing case) for $d=1.084\lambda_o$, $2a=0.6135\lambda_o$, $b=0.1944\lambda_o$, $\varepsilon_r=2.57$, $\theta_i=27.45°$, $g=0.1/\lambda_o$, $z_o=0$, and total slot number 2L+1=51.

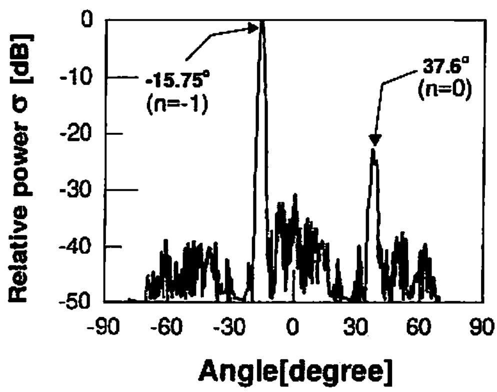


Fig.4. Angular behaviour of σ (Off-Bragg blazing case) for $d=1.133\lambda_o$, $2a=0.6413\lambda_o$, $b=0.5185\lambda_o$, $\varepsilon_r=2.57$, $\theta_i=37.6°$, $g=0.1/\lambda_o$, $z_o=0$, and total slot number 2L+1=61.