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PLANE WAVE DIFFRACTION BY A THICK SLIT - TM CASE -

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1 Introduction

Aperture diffraction problems of electromagnetic waves have been investigated by many authors because of the simple geometry [1-8]. When an infinitely thin slit is composed by two half planes, it would be rather easy to treat this problem [1,2]. However, if the slit has a finite thickness, it will be more difficult to solve it. From a view point of practical applications, it would be very important to see the effect from the thickness of the slit. The solution of Kashyap and Hamid for a thick slit has been derived by Wiener-Hopf and generalized matrix techniques [3]. Hongo et al. have analyzed this problem by using eigen function expansion in terms of Weber-Schafheitlin's discontinuous integrals [4,5]. Auckland and Harrington have utilized the generalized network formulation of coupling through apertures for a thick slit [6]. For pretty wide and thick aperture cases, ray-mode conversion technique has been used to evaluate the diffracton field by a thick slit [7]. Park et al. have formulated the diffractin field from a thick slit in the Fourier spectrum region [8].

In this paper, electromagnetic plane wave diffraction by a loaded thick slit has been formulated using Kobayashi and Nomura's method [2,4,5] , which utilizes the characteristics of the Weber-Schafheitlin's discontinuous integrals. We now extend the method developed by Hongo [4,5] to the case which the aperture of the thick slit is homogeniously filled by some materials. Compared with the previous formulation based on rays [7], the present one is rather suitable for a narrow aperture case. Numerical calculations has been done to obtain scattering patterns for both empty and filled cases.

2 Formulations

As illustrated in Fig.1, E-polarized electromagnetic plane wave:

$$\phi^i(=E_z^i) = \exp[-ik_0(x\cos\theta_0 + y\sin\theta_0)] \tag{1}$$

impinges on the aperture of a perfectly conducting slit with width $2a$, and thickness b. $k_0(=\omega/c)$ denotes free space wavenumber. For later convenience, the region surrounding the slit is divided into three regions: region I; $y>0$, region II; $|x|<a$, $-b<y<0$, region III; $y<-b$. In region I, the total field ϕ^t may be considered as:

$$\phi^t = \phi^i + \phi^r + \phi_I, \tag{2}$$

where $\phi^r(= -\exp[-ik_0(x\cos\theta_0 - y\sin\theta_0)])$ is the reflected wave and ϕ_I is the scattering contribution due to the slit, which may be written with unknown coefficients A_m and B_m as:

$$\phi_I = \sqrt{\frac{\pi u}{2}}\sum_{m=0}^{\infty}\int_0^{\infty}\frac{1}{\sqrt{\xi}}\left\{A_m J_{2m+1}(\xi)J_{-1/2}(\xi u)\right.$$

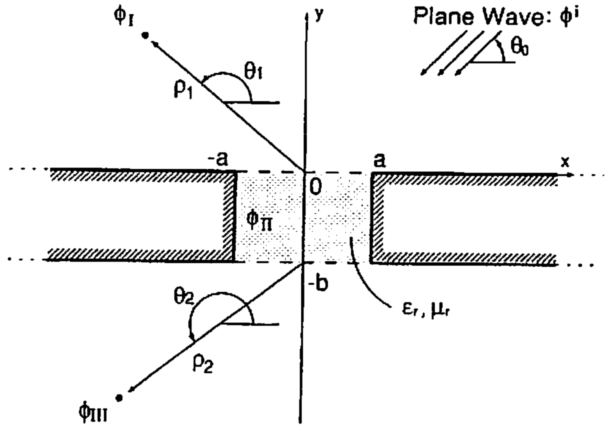

Figure 1: A Thick Slit.

$$+ B_m J_{2m+2}(\xi) J_{1/2}(\xi u)\Big\} e^{-\sqrt{\xi^2-\kappa_0^2}v} d\xi. \qquad (y > 0) \qquad (3)$$

Similarly, there is the diffracted wave in region III, which may be expressed as:

$$\phi_{III} = \sqrt{\frac{\pi u}{2}} \sum_{m=0}^{\infty} \int_0^{\infty} \frac{1}{\sqrt{\xi}} \Big\{ C_m J_{2m+1}(\xi) J_{-1/2}(\xi u)$$
$$+ D_m J_{2m+2}(\xi) J_{1/2}(\xi u)\Big\} e^{+\sqrt{\xi^2-\kappa_0^2}(v+b_0)} d\xi. \qquad (y < -b) \quad (4)$$

where C_m, D_m are unknown coefficients and Cartesian coordinate (x, y) is normalized by a, i.e. $x = au$, $y = av$, $\kappa_0 = k_0 a$, and $b = b_0 a$. The integrals in Eqs.(3),(4) are a class of Weber-Schafheitlin's discontinuous integrals. Considering the characteristics of the above integrals[5], one can automatically satisfy the boundary conditions on both conducting surfaces at $y = 0$ and $y = -b$.

For region II, one can expand the internal field $\phi^{(II)}$ by waveguide eigen modes as

$$\phi^{(II)} = \sum_{n=1}^{\infty} \sin\{\frac{n\pi}{2}(1-u)\}[E_n e^{+ih_n av} + F_n e^{-ih_n av})] \qquad (5)$$

with the modal propagation constant $h_n = \{k^2 - (n\pi/2a)^2\}^{1/2}$ with respect to the $y[v]$ direction, and E_n and F_n are the modal excitation coefficients to be determined.

By using the continuity conditions of the tangential field components at both apertures of the thick slit ($y = 0$ and $y = -b$, $|x| \le a$), one can determine the unknown coefficients A_m, B_m, C_m, D_m, E_n and F_n. Following the procedure discussed in Ref.[5], one may end up with the following simultaneous equations:

$$\sum_{m=0}^{\infty} P_m\{K(2m+1, 2q+1) + KN(2m+1, 2q+1)\} = -2iJ_{2q+1}(\kappa_0 \cos\theta_0)\tan\theta_0, \quad (6)$$

$$\sum_{m=0}^{\infty} Q_m\{K(2m+1, 2q+1) + KP(2m+1, 2q+1)\} = -2iJ_{2q+1}(\kappa_0 \cos\theta_0)\tan\theta_0, \quad (7)$$

$$\sum_{m=0}^{\infty} S_m\{K(2m+2, 2q+2) + KN(2m+2, 2q+2)\} = -2J_{2q+2}(\kappa_0 \cos\theta_0)\tan\theta_0, \quad (8)$$

$$\sum_{m=0}^{\infty} T_m\{K(2m+2, 2q+2) + KP(2m+2, 2q+2)\} = -2J_{2q+2}(\kappa_0 \cos\theta_0)\tan\theta_0, \quad (9)$$

where $P_m = A_m + C_m$, $Q_m = A_m - C_m$, $S_m = B_m + D_m$, $T_m = B_m - D_m$, and function $K(p,q)$ is given by

$$K(p,q) = \int_0^\infty \frac{\sqrt{\xi^2 - \kappa_0^2}}{\xi^2} J_p(\xi) J_q(\xi) d\xi, \tag{10}$$

which can be easily computed by a series expansion[2]. Functions $KN(\cdot,\cdot)$ and $KP(\cdot,\cdot)$ are given respectively by

$$KN(2m+1, 2q+1) = -\frac{\pi}{\mu_r} \sum_{n=0}^{\infty} \frac{(ih_{2n+1}a) J_{2m+1}(\frac{2n+1}{2}\pi) J_{2q+1}(\frac{2n+1}{2}\pi)}{(\frac{2n+1}{2}\pi)^2} \cdot \frac{1 - e^{ih_{2n+1}ab_0}}{1 + e^{ih_{2n+1}ab_0}}, \tag{11}$$

$$KP(2m+1, 2q+1) = -\frac{\pi}{\mu_r} \sum_{n=0}^{\infty} \frac{(ih_{2n+1}a) J_{2m+1}(\frac{2n+1}{2}\pi) J_{2q+1}(\frac{2n+1}{2}\pi)}{(\frac{2n+1}{2}\pi)^2} \cdot \frac{1 + e^{ih_{2n+1}ab_0}}{1 - e^{ih_{2n+1}ab_0}}, \tag{12}$$

$$KN(2m+2, 2q+2) = -\frac{\pi}{\mu_r} \sum_{n=0}^{\infty} \frac{(ih_{2n+2}a) J_{2m+2}((n+1)\pi) J_{2q+2}((n+1)\pi)}{((n+1)\pi)^2} \cdot \frac{1 - e^{ih_{2n+2}ab_0}}{1 + e^{ih_{2n+2}ab_0}}, \tag{13}$$

$$KP(2m+2, 2q+2) = -\frac{\pi}{\mu_r} \sum_{n=0}^{\infty} \frac{(ih_{2n+2}a) J_{2m+2}((n+1)\pi) J_{2q+2}((n+1)\pi)}{((n+1)\pi)^2} \cdot \frac{1 + e^{ih_{2n+2}ab_0}}{1 - e^{ih_{2n+2}ab_0}}. \tag{14}$$

Introducing the cylindrical coordinates (ρ_1, θ_1) for region I and (ρ_2, θ_2) for region III, as in Fig.1. scattering far field ϕ_I and ϕ_{III} may be obtained from Eqs.(3) and (4) by the saddle point method, assuming $k_0\rho_j \gg 1 (j = 1, 2)$, as

$$\phi_I \sim \sqrt{\frac{\pi}{2k_0\rho_1}} e^{i(k_0\rho_1 + \pi/4)} \sum_{m=0}^{\infty} \{A_m J_{2m+1}(k_0 a \cos\theta_1) - iB_m J_{2m+2}(k_0 a \cos\theta_1)\} \tan\theta_1, \tag{15}$$

$$\phi_{III} \sim \sqrt{\frac{\pi}{2k_0\rho_2}} e^{i(k_0\rho_2 + \pi/4)} \sum_{m=0}^{\infty} \{C_m J_{2m+1}(k_0 a \cos\theta_2) - iD_m J_{2m+2}(k_0 a \cos\theta_2)\} \tan\theta_2. \tag{16}$$

3 Numerical Calculation and Discussions

Using the above formulation, numerical calculations have been performed. Fig.2(a) and Fig.2(b) show the normalized scattered patterns for incident angle $\theta_0 = 30°$ and $60°$, respectively. For the cases of the empty slit with thickness $b = 1.6\lambda_0$(———) and $b_0 = 3.2\lambda_0$(— — —), the maximum peak of scattering in region I occurs at the vicinity of the corresponding specular reflection direction for each incidence case. It is also found in these figures that main diffraction lobe in region III changes its direction in accordance with thickness of the slit. This behavior may be understood by GO ray tracing. For the case filled by a lossy material ($\epsilon_r = 2.5 + i0.2, \mu_r = 1.8 + i0.1$) with thickness $b = 1.6\lambda_0$ (- - - -), certain reduction effects due to the loss in the material can be easily observed.

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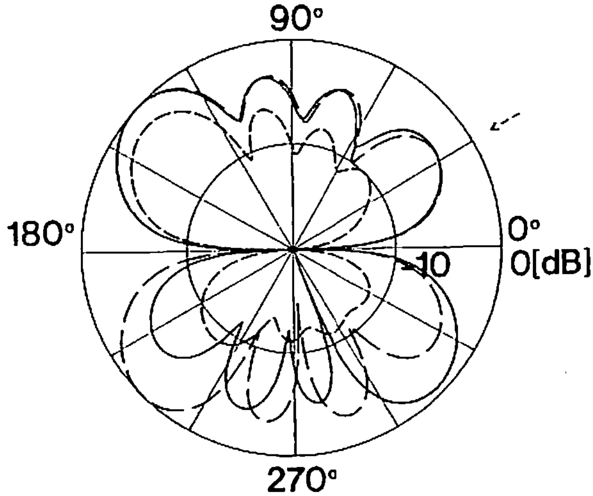
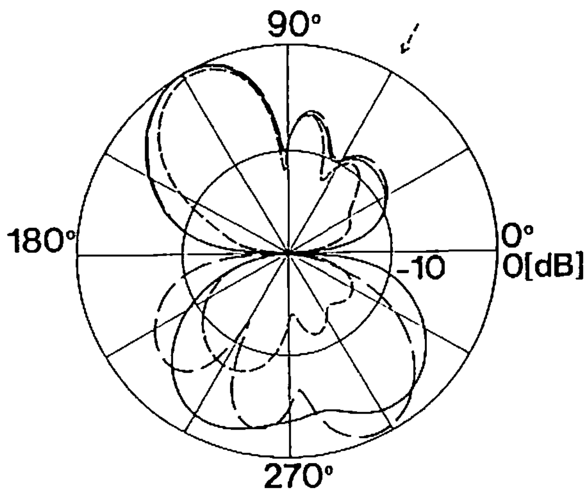


(a)



(b)

Figure 2: Scattered far field pattern by a thick slit. a=$1.1\lambda_0$, ———— :$\epsilon_r = 1.0, \mu_r = 1.0, b = 1.6\lambda_0$; — — — —:$\epsilon_r = 1.0, \mu_r = 1.0, b = 3.2\lambda_0$; - - - - - :$\epsilon_r = 2.5+i0.2, \mu_r = 1.8+i0.1, b = 1.6\lambda_0$. (a) $\theta_0 = 30°$, (b) $\theta_0 = 60°$