A-2-1 GAUSSIAN PATTERN SYNTHESIS BY THE PHASE ADJUSTMENT FOR A MULTIFUNCTIONAL PHASED ARRAY ANTENNA

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INTRODUCTION

Beam broadening technique is often used in phased array radars in order to reduce the time taken to scan through the elevation coverage. It is possible to broaden the beam of a phased array by phase control only. The phase distortion function generally used is a cosine distribution, whose amplitude controls the degree of broadening. However, the broadened pattern produced by the cosine phase distribution has a small amount of ripple across the top of the beam. This ripple causes degradation of the angle detection accuracy in such a phased array radar that target angle detection is made by comparing the target amplitudes of two adjacent beams. Therefore, it is required to shape the beam in one parameter function such as a gaussian pattern.

This paper presents a method that synthesizes the gaussian pattern which has a prescribed principal direction of radiation and has a specified beam width by the adjustment of excitation phases of an array antenna. The required phase distribution is determined by solving the pattern synthesis problem numerically by means of a mathematical programming technique: the mean square error between the normalized power pattern of the array and the desired gaussian one is extremized.

ARRAY ANTENNAS AND ITS FAR-FIELD PATTERN

Figure 1 shows an equally spaced linear array consisting of 2N elements. The element spacing is d, and the directional angle θ is measured from the broadside direction of the array. Point sources are assumed as the elements. Let a(n) be the excitation amplitude of the n-th element, and suppose that a(n)'s distribute symmetrically about the center of the array: a(-n)=a(n). The excitation phase of the n-th element is divided into two components in view of the symmetry about the center of the array: the antisymmetric component $\psi(n)$ plus the symmetric one $\phi(n)$. Hence $\psi(-n)=-\psi(n)$, and $\phi(-n)=\phi(n)$.

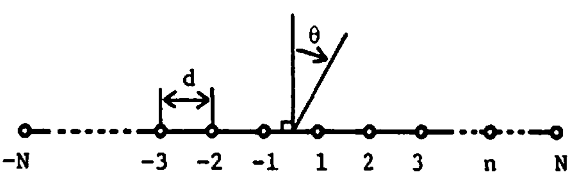


Fig.1 Linear array antenna

The excitation amplitudes are fixed to distribute according to, for example, the Taylor's one. For the purpose of beam scanning the antisymmetric phase components are given as follows.

$$\psi(n) = \frac{2\pi d}{\lambda_0}\left(n - \frac{1}{2}\right)\sin\theta_0 \qquad (n = 1, 2, \cdots\cdots, N), \tag{1}$$

here λ_0 denotes the wavelength in the free space, and $\theta = \theta_0$ is the prescribed principal direction of radiation. The phase distribution given by Eq.(1) corresponds to the cophasal excitation whose principal direction of radiation is $\theta = \theta_0$, when all of the symmetric phase components are equal to zero.

The far-field pattern of the array whose elements are excited as mentioned above can be represented by

$$E(u, \phi) = \sum_{n=1}^{N} a(n)\exp(j\phi(n))\cos\left(n - \frac{1}{2}\right)u, \quad u = \frac{2\pi d}{\lambda_0}(\sin\theta - \sin\theta_0), \tag{2}$$

here ϕ is the abbreviation of N symmetric phase components $\phi(n)$ $(n=1, 2, \ldots, N)$.

POWER PATTERN SYNTHESIS [1]

We are now at the stage to determine the distribution of symmetric phase components, which has been left unspecified up to now, so that the radiation pattern of the array will approximate the desired gaussain one. The latter is given by

$$g(u) = \exp(-(au)^2), \tag{3}$$

here a is the parameter which specifies the beam-width of this gaussian pattern.

The degree of approximation is evaluated by the mean square error between the normalized power pattern of the array and $g(u)$. The former is denoted by $\hat{p}(u, \phi)$, and is defined to be $|E(u, \phi)|^2 / |E(0, \phi)|^2$. Hence the error takes the following form;

$$\varepsilon(\phi) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}(\hat{p}(u, \phi) - g(u))^2\,du. \tag{4}$$

The reason for the use of $\hat{p}(u, \phi)$ in place of $E(u, \phi)$ is that we are interested only in the power pattern $|E(u, \phi)|^2$, while the far field pattern given by Eq. (2) is complex-valued because of the existence of symmetric phase components, and that since only the excitation phases are adjustable in our pattern synthesis, the power pattern without normalization cannot be expected by itself to approximate the desired pattern.

The distribution of symmetric phase components ϕ is determined so that this error would be minimized. This is an optimization problem of nonlinear type, and is solved numerically by means of the Davidon-Fletcher-Powell's algorithm [2]. Notice that the excitation phases are assumed to be able to take any value in the course of this numerical optimization, and that an obtainable solution is not guaranteed to be the grobal optimum one because of nonlinear property of the problem, although it may be locally optimum.

The algorithm requires the evaluation not only of the objective function $\varepsilon(\phi)$ but its gradient $\nabla\varepsilon(\phi)$. For this purpose $\hat{p}(u, \phi)$ and $g(u)$ are expanded into their Fourier serires;

$$\hat{p}(u, \phi) = \sum_{n=0}^{2N-1} \hat{P}(n, \phi)\cos nu, \qquad g(u) = \sum_{n=0}^{+\infty} G(n)\cos nu. \tag{5}$$

$\hat{P}(n,\phi)$ is found to be

$$\hat{P}(0,\phi) = \frac{1}{2} \sum_{k=1}^{N} [\hat{a}(k,\phi)]^2, \tag{6.a}$$

$$\hat{P}(n,\phi) = \sum_{k=1}^{N-n} \hat{a}(n+k,\phi)\hat{a}(k,\phi) \cos(\phi(n+k)-\phi(k))$$
$$+ \sum_{k=s}^{[n/2]} \hat{a}(n-k+1,\phi)\hat{a}(k,\phi) \cos(\phi(n-k+1)-\phi(k)) + \frac{\sigma(n)}{2}\left[\hat{a}\left(\frac{n+1}{2}\right)\right]^2, \tag{6.b}$$

here $\hat{a}(n,\phi)=a(n)/|E(0,\phi)|$ $(n=1,2,...,N)$, $s=1$ $(n<N)$, $=n-N+1$ $(n\geq N)$, and $\sigma(n)=1$ (n=odd), =0 (n=even). Then the required quantities can be represented in terms of expansion coefficients $\hat{p}(n,\phi)$'s and $G(n)$'s as follows.

$$\varepsilon(\phi) = \sum_{n=0}^{2N-1} \delta(n)[R(n,\phi)]^2 + K \tag{7}$$

here $R(n,\phi)=\hat{P}(n,\phi)-G(n)$ $(n=0,1,...,2N-1)$, $\delta(n)=1$ $(n=0)$, $=1/2$ $(n\neq 0)$, and K is a positive constant. The m-th component of gradient $\nabla\varepsilon(\phi)$ is found to be

$$\frac{\partial\varepsilon(\phi)}{\partial\varepsilon(m)} = \hat{a}(m,\phi)\sum_{n=1}^{N}[R(m-n,\phi)+R(m+n-1,\phi)-4S(\phi)]\hat{a}(n,\phi)\sin(\phi(n)-\phi(m)), \tag{8}$$

here

$$R(-n,\phi) \overset{\text{def}}{=} R(n,\phi) \quad (n=1,2,\cdots\cdots,2N-1), \text{ and } S(\phi)=\sum_{n=0}^{2N-1}\delta(n)R(n,\phi)\hat{P}(n,\phi).$$

RESULTS OF NUMERICAL OPTIMIZATION

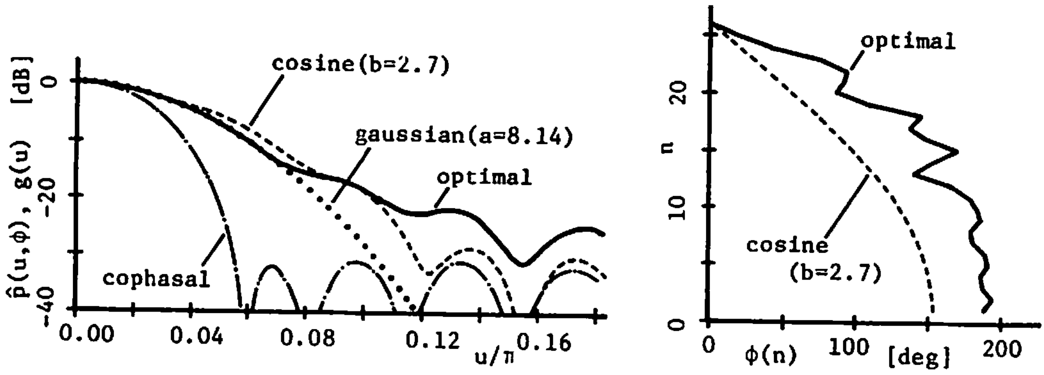
The number of elements 2N is assumed to be 52, and the distribution of excitation amplitudes the Taylor's one whose sidelobe level is -30dB and $\bar{n}=5$. Under these conditions we synthesized the gaussian pattern whose half-power beam width was 1.5, 2.0, and 3.0 times as broad as that of cophasal excitation.

The optimization procedure requires first the initial values of N variables $\phi(n)$'s, for which we adopted the aforementioned cosine distribution given by

$$\phi(n) = b\cos\left(\frac{2n-1}{2N-1}\frac{\pi}{2}\right) \quad (n=1,2,\cdots\cdots,N) \tag{9}$$

The obtained optimal pattern and the corresponding distribution of symmetric phase components in the case that beam broadening ratio (BBR) equals 1.5 are represented by the solid lines in Fig.2 (a) and (b) respectively. The other lines in Fig.2 (a) represent the cophasal pattern (dot-dash line), the desired gaussain one of specified half-power width (dotted line), and the pattern (broken line) by cosine distribution which gives rise to the same BBR (shown by the broken line in Fig.2 (b)). As seen from this figure, the main beam of synthesized optimal pattern coincides with the desired gaussian one in the range from zero to lower than -10dB, and all of the sidelobes of the optimal pattern, including those in the outer region of the figure, are lower than -20dB. Similar results were obtained for the other beam broadening ratios (BBR= 2.0 and 3.0).

Although the excitation phases were assumed to be able to take any value during the above optimization, the quantization, in both cases of 4 and 5 bits, of obtained optimal phase distribution turned out to affect the shaped beam so little that the discrepancy between the resulting main beam (the beam in the range from zero to -10dB) and the desired gaussain one was less than 0.5dB.



(a) patterns (b) distributions of symmetric phase components

Fig.2 Result of numerical optimization (BBR=1.5)

EXPERIMENTAL RESULTS

Experiments were carried out using an S-band equally spaced linear array with 4 bit phase shifters. The measured main beam patterns agree well with the calculated ones in the case that BBR=1.5 and 2.0, although there exists a discrepancy of at most one decibel in the case that BBR=3.0. It was also observed that the variation of the beam broadening ratio for the beam scanning when normalized by the cophasal beam width at each scanning angle, was about 10 percent of the ratio measured at the broadside angle.

CONCLUSION

The gaussian shaped beam of a specified beam-width is synthesized only by the excitation phase adjustment for a multifunctional phased array antenna. The required phase distribution is determined numerically so that the mean square error between the normalized power pattern of the array and the desired gaussian one would be minimized. As a result of such a numerical optimization we obtained a desired gaussian shaped beam with sidelobes lower than -20dB in every case that the specified half-power beam width was 1.5, 2.0, and 3.0 times as broad as that of cophasal excitation. Moreover it was found both numerically and experimentally that even when the obtained analogue optimal phase distribution was quantized according to a usual 4, or 5 bit phase shifter, the discrepancy between the resulting main beam and the desired gaussian one was at most one decibel.

REFERENCES

[1] H. Steyskal, "On antenna power pattern synthesis," IEEE Trans. AP-18, pp.123-4, Jan. 1970

[2] L. C. W. Dixon, "Nonlinear optimization," The English Universities Press Ltd., London, 1972.