# DIFFRACTION COEFFICIENTS OF A CONDUCTING WEDGE BASED ON DUAL INTEGRAL EQUATIONS

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#### 1. Introduction

The geometrical theory of diffraction(GTD)[1] has been played a leading role in analysis of high-frequency electromagnetic diffraction by conducting structures. But its extention to diffraction by dielectric objects is hampered by no rigorous diffraction coefficients of dielectric wedges and cones. In recent, the diffraction by a dielectric wedge has been investigated by employing the formulation of dual integral equations[2]. According to the dual integral equations, the exact diffraction coefficients become zero in the artificially complementary region, in which the material inside(outside) the wedge is replaced by that outside(inside) wedge[3]. To illustrate its physical meaning clearly, the diffraction by a perfectly conducting wedge is reconsidered here by solving the corresponding dual integral equations exactly.

### 2. Dual integral equations

The geometry of a perfectly conducting wedge is shown in Fig. 1. The region of perfectly conducting wedge consists of  $S_1$  and  $S_2$ , which are in the ranges of  $0 \le \theta \le \theta_1$  and  $\theta_2 \le \theta \le 2\pi$ , respectively. When an E-polarized plane wave  $u_i$  is incident on the wedge with angle  $\theta_i$  in the free space region  $S_0$ , the total field u may be written as

$$u(\rho,\theta) = u_i(\rho,\theta) - \int_0^\infty d\rho' \left[ u(\rho',\theta') \frac{1}{\rho'} \frac{\partial G}{\partial \rho'} \right]_{\theta'=\theta_1} + \int_0^\infty d\rho' \left[ u(\rho',\theta') \frac{1}{\rho'} \frac{\partial G}{\partial \rho'} \right]_{\theta'=\theta_2}$$

$$(1)$$

where G denotes 2-dimensional free space Green function.

Following the same procedure as that developed for analyzing the diffraction by a dielectric wedge[2], one may transform Eq.(1) into the dual integral equations as

$$u_{i}(\rho,\theta) - F^{-1}\left[\frac{J_{1}(\alpha,\beta) + J_{2}(\alpha,\beta)}{\alpha^{2} + \beta^{2} - k_{0}^{2}}\right] = \begin{bmatrix} u(\rho,\theta) & \text{in } S_{0} \\ 0 & \text{in } S_{1} + S_{2} \end{bmatrix}$$
(2a)

where  $k_0$  denotes the wavenumber in  $S_0$ , and the operator  $F^{-1}$  denotes the operator of 2-dimensional inverse Fourier integral. And the spectral functions  $J_1$  and  $J_2$  are given by

$$J_{1}(\alpha,\beta) = \int_{0}^{\infty} d\rho \, \frac{1}{\rho} \, \frac{\partial}{\partial \theta} \, u(\rho,\theta_{1}) e^{-i\rho(\alpha\cos\theta_{1} + \beta\sin\theta_{1})} \tag{3a}$$

$$J_2(\alpha,\beta) = -\int_0^\infty d\rho \, \frac{1}{\rho} \, \frac{\partial}{\partial \theta} \, u(\rho,\theta_2) e^{-i\rho(u\cos\theta_2 + \beta\sin\theta_2)} \tag{3b}$$

### 3. Diffraction coefficients outside wedge region

The exact solution  $u(\rho, \theta)$  is well established in series form[4] as

$$u(\rho,\theta) = \frac{4}{\nu} \sum_{n=1}^{\infty} (-i)^{\frac{n}{\nu}} \sin \frac{n(\theta-\theta_1)}{\nu} \sin \frac{n(\theta_i-\theta_1)}{\nu} J_{\frac{n}{\nu}}(k_0\rho) \tag{4}$$

where  $I_{\frac{n}{\nu}}$  denotes the  $\frac{n}{\nu}$ -th order Bessel function. The value  $\nu$  is defined to satisfy the edge condition at the tip of the wedge[5] as

$$\nu = \frac{\theta_2 - \theta_1}{\pi} \tag{5}$$

Applying Eq.(4) into Eq.(3), and then performing the discontinuous Weber-Schafheitlin integral [6], one may obtain  $J_1(\alpha, \beta)$  and  $J_2(\alpha, \beta)$  as

$$J_{1}(\alpha,\beta) = \frac{4}{\nu} \sum_{m=1}^{\infty} (-i)^{\frac{n}{\nu}} \sin \frac{n(\theta_{i} - \theta_{1})}{\nu} e^{\frac{n}{\nu} \cos^{-1}(\frac{\alpha \cos \theta_{1} + \beta \sin \theta_{1}}{k_{3}})}$$
(6a)

$$J_2(\alpha,\beta) = -\frac{4}{\nu} \sum_{m=1}^{\infty} (-i)^{\frac{n}{\nu}} \sin \frac{n(\theta_i - \theta_2)}{\nu} e^{\frac{n}{\nu} \cos^{-1}(\frac{\alpha \cos \theta_2 + \beta \sin \theta_2}{k_0})}$$
(6b)

Applying Eq.(6) into Eq.(2a), and then performing the 2-D inverse Fourier integral, one may obtain the asymptotic integral form of  $u(\rho, \theta)$  as

$$u(\rho,\theta) = u_i(\rho,\theta) - \frac{i}{4\pi} \int_P dw \, p(w) e^{ik_0 \rho \cos(w-\theta)} \quad , \quad \text{in } S_0$$
 (7)

where P denotes the Sommerfeld integral path[7]. If the region outside the wedge  $S_0$  is defined in the range of  $\theta_1 \le w \le \theta_2$ , the diffraction coefficients p(w) are expressed by

$$p(w) = p_1(w) + p_2(w) , \quad \text{in } \theta_1 \le w \le \theta_2$$
 (8a)

$$p_1(w) = \frac{1}{\nu} \left[ \cot(\frac{w - \theta_i - \pi}{2\nu}) - \cot(\frac{w - 2\theta_1 + \theta_i - \pi}{2\nu}) \right]$$
 (8b)

$$p_2(w) = \frac{1}{\nu} \left[ -\cot(\frac{w - \theta_i + \pi}{2\nu}) + \cot(\frac{w - 2\theta_2 + \theta_i + \pi}{2\nu}) \right]$$
 (8c)

It is well known that the diffraction coefficients in Eq.(8) are theoretically exact in the actual free-space region outside the perfectly conducting wedge.

# 4. Diffraction coefficients in complimentary wedge region

Applying Eq.(6) into Eq.(2b), and then performing the 2-D inverse Fourier integral, one may obtain the same form as that in Eq.(7)

$$u_i(\rho,\theta) - \frac{i}{4\pi} \int_P dw \, p(w) e^{ik_0 \rho \cos(w-\theta)} = 0 \quad , \quad \text{in } S_1 + S_2$$
 (9)

The remaining problem may be whether the diffraction coefficients p(w) of Eq.(8) satisfy Eq.(9) or not. To answer the above question, we must define the regions of  $S_1$  and  $S_2$  in advance. Assume that the complementary wedge regions  $S_1$  and  $S_2$  are defined in the range of  $0 \le w < \theta_1$  and  $\theta_2 < w \le 2\pi$ , respectively. But one may easily proved that the diffraction coefficients p(w) of Eq.(8) cannot satisfy Eq.(9). As a typical example of  $\theta_1 = 60^\circ$ ,  $\theta_2 = 270^\circ$ , and  $\theta_i = 180^\circ$ , Fig. 2 illustrates that p(w) in Eq.(8) becomes nonzero in  $0 \le w < \theta_1$  and  $\theta_2 < w \le 2\pi$ .

The above discrepancy may be arisen from the fact that the period of p(w) is not  $2\pi$  but  $2\pi\nu$ . In a physical point, the induced sources  $J_1(\alpha,\beta)$  at  $\theta=\theta_1$  and  $J_2(\alpha,\beta)$  at  $\theta=\theta_2$  generate  $p_1(w)$  and  $p_2(w)$ , respectively, in the actually free space region of  $\theta_1 \le w \le \theta_2$ . Both  $p_1(w)$  and  $p_2(w)$  are  $2\pi\nu$ -periodic functions. Hence  $p_1(w)$  and  $p_2(w)$  can exist only in the ranges of  $\theta_1 \le w$  and  $w \le \theta_2$ , respectively, as shown in Fig. 3. Therefore, when  $S_1$  and  $S_2$  are defined in the range of  $0 \le w < \theta_1$  and  $\theta_2 < w \le 2\pi$ , respectively, p(w) should be changed as

$$p(w) = \int_{-p_1(w+2\pi) + p_2(w), \text{ in } 0 \le w < \theta_1} p_1(w) + p_2(w), \text{ in } \theta_1 \le w \le \theta_2 \\ p_1(w) + p_2(w-2\pi), \text{ in } \theta_2 < w \le 2\pi$$
 (10)

where  $p_1(w)$  and  $p_2(w)$  are given by Eqs. (8b) and (8c), respectively. It can be easily proved that p(w) in Eq.(10) satisfies Eq.(9) in the complimentary wedge region of  $0 \le w < \theta_1$  and  $\theta_2 < w \le 2\pi$ . For the same example of  $\theta_1 = 60^\circ$ ,  $\theta_2 = 270^\circ$ , and  $\theta_i = 180^\circ$ , Fig. 4 illustrates that p(w) in Eq.(10) becomes zero in  $0 \le w < \theta_1$  and  $\theta_2 < w \le 2\pi$ .

## 5. Conclusion

According to the formulation of dual integral equations, the exact diffraction coefficients of a perfectly conducting wedge have to become zero in the artificially complimentary region inside the wedge, of which material is substituted by the same material as that outside the wedge. To satisfy such a null-field condition in the extended region, the angular range of the complimentary wedge region is defined differently according to the components of the diffraction coefficients because the diffraction coefficients satisfy the edge condition at the tip of the wedge. We are now trying to solve the diffraction from a wedge composed of conductor and lossless dielectric by applying the presented null-field condition to the corresponding dual integral equations.

#### References

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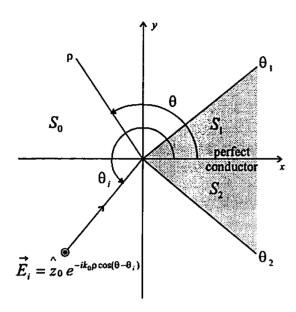
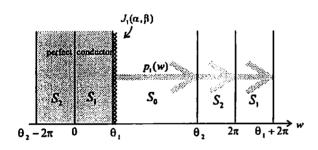


Fig. 1 Geometry of a perfectly conducting wedge illuminated by an E-polarized plane wave



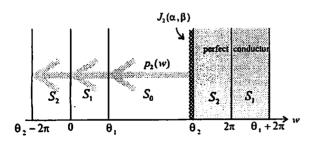


Fig. 3 Angular distribution ranges of  $p_1(w)$  and  $p_2(w)$  radiated from the induced sources  $J_1(\alpha, \beta)$  at  $\theta = 0$  and  $J_2(\alpha, \beta)$  at  $\theta = \theta_c$ , respectively

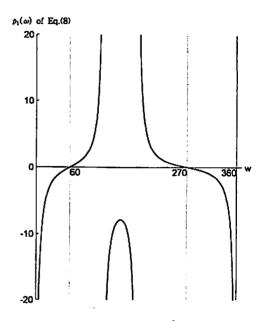


Fig. 2 Diffraction coefficients defined by Eq.(8) for  $\theta_1 = 60^\circ$ ,  $\theta_2 = 270^\circ$ , and  $\theta_i = 180^\circ$ 

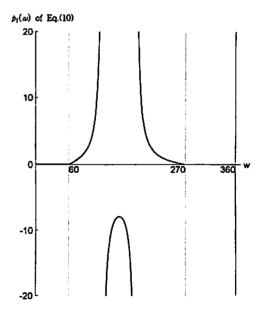


Fig. 4 Diffraction coefficients defined by Eq.(10) for  $\theta_1 = 60^\circ$ ,  $\theta_2 = 270^\circ$ , and  $\theta_i = 180^\circ$