AN ANALYSIS OF THE ELLIPTICAL CORRUGATED HORN FOR CIRCULARLY POLARIZED WAVE FEED

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Introduction

In a broadcasting satellite, a circularly polarized wave is used and elliptical beams are mostly required. When the single antenna is used for transmitting and receiving, the requirements for characteristics of circular polarization should be satisfied in both down and feederlink bands. In such cases, an elliptical corrugated horn is suitable as a primary feed from the view point of low cross-polarized radiation in broad frequency bands and efficient illumination of the elliptical paraboloidal reflector.

It is known that the elliptical balanced hybrid mode radiates no cross-polarized waves(1). Corrugations on the conducting wall are one of the ways to realize a balanced hybrid mode. However, the frequency characteristics of the elliptical corrugated horn are analytically not known at present. Since the balanced hybrid mode is an idealized one, in order to obtain the frequency characteristics the analysis under the condition other than the balanced hybrid mode must be done. The purpose of this paper is to present a design and analysis method for an elliptical corrugated horn. The k-β curves are calculated by using the analysis method and the results are compared with measured ones.

An Analysis of the Aperture Field of an Elliptical Corrugated Waveguide

Though an analysis of an elliptical corrugated horn is a three dimensional problem, for simplicity, a horn is approximated to a waveguide. Fig. 1 shows the transversal cross-section of an elliptical corrugated waveguide and elliptical coordinate system. The ellipse in which $\xi=\xi_1$ is the inner boundary(the tip of the corrugation): and the one in which $\xi=\xi_0$ is the outer boundary(the bottom of the corrugation). Fig. 2 shows the longitudinal cross-section of the elliptical corrugated waveguide. w is the width and p is the pitch of the corrugation.

Because of the asymmetry of the elliptical aperture there are even and odd modes which correspond to fields excited electrically in minor and major axis direction respectively. The inner fields ($\xi<\xi_1$) are expressed by the sum of the elliptical hybrid modes which are composed of elliptical TM and TE modes. A time dependence $e^{j\omega t}$ and the z-dependence $e^{-j\beta z}$ are assumed, but omitted in the equations. The outer fields ($\xi_1<\xi<\xi_0$) are in z-independent TM radial line modes. If the width of corrugation w is sufficiently small such that TE modes are cut-off in the corrugation, $E\eta=0$ at the inner boundary $\xi=\xi_1$. By introducing the boundary condition in which the tangential components of the electric and magnetic fields E_z, E_η, H_η are continuous at $\xi=\xi_1$, the boundary condition equations are obtained as follows(2).

Even mode

$$b_p Jo_p(\xi_1, q) = \sum_m c_m \{Jo_m(\xi_1, q') No_m(\xi_o, q') - No_m(\xi_1, q') Jo_m(\xi_o, q')\} \gamma_{mp} \tag{1}$$

$$a_p J'e_p(\xi_1, q) - \overline{\beta} \sum_m b_m Jo_m(\xi_1, q) \nu_{mp} = 0 \tag{2}$$

$$\overline{\beta} \sum_m a_m Je_m(\xi_1, q) \chi_{mp} + b_p J'o_p(\xi_1, q) = \frac{k_c^2}{k^2} \sum_t c_m \{J'_{ot}(\xi_1, q') No_t(\xi_0, q') - No'_t(\xi_1, q') Jo_t(\xi_0, q')\} \gamma_{tp} \tag{3}$$

Odd mode

$$a'_p Je_p(\xi_1, q) = \sum_m c'_m \{Je_m(\xi_1, q') Ne_m(\xi_0, q') - Ne_m(\xi_1, q') Je_m(\xi_0, q')\} \alpha_{mp} \tag{4}$$

$$b'_p J'o_p(\xi_1, q) - \overline{\beta} \sum_m a'_m Je_m(\xi_1, q) \chi_{mp} = 0 \tag{5}$$

$$\overline{\beta} \sum_m b'_m Jo_m(\xi_1, q) \nu_{mp} + a'_p J'e_p(\xi_1, q) = \frac{k_c^2}{k^2} \sum_t c'_t \{J'e_t(\xi_1, q') Ne_t(\xi_0, q') - Ne'_t(\xi_1, q') Je_t(\xi_0, q')\} \alpha_{tp} \tag{6}$$

where $Je(\xi, q)$, $Jo(\xi, q)$: the radial first Mathieu functions
$Ne(\xi, q)$, $No(\xi, q)$: the radial second Mathieu functions

$\overline{\beta} = \beta/k$, $q = (k_c h)^2/4 = (kh)^2(1 - \overline{\beta}^2)/4$, $q' = (kh)^2/4$

a_m, b_m, c_m, a'_m, b'_m, c'_m :coefficients of modes

$$\chi_{mn} = -\frac{\sum_{k=0}^{\infty} (2k+1) De_{2k+1}^{m} \; Do_{2k+1}^{n}}{\sum_{k=0}^{\infty} [Do_{2k+1}^{n}]^2} \tag{7}$$

$$\nu_{mn} = \frac{\sum_{k=0}^{\infty} (2k+1) De_{2k+1}^{n} Do_{2k+1}^{m}}{\sum_{k=0}^{\infty} [De_{2k+1}^{n}]^2} \tag{8}$$

α_{mn}, γ_{mn} are obtained by solving the following equations respectively.

$$De_{2k+1}^{m}(q') = \sum_n \alpha_{mn} De_{2k+1}^{n}(q) \tag{9}$$

$$Do_{2k+1}^{m}(q') = \sum_n \gamma_{mn} Do_{2k+1}^{n}(q) \tag{10}$$

De, Do : coefficients of Mathieu functions

m, n, p, t = 1, 3, 5 ……

The infinite series expressing the boundary condition must be truncated in order to be solved numerically. In this paper, the field is assumed to be composed of TM and TE waves of the orders 1,3,5,7,9 and 11.

The Design of the Elliptical Corrugation and k-β Characteristics

The balanced hybrid condition is $E_\eta=0$ and $H_\eta=0$ at $\xi=\xi_1$. Under this condition a single phase constant β satisfies the boundary equations for even and odd mode, and the amplitudes of the radiation fields from the balanced hybrid mode are also equal. In this paper, the corrugation depth ($\xi_0-\xi_1$) is chosen such that the H_η's of the TM_{11} radial line mode are zero at $\xi=\xi_1$ for even and odd modes respectively and the mean value of the two depths is then taken. The parameters for the corrugation shape designed for 12 GHz are as follows. (λ=25mm)

inner ellipse major axis $2a_1$=86mm=3.44λ
minor axis $2b_1$=37.5mm=1.5λ eccentricity e_1=0.9
outer ellipse major axis $2a_0$=92.85mm
minor axis $2b_0$=51.3mm eccentricity e_0=0.834
pitch p=3.3mm width w=3mm

By solving the boundary condition equations (1)-(6), β is obtained. Fig. 3 shows the calculated and measured k-β characteristics for the even and odd dominant hybrid mode HE_{11}. The β_b curve in balanced hybrid mode and the difference between β_b and β_e, β_o are also shown in Fig. 3. Frequency bands are from 10.5 GHz to 14.5 GHz which include the down and feeder link bands for operational Japanese Broadcasting Satellite. For the even mode, the β_e curve crosses the β_b curve at 12 GHz as designed and changes largely depending on frequency. For the odd mode, the β_o curve is close to the β_b curve and the difference between β_b and β_o is small in broad bands. The measured curve coincides considerably with the calculated one for the even mode, but for the odd mode the measured curve slightly differs from the calculated one.

Summary

An analysis method of an elliptical corrugated horn was introduced. By using it the propagation characteristics were obtained and an elliptical corrugated horn can be estimated from the viewpoint of phase characteristics. The aperture field and the radiation pattern from the elliptical corrugated horn excited by circularly polarized waves are obtainable by applying this method for the condition other than the balanced hybrid mode.

References

(1) Jansen,J.K.M. and Jeuk,H.E.J. : "Circularly Polarized Horn Antenna with an Asymmetrical Pattern", Paper presented at Fifth Coll. On Microwave Communication Budapest, 21-30 June 1974.

(2) Shogen,K. and Toyama,N. : "An Analysis of Elliptical Corrugated Horn for Broadcasting Satellite", 14th ISTS, Chap.8, pp.879-886, 1984.

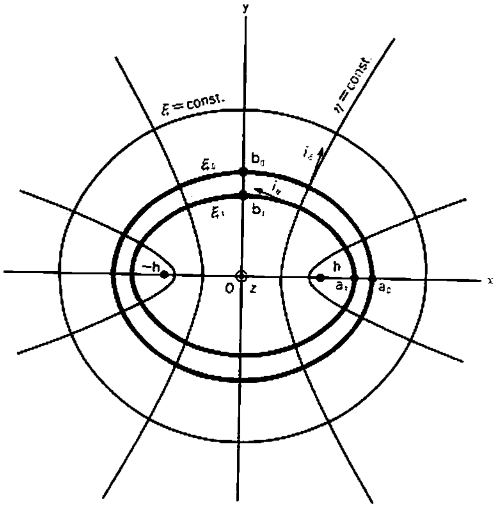

Fig. 1 Elliptical coordinate system and transversal cross-section of elliptical corrugated waveguide.

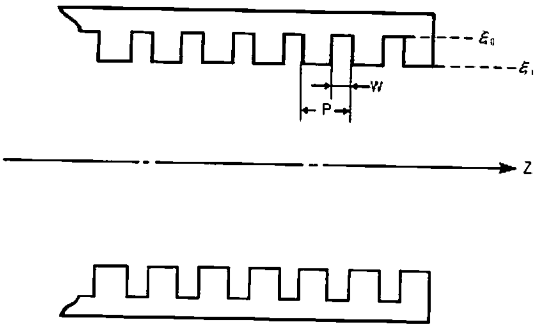

Fig. 2 Longitudinal cross-section of elliptical corrugated waveguide.

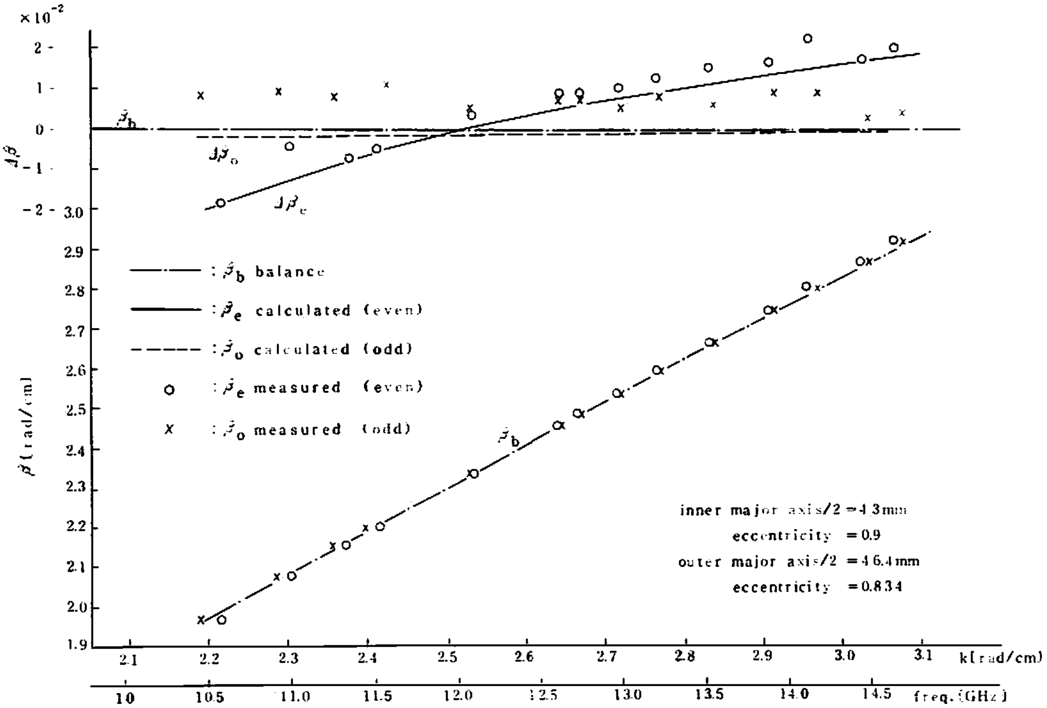

Fig. 3 Phase characteristics of an elliptical corrugated waveguide