## A–2–4 MOMENT METHOD CALCULATION OF REFLECTION COEFFICIENT FOR WAVEGUIDE ELEMENTS IN A FINITE PHASED ARRAY*

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The interior problem of the reflection coefficient for probe-fed cavity-backed slot antennas in a phased array configuration is considered in this paper. The exterior coupling problem is also considered. A probe-fed waveguide, in an infinite ground plane, is shown in Fig. 1. The solution utilizes the method of moments to find the induced aperture distribution due to the probe source[1]. This aperture distribution is then used to find the reflected fields in the waveguide. The reflection coefficient is found by taking the ratio of the reflected and incident fields at a point equidistant between the probe and the aperture.

The first step in a solution to this problem is to cover the aperture with a perfect electric conductor and place magnetic current sheets $\overline{M}_s$ and $-\overline{M}_s$ on the waveguide and half-space sides, respectively. The magnetic current $\overline{M}_s$ is unknown but is related to the electric field in the aperture by taking the cross product with the unit normal. These currents satisfy the condition that the tangential electric field be continuous at the boundary between the waveguide and half-space regions. An integral equation involving the unknown current $\overline{M}_s$ can be obtained by satisfying the boundary condition that the tangential magnetic field be continuous at the aperture. The tangential component of the magnetic field is seen from Fig. 1 to be x-directed due to the probe source orientation. The contribution to the magnetic field in the waveguide region at the aperture is the incident field $H_x^i$ from the probe source and the scattered field $H_x^{wg}$ from $\overline{M}_s$. The magnetic field $H_x^{hs}$ in the half-space region is due to $-\overline{M}_s$ alone. Equating the magnetic field on both sides of the aperture yields[2]

$$H_x^{wg}(\overline{M}_s) + H_x^{hs}(\overline{M}_s) = - H_x^i \quad . \tag{1}$$

To obtain an approximate solution for $\overline{M}_s$ introduce a set of expansion functions $\{\overline{M}_n, n=1,2,\cdots,N\}$ and let

$$\overline{M}_s = \sum_n V_n \frac{\overline{M}_n}{K^n} \tag{2}$$

where $V_n$ is the unknown coefficient to be determined and

$K^n$ is the terminal current for $\overline{M}_n$.

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The expansion functions that are used in this solution have piecewise sinusoidal current distribution along the direction of current flow and are uniform along the transverse direction.

Next, consider the inner product

$$<\overline{M},\overline{H}> = \iint_{\text{aperture}} \overline{M}\cdot\overline{H}\, ds \tag{3}$$

and a set of testing functions $\{\overline{W}_m,\ m=1,2,\cdots,N\}$ equal to the expansion functions (Galerkin's method). Substituting Equations (2) and (3) into (1) and taking the inner product with each testing function $\overline{M}_m$ leads to the set of simultaneous equations

$$\sum_n V_n \frac{<M_m,H_x^{wg}(M_n)>}{K^m K^n} + \sum_n V_n \frac{<M_m,H_x^{hs}(M_n)>}{K^m K^n} = - \frac{<M_m,H_x^i>}{K^m} \tag{4}$$

$m=1,2,\cdots,N.$

Recognizing the quantities $\frac{<M_m,H_x^{wg}(M_n)>}{K^m K^n}$ and $\frac{<M_m,H_x^{hs}(M_n)>}{K^m K^n}$ as admittances and the quantity $-\frac{<M_m,H_x^i>}{K^m}$ as current enables Equation (4) to be written as

$$\sum_n V_n \left(Y_{mn}^{wg} + Y_{mn}^{hs}\right) = I_m \qquad m=1,2,\cdots,N \quad . \tag{5}$$

In matrix notation the solution for the coefficients $V_n$ becomes

$$(V) = [[Y^{wg}] + [Y^{hs}]]^{-1}(I) \quad . \tag{6}$$

These coefficients are then substituted into Equation (2) to determine $\overline{M}_S$. Now that $\overline{M}_S$ has been determined the fields inside the waveguide can be found.

The voltage reflection coefficient at the equidistant point for the $TE_{mn}$-th mode is calculated by the equation

$$\Gamma_{mn} = - \frac{H_{x_{mn}}^{wg} + H_{x_{mn}}^{image}}{H_{x_{mn}}^{i}} \tag{7}$$

where

$H_x^{wg}$ is the scattered field due to $\overline{M}_s$ and

$H_x^{image}$ is the image of the incident field due to the infinite ground plane.

The above magnetic fields are determined by imaging the sources in the waveguide walls, resulting in an infinite array, and then applying the Poisson sum formula to obtain a plane wave expansion. The reflection coefficient at the aperture is then found by introducing an appropriate phase shift. Reflection coefficients for various sizes of rectangular guides were computed and found to be in excellent agreement with data obtained from the literature.

To apply this technique to a finite phased array of rectangular waveguides it is necessary to include the coupling between apertures. This affects only the half space admittance calculation. The number of elements that can be analyzed is restricted by the size of the admittance matrix. Fortunately a good deal of symmetry exists resulting in an admittance matrix which is block Toeplitz. This greatly reduces the numer of elements required to be stored.

Shown in Fig. 2 are results obtained with our method compared to results in the literature[3]. The agreement is seen to be excellent. Similar agreement has been obtained for the $TE_{10}$ mode to $a/\lambda=2$ when compared against unpublished data by Zaghloul and MacPhie. We have also obtained reflection coefficient results for the $TE_{30}$ mode which are not yet verified.

Figure 3 shows the variation of the reflection coefficient with scan angle for two elements in a 5 by 11 array of rectangular waveguide apertures. One element is the center most element and the other is the edge element in the same column in the direction opposite the scanning direction of the main beam. It can be seen that there is a significant difference in the behaviour of the two elements.

Results will be presented for various array configurations of rectangular and square elements. Advantages and limitations of the method will also be discussed.

REFERENCES

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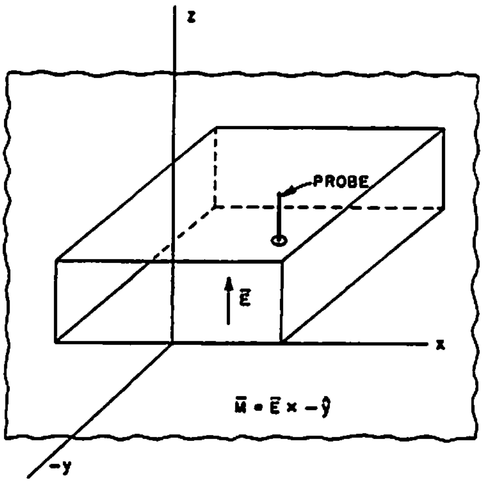


Fig. 1. Probe-fed waveguide element.

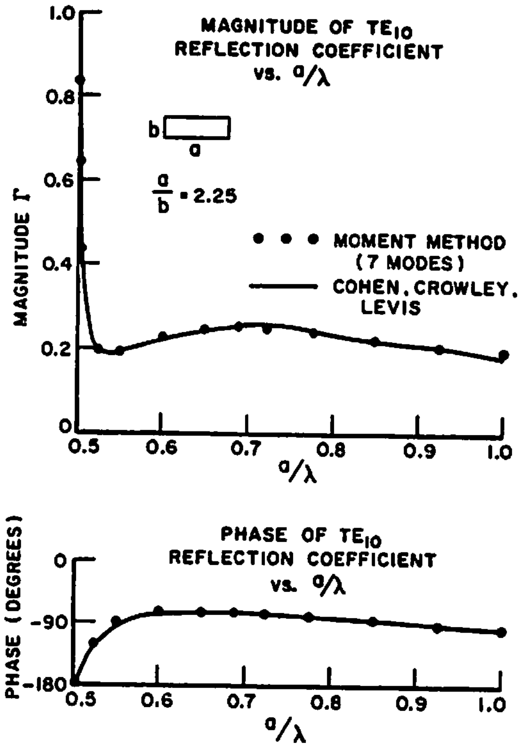


Fig. 2. Reflection coefficient data.

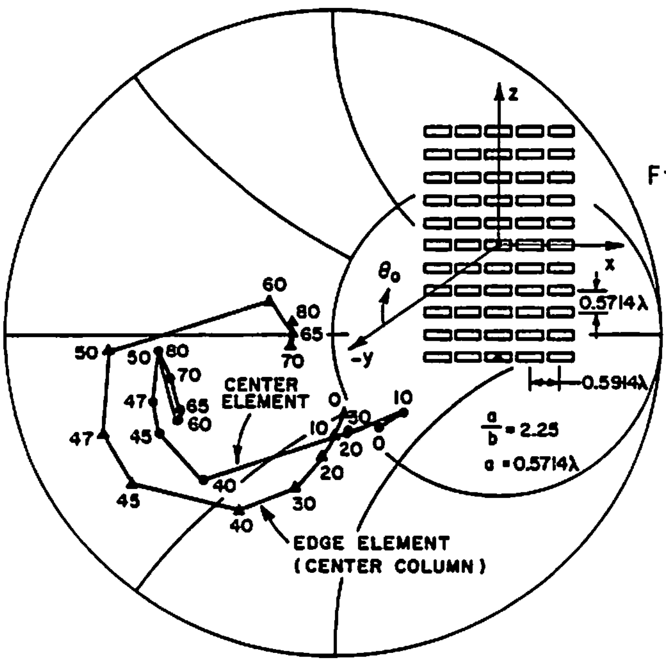


Fig. 3. Reflection coefficient variation with scan angle $\theta_0$ for two elements in a 5 x 11 planar array.