1-1 C4

ANALYSIS OF ELLIPTIC LOOP ANTENNA

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§1 INTRODUCTION

In this paper, the characteristics of the elliptic loop antenna are discussed by expressing the electromagnetic field of the antenna in the elliptic-cylindrical coordinates.

§2 ELLIPTIC-CYLINDRICAL COORDINATES

The variables(ξ,η,Z) are called the elliptic-cylindrical coordinates.

They are related to the rectangular coordinates by the eq.(1).

$$x = a\cosh\xi\cdot\cos\eta,\quad y = a\sinh\xi\sin\eta,\quad Z = Z \qquad (1)$$

where

$$0 \leqq \xi \leqq \infty,\ -\pi \leqq \eta \leqq \pi,\ -\infty \leqq Z \leqq \infty$$

The coordinates surface, ξ=constant, is a cylinder of elliptic cross section, whose foci are P1 and P2. The surface, η=constant, represents a family of conforcal hyperbolic cylinders of two sheets as shown Fig.1.

§3 ELECTROMAGNETIC FIELD

On the assuming that a radius of the antenna is very small, the current source $I(\eta)$ exsists in the center of the antenna wire and is a filament current having η component only.

$P(\xi,\eta,Z)$ on the elliptic-cylindrical coordinates is a point of the current source and $P(\xi',\eta',Z')$ is a observation-point.

Electric field at the observation-point is shown as the eq.(2).

$$E_\eta(\xi',\eta',Z') = \int_{-\pi}^{\pi} I(\eta)\, G(\eta\ \eta')\, d\eta \qquad (2)$$

where

$$G(\eta,\eta') = \frac{1}{j4\pi\omega\varepsilon}\frac{1}{\sqrt{\sinh^2\xi'+\sin^2\eta'}}\Big\{a^2k^2(\sinh\xi'\cos\eta' \times \sinh\xi\cos\eta - \cosh\xi'\sin\eta'\cosh\xi\sin\eta) - \frac{\partial}{\partial\eta'}\frac{\partial}{\partial\eta}\Big\}\frac{1}{r}e^{-jkr}$$

r is a distance between a current source-point and a observation-point, ω is the angular frequency, k is the propagation constant, and a is the focal distance of the ellipse.

§4 FOURIER SERIES SOLUTION OF THE CURRENT DISTRIBUTION

The induced electric field by the driving potential difference V applied across the gap of the antenna is expanded to the Fourier series as shown the eq.(3).

$$E_{\eta'}(\xi',\eta',Z') = \sum_{m=-\infty}^{\infty} V_m e^{-jm\eta'} \qquad (3)$$

where V_m is the Fourier coefficient.

In the eq.(2), $I(\eta)$ and $G(\eta,\eta')$ are expanded to the Fourier series as shown the eq.(4),(5).

$$I(\eta) = \sum_{n=-\infty}^{\infty} I_n e^{jn\eta} \qquad (4)$$

$$G(\eta,\eta') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} A_{m,n} e^{jm\eta'} e^{-jn\eta} \qquad (5)$$

where I_n and $A_{m,n}$ are the Fourier coefficients respectively.

From the eq.(2),(3),(4),(5), the coefficients of the Fourier series solution are given as the eq.(6)

$$[I] = \frac{1}{2\pi}[A]^{-1}[V] \qquad (6)$$

where

- $[I]$: column vector consisted of element I_n
- $[V]$: column vector consisted of element V_m
- $[A]^{-1}$: inverse matrix of matrix $[A]$ consisted of element $A_{m,n}$

Fig.(2),(3),(4),(5) are the examples of the current distribution and the radiation pattern respectively.

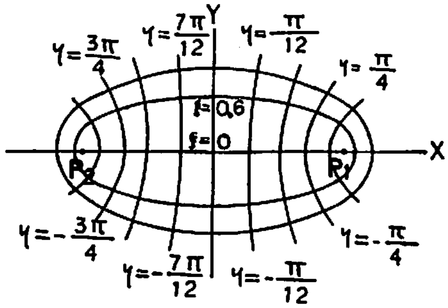

Fig.1 Elliptic-cylindrical coordinates

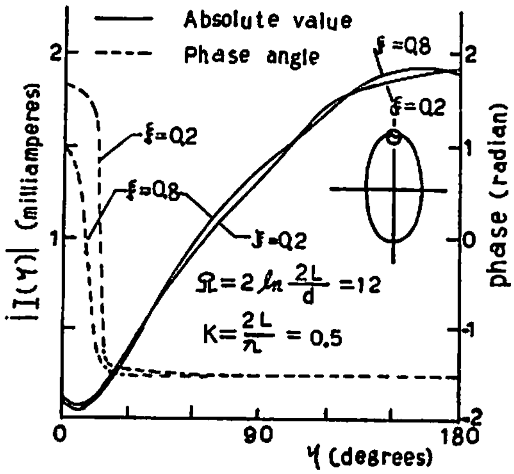

Fig.2 Absolute value and phase angle of I(ψ)

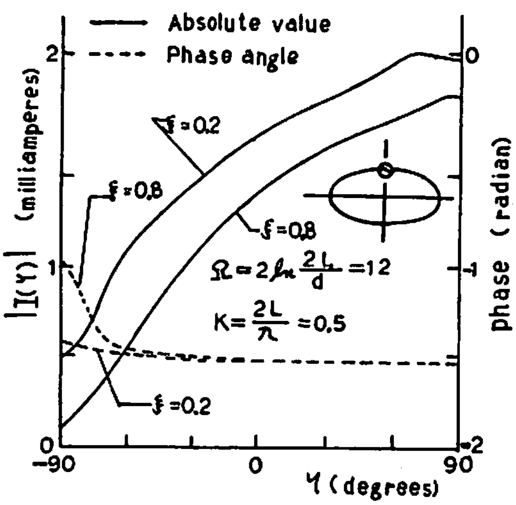

Fig.3 Absolute value and phase angle of I(ψ)

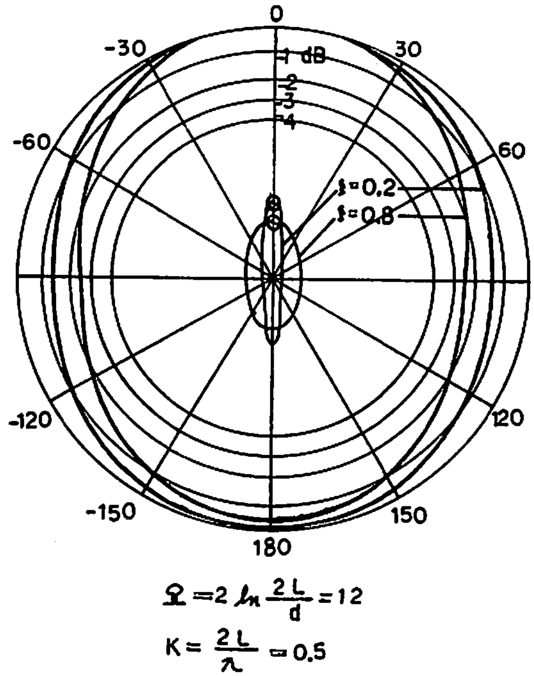

Fig.4 Horizontal electric field pattern

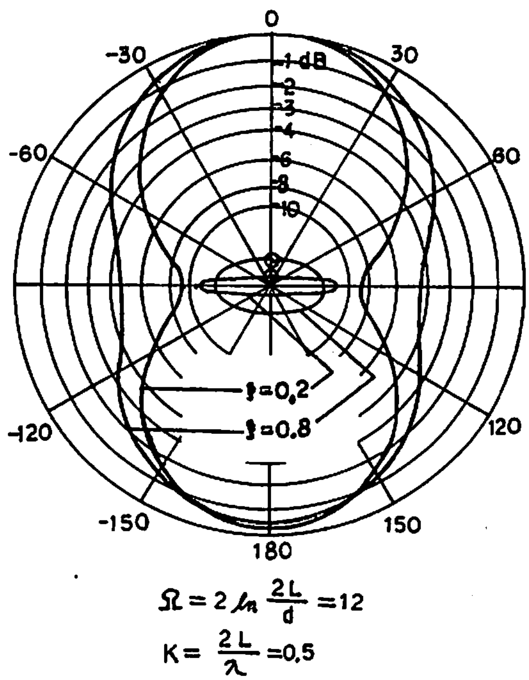

Fig.5 Horizontal electric pattern