Multi-Reflector Offset Antennas Eliminating Cross-Polarization Component Based on Beam Mode Analysis

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1. Introduction

Offset antennas have more efficient and desirable wide-angle radiating characteristics than rotationally symmetric parabolic antennas and Casegrain antennas because there is no blocking due to the conical horn, sub-reflector, and supporting members. However, in space communications and microwave relay links, two orthogonal polarizations are used. In that case, the cross-polarization component that appears in the transverse plane due to the asymmetric reflectors is a problem. The cross-polarization elimination condition for the trireflector offset antennas was already derived [1].

In this paper, the general cross-polarization elimination condition for the multireflector offset antennas is derived by considering the phase of the cross-polarization component generated by each reflector. It is clarified that the derived condition is a geometrical relationship independent of the frequency. Further, the possible configurations obtained by applying this condition to a four-reflector beam waveguide feed are shown.

2. Derivation of the cross-polarization elimination condition

In a multireflector offset antenna consisting of a conical horn and many reflectors as shown in Fig. 1, if the unit vector $\mathbf{i_n}$ is taken on the plane that contains all foci of each reflector, $\mathbf{k_n}$ and $\mathbf{i_n}$ lie in the same plane, and $\mathbf{j_n}$ is perpendicular to the plane. Furthermore, let ω_n be the beam radius of the nth reflector, and R_n and R_n' be the radii of curvature of the wave front of the incident and reflected side of the nth reflector; then, the focal length f_n of the nth reflector is related by

$$\frac{1}{f_n}=\frac{1}{R_n}-\frac{1}{R_n'} \tag{1}$$

where, n=1,2,⋯,N. When the focal point is on the propagating direction of the ray, the signs of R_n and R_n' become negative (-). The reflector then becomes concave if f_n is positive and convex if f_n is negative. The distance d_n and the angle σ_n are as defined in Fig. 1. From the results in [2], the ratio C_N of the peak value of the cross-polarization component to that of the co-polarization component is given by

$$\begin{aligned} C_N &= \frac{1}{\sqrt{2e}}\left\{\sum_{n=1}^{N}(-1)^n \frac{q_n w_n}{f_n}\tan\frac{s_n}{2}\left(\prod_{i=1}^{n-1} e^{-jq_i}\right)\right\} \\ &= \frac{1}{\sqrt{2e}}\left\{a_1+\sum_{n=2}^{N} a_n\left(\prod_{i=1}^{n-1} X_i\right)\right\} \end{aligned} \tag{2}$$

where

$$
\begin{aligned}
a_n &= (-1)^n \frac{q_n}{f_n} \tan \frac{s_n}{2} \\
q_n &= sign(j_1 \cdot k_n \times k_{n+1}) \\
X_n &= \frac{w_{n+1}}{w_n} e^{-jq_n} \\
e^{-jq_n} &= \frac{1}{u_n}\left(e_n \sqrt{u_n^2 - 1} - j\right) \qquad (3) \\
u_n &= \frac{p w_n w_{n+1}}{l d_n} \\
e_n &= sign\left(\frac{1}{R_n'} + \frac{1}{d_n}\right)
\end{aligned}
$$

and λ is the free-space wavelength.

The relation between ω_n and ω_{n+1} is derived by means of the relationship among the beam mode parameters [3] as follows:

$$
\frac{w_n^2}{w_{n+1}^2} = \frac{n_n' R_n'}{n_{n+1} R_{n+1}} = \frac{1 + n_n'^2}{n_{n+1}^2} \qquad (4)
$$

where

$$
\frac{n_{n+1}}{n_n'} = 1 + \frac{d_n}{R_n'}\left(1 + \frac{1}{n_n'^2}\right). \qquad (5)
$$

The following relations are derived using this result:

$$
\begin{aligned}
\frac{w_n^2}{w_{n+1}^2} &= \frac{Y_n}{Y_n'} \\
Y_n &= 1 - \frac{d_n}{R_{n+1}} \qquad (6) \\
Y_n' &= 1 + \frac{d_n}{R_n'}
\end{aligned}
$$

and also

$$
\begin{aligned}
\frac{e_n \sqrt{u_n^2 - 1}}{u_n} &= \frac{w_n}{w_{n+1}} \cdot Y_n' \qquad (7) \\
\frac{1}{u_n^2} &= 1 - Y_n Y_n'.
\end{aligned}
$$

Therefore, X_n in (3) is rewritten as follows:

$$
\begin{aligned}
X_n &= Y_n' - jZ_n \qquad (8) \\
Z_n &= \frac{w_{n+1}}{w_n} \cdot \frac{1}{u_n}.
\end{aligned}
$$

Then, from (8) and the fifth equation in (3), we obtain

$$
\begin{aligned}
X_{n-1} \cdot X_n &= \left(Y_{n-1}' - jZ_{n-1}\right)\left(Y_n' - jZ_n\right) \\
&= h_n X_{n-1} - \frac{d_n}{d_{n-1}} \qquad (9) \\
h_n &= 1 + \frac{d_n}{d_{n-1}} - \frac{d_n}{f_n}.
\end{aligned}
$$

From the first equation in (9), we obtain

$$
\begin{aligned}
X_1 \cdot X_2 &= h_2 X_1 - \frac{d_2}{d_1} \qquad (10) \\
X_1 \cdot X_2 \cdot X_3 &= \left(h_3 \cdot h_2 - \frac{d_3}{d_2}\right) X_1 - h_3 \cdot \frac{d_2}{d_1}.
\end{aligned}
$$

Therefore, the following equation can be expressed:

$$
\prod_{i=1}^{n-1} X_i = P_n X_1 + Q_n . \qquad (11)
$$

Then, from (11) and the first equation in (9), we obtain

$$
\begin{aligned}
\prod_{i=1}^{n} X_i &= P_{n+1} X_1 + Q_n = X_n \prod^{n-1} X_i \\
&= \left(h_n - \frac{d_n}{d_{n-1}} \cdot \frac{1}{X_{n-1}}\right) \prod_{i=1}^{n-1} X_i \qquad (12) \\
&= h_n \prod_{i=1}^{n-1} X_i - \frac{d_n}{d_{n-1}} \prod_{i=1}^{n-2} X_i .
\end{aligned}
$$

By means of the relationship between (11) and (12), we obtain the following recursion relation:

$$
\begin{aligned}
P_{n+1} &= h_n P_n - \frac{d_n}{d_{n-1}} \cdot P_{n-1} \qquad (13) \\
Q_{n+1} &= h_n Q_n - \frac{d_n}{d_{n-1}} \cdot Q_{n-1}
\end{aligned}
$$

where $P_1=0$, $Q_1=1$ and $P_2=1$, $Q_2=0$ are derived from (2) and (11).

From these results, equation (2) is represented by

$$
\begin{aligned}
C_N &= \frac{w_1}{\sqrt{2e}} \left\{ a_1 + \sum_{n=2}^{N} a_n \left(P_n X_1 + Q_n\right) \right\} \qquad (14) \\
&= \frac{w_1}{\sqrt{2e}} \left\{ a_1 + \sum_{n=2}^{N} a_n Q_n + \left(\sum_{n=2}^{N} a_n P_n \right) X_1 \right\}.
\end{aligned}
$$

The condition for designing multireflector offset antennas without a cross-polarization component is C=0. From equation (14), we obtain the following equations:

$$\sum_{n=1}^{N} a_n P_n = 0 \qquad (15)$$
$$\sum_{n=1}^{N} a_n Q_n = 0$$

where

$$a_n = (-1)^n \frac{q_n}{f_n} \tan\frac{\sigma_n}{2}$$
$$P_1 = 0 \qquad Q_1 = 1$$
$$P_2 = 1 \qquad Q_2 = 0$$
$$P_{n+1} = h_n P_n - \frac{d_n}{d_{n-1}} \cdot P_{n-1} \qquad (16)$$
$$Q_{n+1} = h_n Q_n - \frac{d_n}{d_{n-1}} \cdot Q_{n-1}$$
$$h_n = 1 + \frac{d_n}{d_{n-1}} - \frac{d_n}{f_n}.$$

The cross-polarization elimination condition in equation (15) is a geometrical relationship independent of frequency because q_n, a_n, d_n, and f_n are constant. Hence, in the reflector system satisfying this relationship, the cross-polarization component is not generated regardless of frequency.

3. Four-reflector beam waveguide feeds

Figure 2 shows a typical beam waveguide feed using three focusing reflectors, a plane reflector and a corrugated horn. Substituting N=4 into (15), we obtain the following equations:

$$a_1 = \frac{d_2}{d_1}(a_3 + h_3 a_4) \qquad (17)$$
$$a_2 = \frac{d_3}{d_2} \cdot a_4 - h_2 \cdot \frac{d_1}{d_2} \cdot a_1$$

where

$$a_1 = -\frac{q_1}{f_1}\tan\frac{\sigma_1}{2} \qquad a_2 = \frac{q_2}{f_2}\tan\frac{\sigma_2}{2}$$
$$a_3 = -\frac{q_3}{f_3}\tan\frac{\sigma_3}{2} \qquad a_4 = \frac{q_4}{f_4}\tan\frac{\sigma_4}{2} \qquad (18)$$
$$h_2 = 1 + \frac{d_2}{d_1} - \frac{d_2}{f_1} \qquad h_3 = 1 + \frac{d_3}{d_2} - \frac{d_3}{f_3}.$$

The cross-polarization elimination condition of these beam waveguide feeds can be derived by substituting $1/f_4=0$ into (18) as follows. This equation means a geometrical relationship not related to the frequency.

$$f_1 = \frac{q_1 d_1 f_3 \tan(\sigma_1/2)}{q_3 d_2 \tan(\sigma_3/2)} \qquad (19)$$
$$f_2 = \left\{\frac{q_2 f_3 \tan(\sigma_2/2)}{q_3 d_2 \tan(\sigma_3/2)}\right\} \Big/ \left(\frac{1}{d_1} + \frac{1}{d_2}\right).$$

From (19), since d_1, d_2, $\tan(\sigma_1/2)$ and $\tan(\sigma_2/2)$ are positive, we can then define the following:

$$sign(f_1) = sign(q_1 q_3 f_3) \qquad (20)$$
$$sign(f_2) = sign\left\{\frac{q_2 q_3 f_3 \tan(\sigma_2/2)}{d_2 \tan(\sigma_3/2)} + 1\right\}.$$

Therefore, from the first equation in (20), in Fig. 2a, the first focusing reflector is convex if the third focusing reflector is concave and is concave if the third focusing reflector is convex. Furthermore in Fig. 2b, the first focusing reflector is concave if the third focusing reflector is concave and is convex if the third focusing reflector is convex.

From the second equation in (20), the configuration of the second focusing reflector is classified as follows:

(i) For the the concave third reflector,

in Fig. 2a, the second focusing reflector always becomes concave.

In Fig. 2b, the second focusing reflector becomes convex when the following equation is satisfied:

$$\frac{f_3}{d_2} > \frac{\tan(\sigma_3/2)}{\tan(\sigma_2/2)}. \qquad (21)$$

Also, the second focusing reflector becomes concave when the following equation is satisfied:

$$\frac{f_3}{d_2} < \frac{\tan(\sigma_3/2)}{\tan(\sigma_2/2)}. \qquad (22)$$

(ii) For the convex third reflector,

in Fig. 2a, the second focusing reflector becomes convex when the following equation is satisfied:

$$\frac{f_3}{d_2} < -\frac{\tan(\sigma_3/2)}{\tan(\sigma_2/2)}. \qquad (23)$$

Also, the second focusing reflector becomes concave when the following equation is satisfied:

$$\frac{f_3}{d_2} > -\frac{\tan(\sigma_3/2)}{\tan(\sigma_2/2)}. \qquad (24)$$

In Fig. 2b, the second focusing reflector always becomes concave. From the above, it is clarified that the four-reflector beam waveguide feed has the six possible configurations shown in Fig. 3.

4. Conclusions

A general cross-polarization elimination condition was derived for multireflector offset antennas. It was proven that this condition is a geometrical relationship independent of frequency. The necessary conditions for four-reflector beam waveguide feed configurations with no cross-polarization component were also presented. In actual design, the possible configurations are restricted by the values of design parameters as defined in Fig. 2, and the best configuration must be selected.

References

[1]T. Furuno, S.Urasaki, and T.Katagi,“Tri-Reflector Antennas Eliminating Cross-Polarized Component Based on Beam-Mode Analysis,” I.E.I.C.E. Trans. (B-II), J78-B-II,9,pp.585-592(Sept. 1995).

[2]T. Katagi, S. Urasaki, S. Ebisui, and S. Betsudan,“Analysis and Design Method of Beam Waveguide Feeds by Beam Mode Expansion,” I.E.I.C.E. Trans. (B), J66B, 3, pp.305-312 (March 1983).

[3]S. Betsudan, T. Katagi, and S. Urasaki,“Design Method of Four-Reflector Type Beamwaveguide Feeds,” I.E.I.C.E. Trans. (B), J67-B, 6, pp.622-629 (June 1984).

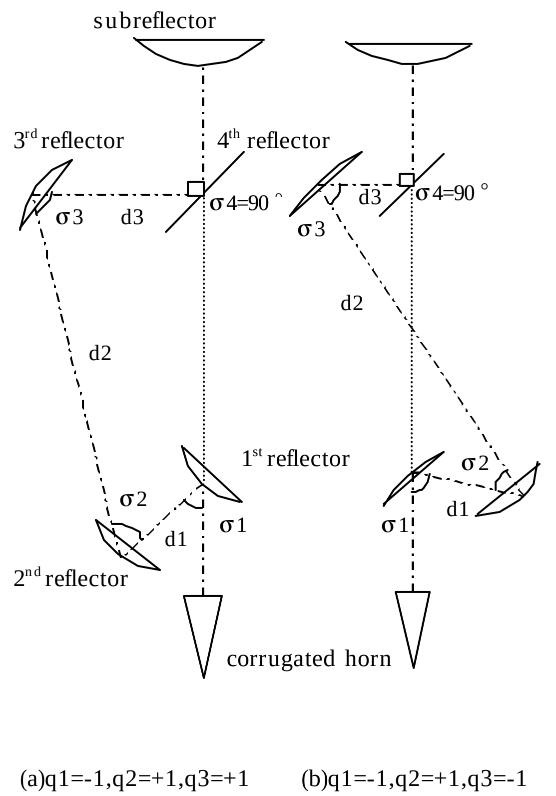

Fig.2 Four-reflector beam waveguide feeds.

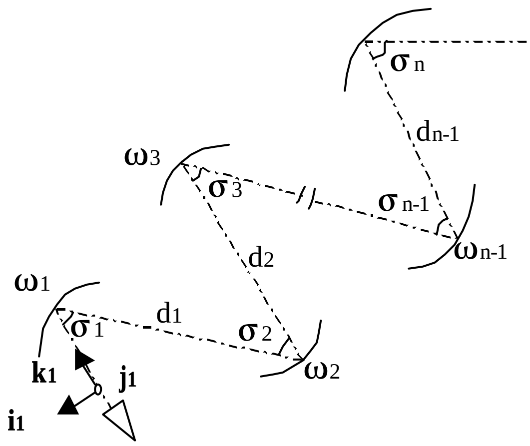

Fig. 1 Multireflector offset antennas.

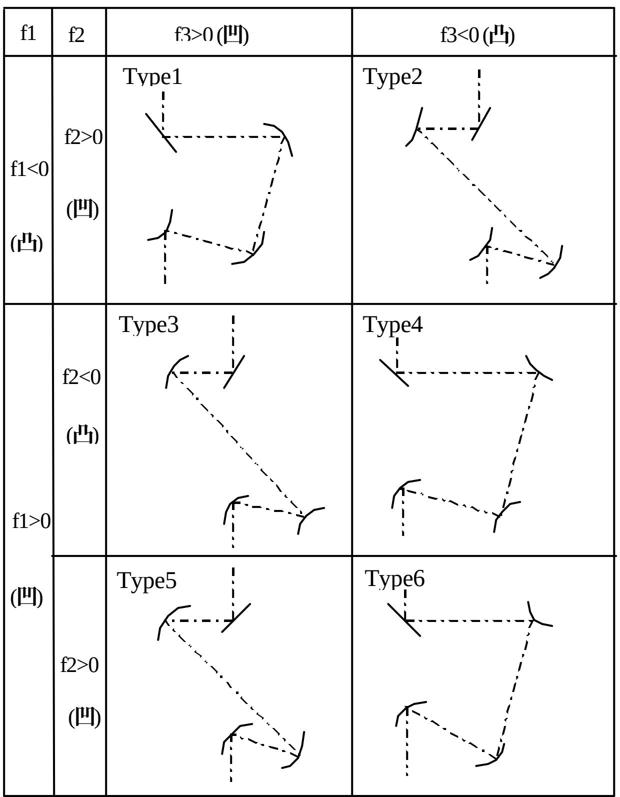

Fig. 3 Possible configurations of a four-reflector beam waveguide feed.