Diffraction of Electromagnetic Plane Wave by an Infinitely Long Conducting Strip on Dielectric Slab

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1. Introduction

The diffraction of electromagnetic plane wave by an infinite conducting strip located on the dielectric slab is formulated rigorously using Weber-Schafheitlin's type discontinuous integrals. (This method is also called method of Kobayashi Potential (Sneddon, 1966)). This kinds of problems have been studied using other methods (Butler, 1987,Tsanlamengas and Fikioris, 1991, 1993). According to the present method, the fields are divided into geometrical optic and diffracted components. The diffracted fields in the upper and lower half-space and in the slab region are expanded by the functions which are given by Fourier sine and cosine transforms. The relations among the weighting functions in the Fourier transform are obtained by imposing the boundary conditions on the interface of each region. Furthermore we impose the condition on the interface, where the strip exists, that the difference between the upper and lower side values of tangential component of magnetic field reduces to Weber-Schafheitlin's type discontinuous integrals. With this choice the tangential component of the magnetic field is continuous on the dielectric part and discontinuous on the strip on the upper interface. Since the resulting functions satisfy a part of required boundary conditions automatically, they may be regarded as the modal functions for the structure. This is one of the main advantages of this method. The problem is reduced to simultaneous equations when the rest of boundary conditions are imposed to the assumed field expression. The coefficients of the equations are given by infinite integrals which is difficult to express in closed form. We also discuss how to compute effectively these integrals numerically. Some numerical results including the effects of depth and permittivity of the dielectric slab on the current distributions and radiation patterns are presented. Discussion on the results will be made in presentation.

2. Field Expression for the Strip on the Slab

The geometry of the problem and the coordinate system are shown in Fig.1. The conducting strip is located on dielectric slab with thickness d. The problem is to obtain the expression for the field diffracted by the strip when plane wave is incident from the upper half space. Both cases of E-polarization and H-polarization are considered, but we show only for E-polarization for sake of saving space. The case of H-polarization may be treated in a similar manner.

Incident wave E_z^i, reflected wave E_z^r, wave in the slab E_z^s, and transmitted wave E_z^t are given by

$$\begin{aligned} E_z^i &= \exp[jk_0(x\sin\alpha + y\cos\alpha)] \\ E_z^r &= R\exp[jk_0(x\sin\alpha - y\cos\alpha)] \\ E_z^s &= \exp[jkx\sin\beta]\{A\exp[jky\cos\beta] + B\exp[-jky\cos\beta]\} \\ E_z^t &= T\exp\{jk_0[x\sin\alpha + (y+d)\cos\alpha]\} \end{aligned} \tag{1}$$

where R and T are reflection and transmission coefficients of the dielectric slab when strip is absent. From Snell's law we have the relation $k\sin\beta = k_0\sin\alpha$ which determines β in terms of the incident angle α. Magnetic field H_x is obtained from the relation $H_x = -\frac{1}{j\omega\mu}\cdot\frac{\partial E_z}{\partial y}$. The expressions for the scattered fields E_z^{su} for the upper half-space, E_z^{sd} in the slab, and E_z^{sl} for the lower half space are assumed in the forms

$$E_z^{su} = \int_0^\infty [f_1(\xi)\cos(\xi x_n) + g_1(\xi)\sin(\xi x_n)]\exp[-uy_n]\,d\xi \qquad y > 0 \tag{2a}$$

$$E_z^{sd} = \int_0^\infty \cos(\xi x_a)\{f_2(\xi)\exp[vy_a] + f_3(\xi)\exp[-vy_a]\}\,d\xi$$
$$+ \int_0^\infty \sin(\xi x_a)\{g_2(\xi)\exp[vy_a] + g_3(\xi)\exp[-vy_a]\}\,d\xi \qquad -d < y < 0 \tag{2b}$$
$$E_z^{sl} = \int_0^\infty [f_4(\xi)\cos(\xi x_a) + g_4(\xi)\sin(\xi x_a)]\exp[u(y_a + d_a)]\,d\xi \qquad y < -d \tag{2c}$$
$$u = \sqrt{\xi^2 - \kappa_0^2}, \quad v = \sqrt{\xi^2 - \kappa^2}, \qquad x_a = \frac{x}{a} \quad y_a = \frac{y}{a}, \quad d_a = \frac{d}{a} \quad \kappa_0 = k_0 a, \quad \kappa = ka \tag{2d}$$

where $f's$ adn $g's$ are teh weighting functions.

Required boundary conditions of the present problem are:

[i] E_z is continuous on the whole upper boundary ($y = 0$ for all x)
[ii] E_z and H_x are continuous on the whole lower boundary ($y = -d$ for all x)
[iii] H_x is continuous on the dilectric part of the upper interface ($y = 0$ and $|x| > a$)
[iv] $E_z^i + E_z^r + E_z^{su} = 0$ on the strip ($y = 0$ and $|x| < a$)

From boundary conditions [i] and [ii], we can obtain the relations among the unknown weighting functions $f's$ and $g's$. To make satisfy the condition [iii], we impose the condition that $H_x^+ - H_x^-$ reduces to Weber-Schafheitlin's discontinuous integrals, where superscripts + and − mean upper and lower sides of the interface. Then the functions $f_1(\xi)$ and $g_1(\xi)$, which are required for the diffracted field in the upper space, may be written as

$$\begin{pmatrix} f_1(\xi) \\ g_1(\xi) \end{pmatrix} = \frac{u + v + (v - u)\exp(-2p)}{(v + u)^2 - (v - u)^2\exp(-2p)} \sum_{m=0}^{\infty} \begin{pmatrix} A_m J_{2m}(\xi) \\ B_m J_{2m+1}(\xi) \end{pmatrix} \tag{3}$$

where A_m and B_m are expansion coefficients and these are determined from the condition [iv]. Thus the unknown weighting functions are changed to the unknown expansion coefficients. Using these results, current flowing on the strip becomes

$$J_z = -H_x\Big|_{y=0+} + H_x\Big|_{y=0-} = \frac{1}{j\omega\mu}\sum_{m=0}^{\infty} A_m \int_0^\infty J_{2m}(\xi)\cos\frac{\xi x}{a}d\xi + \frac{1}{j\omega\mu}\sum_{m=0}^{\infty} B_m \int_0^\infty J_{2m+1}(\xi)\cos\frac{\xi x}{a}d\xi$$
$$= \frac{1}{j\omega\mu}\left(1 - x_a^2\right)^{-\frac{1}{2}} \sum_{m=0}^{\infty} \left\{A_m \cos\left[2m\sin^{-1}x_a\right] + B_m \sin\left[(2m+1)\sin^{-1}x_a\right]\right\} \tag{4}$$

Functions $(1 - x_a^2)^{-\frac{1}{2}}\cos[2m\sin^{-1}x_a]$ and $(1 - x_a^2)^{-\frac{1}{2}}\sin[(2m+1)\sin^{-1}x_a]$ in the above equation may be regarded as expansion functions for the current distribution and they show an expected singularity at $x_a = 1$ associated with the edge conditions. The expression for J_x in the H-polarization is derived similarly, and the corresponding expansion functions become $\frac{1}{2m+1}\cos[(2m+1)\sin^{-1}x_a]$ and $\frac{1}{2m+2}\sin[(2m+2)\sin^{-1}x_a]$. When x_a approaches 1, $\sin^{-1}x_a$ becomes $\frac{1}{2}\pi$, and these expansion functions are found to approach zero at the edge. This is consistent with the edge condition for J_x. The determinantal equations for the expansion coefficients A_m and B_m are obtained as follows. The resulting relation of condition [iv] is projected into the functional space and each component is identified. We use Jacobi's polynomials as set of functions. The results are given by

$$\sum_{m=0}^{\infty}\left\{\begin{matrix} A_m GG(2m, 2n) \\ B_m GG(2m+1, 2n+1) \end{matrix}\right\} = -(1+R)\left\{\begin{matrix} J_{2n}(\kappa\sin\alpha) \\ J_{2n+1}(\kappa\sin\alpha) \end{matrix}\right\}, \quad R = -\frac{(1 - Q^2)\sin\Delta}{(1 + Q^2)\sin\Delta - j2Q\cos\Delta} \tag{5}$$

where $Q = \frac{Y_0\cos\alpha}{Y\cos\beta}$ and $\Delta = kd\cos\beta$. $Y_0 = \sqrt{\frac{\epsilon_0}{\mu}}$ and $Y = \sqrt{\frac{\epsilon}{\mu}}$ are the intrinsic admittance in free space and dielectric medium, respectively. In eq.(5), $GG(\nu,\mu)$ is defined by

$$GG(\nu,\mu) = \int_0^\infty \frac{u + v + (v - u)\exp(-2p)}{(v + u)^2 - (v - u)^2\exp(-2p)} J_\nu(\xi)J_\mu(\xi)d\xi$$
$$= \frac{1}{2}\int_0^\infty \frac{J_m(\xi)J_n(\xi)}{v}d\xi + \int_0^\infty \frac{v^2 - u^2 + (v - u)(3v - u)\exp(-2p)}{2v[(v + u)^2 - (v - u)^2\exp(-2p)]} J_m(\xi)J_n(\xi)d\xi \tag{6}$$

The first integral can be calculated using the series expression (Hongo and Ishii, 1978). Since the integrands of the second integrals include two branch point at $\xi = \kappa_0$ and $\xi = \kappa$, and pole at $\xi = \xi_p$, we deform the contour of integration in the complex $\xi-$ plane to make the integrands smooth. The contribution from the poles are taken into account in the deformed integrals automatically. It is readily shown that the integrand of the second integral decreases as ξ^{-4} for large ξ, so that it can be computed readily.

3. Far Field Expression

For perfectly conducting strip or slit between two conducting half-planes, the near field expression can be derived in the form convenient for applying FFT (Hongo and Kaneda, 1980). But in the present problem it seems to be difficult to derive such an expression. In this section we will show the derivation of the far field expressions using saddle point method of integration. Since the integrand of the integral for E_z^d is even function of variable ξ, the limit of the integration can be extended to $(-\infty, \infty)$. This is represented in the form

$$E_z^d = \int_{-\infty}^{\infty} P(\xi)\exp[-jx_a\xi - \sqrt{\xi^2 - \kappa^2}y_a]d\xi$$
$$= -\kappa \int_C P(\beta)\exp[-jk\rho\cos(\phi - \beta)]\cos\beta d\beta \tag{7}$$

where transformations of variables $\xi = \kappa\sin\beta$, $x = \rho\sin\phi$ and $y = \rho\cos\phi$ are introduced. C is the contour of the integration running from $\pi + j\infty$ to $-j\infty$. Integrand has poles at $\xi = \xi_p, (\kappa_0 < \xi_p < \kappa)$, which are solutions of

$$[2\xi^2 - (1+\epsilon_r)\kappa_0^2]\sin q + 2\sqrt{\xi^2 - \kappa_0^2}\sqrt{\epsilon_r\kappa_0^2 - \xi}\cos q = 0, \qquad q = \sqrt{\epsilon_r\kappa_0^2 - \xi^2}d_a \tag{8a}$$

The poles ξ_p are expressed in $\phi-$plane as

$$\xi_p = \kappa_0\sin\phi_p, \qquad \phi_p = \frac{\pi}{2} + j\gamma_p \tag{8b}$$

The saddle point is readily found to be located at $\beta = \phi$. For $\phi < \phi_0 = \sin^{-1}\left(\frac{1}{\cosh\gamma_p}\right)$, application of the standard saddle point method of integration to eq.(7) leads approximate solution given by

$$E_z^d = \kappa\sqrt{\frac{2\pi}{k\rho}}P(\phi)\cos\phi\exp\left[j\frac{\pi}{4} - jk\rho\right] \tag{9}$$

For $\phi > \phi_0$ we have to add the contribution from the poles which correspond to surface wave propagating along the slab. The results are expressed by

$$E_z^d = 2\pi j\lim_{\xi\to\xi_p}(\xi - \xi_p)P(\xi)\exp[-jx_a\xi - \sqrt{\xi^2 - \kappa^2}y_a] \tag{10}$$

Pattern function $P(\phi)$ is expressed as

$$P(\phi) = \frac{\sqrt{\epsilon_r - \sin^2\phi} + \cos\phi + \left(\sqrt{\epsilon_r - \sin^2\phi} - \cos\phi\right)\exp\left(-j2k_0d\sqrt{\epsilon_r - \sin^2\phi}\right)}{\left(\sqrt{\epsilon_r - \sin^2\phi} + \cos\phi\right)^2 - \left(\sqrt{\epsilon_r - \sin^2\phi} - \cos\phi\right)^2\exp\left(-j2k_0d\sqrt{\epsilon_r - \sin^2\phi}\right)}$$
$$\sum_{m=0}^{\infty}[A_mJ_{2m}(\kappa_0\sin\phi) + jB_mJ_{2m+1}(\kappa_0\sin\phi)] \tag{11}$$

where A_m and B_m are solutions of eq.(5).

4. Diffraction by s Strip on Dielectric Half-Space

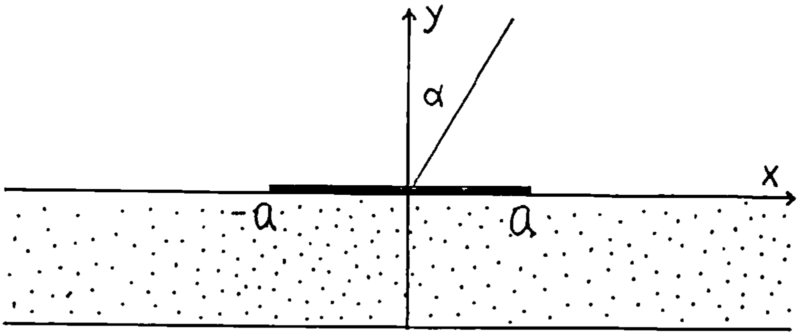

Fig.1 Geometry and coordinate system of the problem. Incident angle is α, dielectric constant is ϵ, thickness of the slab is d.

As a special case, we will consider a diffraction of electromagnetic plane wave by a conducting strip located on the interface of dielectric half space. The solution is derived in a manner similar to the case of dielectric slab discussed above. The solution may also be obtained by taking limit $d \to \infty$. We will present only the summary of the results for E-polarization

(i) Expressions for the functions $f_1(\xi)$ and $g_1(\xi)$

$$f_1(\xi) = \frac{1}{u+v}\sum_{m=0}^{\infty} A_m J_{2m}(\xi), \qquad g_1(\xi) = \frac{1}{u+v}\sum_{m=0}^{\infty} B_m J_{2m+1}(\xi) \tag{12}$$

(ii) Matrix Equations for Expansion Coefficients A_m and B_m: has the same form as eqs.(5) except GG and the reflection coefficients R are replaced by

$$GG(\nu,\mu) = \int_0^\infty \frac{1}{u+v} J_\nu(\xi)J_\mu(\xi)d\xi = \frac{1}{2}\int_0^\infty \frac{J_\nu(\xi)J_\mu(\xi)}{v}d\xi + \frac{1}{2}\int_0^\infty \frac{v-u}{v(u+v)}J_\nu(\xi)J_\mu(\xi)d\xi$$

$$R = -\frac{k\cos\beta - k_0\cos\alpha}{k\cos\beta + k_0\cos\alpha} \tag{13}$$

(iii) Far Field Expressions for the Diffracted Wave are given by

$$P(\phi) = \frac{1}{\sqrt{\epsilon_r - \sin^2\phi} + \cos\phi}\sum_{m=0}^{\infty}[A_m J_{2m}(\kappa_0 \sin\phi) + jB_m J_{2m+1}(\kappa_0\sin\phi)]$$

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