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# PROCESSING OF INFORMATION AVAILABLE AT THE TERMINALS OF A MULTI-PORT ANTENNA

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The paper considers a receiving situation where a distant signal arrives at a site as several components from different directions and with different time (phase) delays. The components are mutually coherent and add vectorially to form a standing wave pattern over the site. The site is a horizontal plane with positions specified by x-y coordinates. The field over the site is sampled as by elements of an array antenna. The goal is to determine from these measurements the number of the multipath components and the amplitude and the directions of each component.

Conventionally this is done by connecting the elements together to form an array and by steering the main beam of the radiation pattern over the hemisphere. Watterson[1], using antenna aperture synthesis and measuring both amplitude and phase of the field, was able to identify the various components by analysis of the data. This paper demonstrates that it is possible to do so using amplitude measurements alone.

In a model of n multipath components consider the i-th component: its azimuthal angle of arrival is $\varphi_i$, measured from +Y axis; its elevation angle of arrival is $\delta_i$. The phase, $\psi_i$, of the i-th component is measured at the origin. The field at x, y due to the i-th component is then

$$E_i(x,y) = E_i \exp j[2\pi/\lambda(x \sin\varphi_i + y\cos\varphi_i)\cos\delta_i + \psi_i] \; . \quad (1)$$

Since the total field at the point x, y is the vector sum of the n coherent components, it is given simply by summation

$$\overline{E}(x,y) = \sum_{i=1}^{n} E_i \exp j[2\pi/\lambda(x \sin\varphi_i + y\cos\varphi_i)\cos\delta_i + \psi_i] \; . \quad (2)$$

A crucial point in this presentation is a demonstration that the standing wave pattern of the received field is more tractable to analysis if expressed in terms of field intensity (power incident per unit area) than as field strength. Conventionally, field strength can be converted to intensity by multiplying it by its conjugate. That is done here except that in the process of multiplication the field strength is left in the form of polynomial (2), and its conjugate is cast in the form of another polynomial each term of which is a conjugate of the corresponding term in the expression for the field. Thus,

$$E^*(x,y) = \sum_{i=1}^{n} E_i \exp -j[2\pi/\lambda(x \sin\varphi_i + y\cos\varphi_i)\cos\delta_i + \psi_i] \; . \quad (3)$$

When multiplied out term by term, the resulting expression is in the form

$$\begin{aligned} P(x,y) = \overline{E}(x,y)\cdot E^*(x,y) &= E_i^2 + E_j^2 \\ &+ E_iE_j \cdot \exp j[2\pi/\lambda(x(\sin\varphi_j\cos\delta_j - \sin\varphi_i\cos\delta_i) + y(\cos\varphi_j\cos\delta_j - \cos\varphi_i\cos\delta_i)) + (\psi_j - \psi_i)] \\ &+ E_iE_j \cdot \exp -j[2\pi/\lambda(x(\sin\varphi_j\cos\delta_j - \sin\varphi_i\cos\delta_i) + y(\cos\varphi_j\cos\delta_j - \cos\varphi_i\cos\delta_i)) + (\psi_j - \psi_i)] \; . \end{aligned} \quad (4)$$

Only two terms, i-th and j-th, are shown to avoid clutter. The two cross-products of the i-th and j-th terms are readily seen to be combinable into a cosine term. Extending this to n terms, the field intensity, P(x, y), is seen to be in the form

$$P(x,y) = \sum_{i=1}^{n} E_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 2 E_i E_j$$

$$\cos[2\pi/\lambda(x(\sin\varphi_j \cos\delta_j - \sin\varphi_i \cos\delta_i) + y(\cos\varphi_j \cos\delta_j - \cos\varphi_i \cos\delta_i)) + (\psi_j - \psi_i)] \quad . \quad (5)$$

Examination of the above expression shows that the field intensity pattern consists of a constant term plus a series of cosine terms. The constant term equals the algebraic sum of the individual component intensities. The amplitudes of the cosine terms are twice the product of amplitudes of two of the components; their arguments are determined by the direction angles of the two components; their phase is the difference in the phases of the two components. Since the sinusoidal terms are formed by combining components, two at a time, the total number of sinusoidal periodicities is n(n-1)/2.

The form of the standing wave pattern, a sum of sinusoidal periodicities, permits analysis using the search for hidden periodicities. Techniques for this are described in published literature (e. g. Hildebrand[2]). In outline form, the procedure is: (1) Measure the standing wave pattern in terms of power, P(x, y), at equal intervals along a line y=a+bx. (2) Tabulate differences, F(i), between successive measurements. This is to eliminate the constant term. (3) From this table form a matrix equation

$$C \cdot X = D \quad . \qquad (6)$$

Both C and D matrices are formed from F(i) tabulation. If the search is made for six periodicities (n=4) then

$$C_{ij} = F(i+j) + F(i+12-j)$$

$$D_i = F(i) + F(i+12)$$

Matrix eq. (6) is solved for X using the method of least squares (e. g., Lanczos[3])

$$X = [\,trn(C) \cdot C\,]^{-1} [\,trn(C) \cdot D\,] \quad . \qquad (7)$$

Solutions for X are used to form coefficients for a 6-th degree equation. The roots of the equation are cosines of the periodicities projected on the line y=a+bx.

With periodicities solved, another matrix equation is set up

$$C \cdot A = B \quad . \qquad (8)$$

Matrix C has thirteen columns, the first is unity, the others are cosine and sine values for the six periodicities. B is a vector matrix consisting of measured values of P(x, y).

Solution for A gives amplitudes and phases and is obtained similarly to that for X, using least squares

$$A = [\,trn(C) \cdot C\,]^{-1} \cdot [\,trn(C) \cdot B\,] \quad . \qquad (9)$$

Data measured along one line y=a+bx gives sufficient information to identify the amplitudes of the components. However, measurements along one line are insufficient to determine the directions-of-arrival for the components. To get these it is necessary to make measurements along several intersecting lines.

The procedure is illustrated by a numerical example in which three components are assumed and the standing wave pattern computed. The computed values are used as "measured" data and processed to identify and evaluate the four components. The fourth component is found to have negligible amplitude.

REFERENCES:

(1) Watterson, C. C. "A method of measuring the multipath components of a field," ESSA Tech. Report IER6-ITSA 6, August 1966.

(2) Hildebrand, F. B., Introduction to Numerical Analysis, McGraw-Hill Book Company, Inc., New York, 1956, pp. 378-386.

(3) Lanczos, C., Applied Analysis, Prentice Hall, Inc., Englewood Cliff, N. J., 1956, pp. 156-161.