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A TIME DOMAIN INTEGRAL EQUATION SOLUTION FOR THIN WIRE DIPOLES

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The behavior of a thin wire under the excitation of an arbitrary incident electric field E^{inc} can be described by a time domain integral equation which contains no derivatives at all. The solution of this integral equation, which corresponds to Hallen's integral equation, which corresponds to Hallen's integral equation in the frequency domain, is carried out by numerical methods on a digital computer, and has advantages in terms of speed and accuracy over the solution of an integro-differential equation method.[1,2]

For a straight thin wire, the integral equation is of the form

$$\int_0^L \frac{I(z,t-\frac{|z-z'|}{c})}{4\pi\sqrt{(z-z')^2+a^2}}dz' = \frac{1}{2\eta}\int_0^L E^{inc}(z',t-\frac{|z-z'|}{c})dz' + f_1(ct-z)+f_2(ct+z), \tag{1}$$

where $I(z,t)$ is the current on the antenna at time t, position z, $\underline{a}$ is the radius, L is the length of the wire, and η is the wave impedance of the medium. The two arbitrary functions, f_1 and f_2 arise from the solution to the homogeneous wave equation and are evaluated subject to the boundary condition that current vanishes at both ends of the wire. The integral involving the current is numerically approximated by a summation. The current I at a point (z_1, t_1) can be obtained if the current values prior to time t_1 along the two characteristic curves $t = t_1 - \frac{z_1-z}{c}$ for $z < z_1$ and $t = t_1 + \frac{z_1-z}{c}$ for $z > z_1$ are known. This procedure thus yields a step-by-step time marching solution, which is started with the most common initial condition that the wire is initially relaxed.

The calculated transient response results can be Fourier transformed to compare with the frequency domain results. As an example, the transient current response due to a Gaussian input waveform is converted to yield the input admittence of the center-driven wire as a function of frequency. The result is given in Fig. 1 for an antenna with $\Omega = 2\log L/a = 10$ and shows good agreement with that calculated by the frequency domain method.[3]

This method is extended to the case of coupled parallel dipoles. For N dipoles, the integral equation becomes

$$\int_{L_j}\frac{I_j(z',t-\frac{|z-z'|}{c})}{4\pi\sqrt{(z-z')^2+a_j^2}}dz' + \sum_{\substack{i=1\\i\neq j}}^{N}\int_{L_i}\frac{I_i(z',t-\frac{1}{c}\sqrt{(z-z')^2+b_{ij}^2})}{4\pi\sqrt{(z-z')^2+b_{ij}^2}}dz' = \frac{1}{2\eta}\int_{L_j}E^{inc}(z',t-\frac{|z-z'|}{c})dz' + f_{1j}(ct-z)+f_{2j}(ct+z)$$

$$j = 1, \ldots, N \quad, \tag{2}$$

where the subscript j denotes the dipole on which z is located, b_{ij} is the perpendicular separation between wires i and j, and a_j is the radius of wire j. This equation can be solved numerically in a manner similar to that of (1). As an example, the case of a pair of non-staggered, identical, parallel coupled

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dipoles separated at a distance equal to half the length of the dipoles is considered. With antenna 1 center-driven by a Gaussian input waveform and antenna 2 short-circuited at the center, the self and mutual admittances are related to the Fourier transforms of the currents at the centers of antenna 1 and antenna 2, respectively. The result is presented in Fig. 2 and shows good agreement with frequency domain calculation by Chang and King.[4]

The time domain integral equation method is not only good for studying transient responses of dipoles, but is also an efficient method to study dipole antennas in the frequency domain. As an example of its efficiency, the computation time for the result presented in Fig. 1, which is actually reliable up to $L/\lambda = 4$ with 64 data points, takes approximately 32 seconds on a CDC 6400 computer. This is a saving by a factor of at least 5 over the frequency domain method of comparable accuracy.

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FIGURE CAPTIONS

Fig. 1 Input admittance of a center-fed antenna with $\Omega = 2 \log L/a = 10$, where L is the length and a is the radius of the antenna.

Fig. 2 (a) Self admittance and (b) Mutual admittance of a pair of nonstaggered, identical, coupled parallel antennas separated at a distance equal to half the length of the antennas. $\Omega = 2 \log L/a = 8.5$, where L is the length and a is the radius of the antennas.

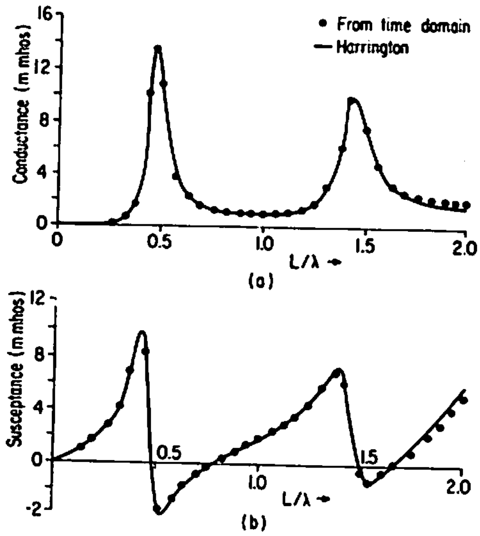

Fig. 1.

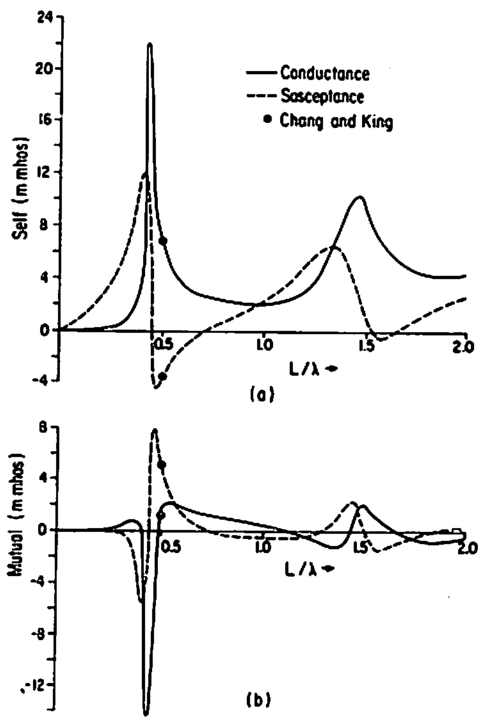

Fig. 2.