PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

EFFECTS OF RANDOM DISPLACEMENT OF THE ARRAY ELEMENTS ON THE PERFORMANCE OF DIRECTION FINDING.

Byung-Seub Lee*, Y.Bar-Ness**
* Electronics and Telecommunication Research Institute
Dae-Dog Science Town P.O.Box 8 Daejon,Korea
** New Jersey Institute of Technology
Newark, N.J. U.S.A.

1. Introduction.

Uniform Regular Array(URA); that is an array whose M elements are located at an equal distances(customarily = $\lambda/2$) from each other, was intensively used in the literature for direction finding of multi-sources. Different algorithms were used to extract the directions of these sources, Some are known as classical power methods, like the Bartlett estimator[1], others are classified as superresolution methods. Among the latter are, the linear prediction method[2],the maximum likelihood method[3] and the eigenstructure methods.

Effects of random displacement of the array elements has been considered in the literature dealing with the interference cancellation problem[6]. It is obvious that element displacement from a nominal location will cause perturbations of the covariance matrix elements. This perturbation may cause an error in the estimation of the direction of the signals, particularly if the correlation matrix is obtained from the mean of finite number of snapshots.

The purpose of this paper is to study the effect of these perturbation on the entries of the covariance matrix and conclude on the effect of these on the performance of a direction finder.

2. Variation of the Covariance Matrix due to perturbation in the element location.

We can write for the entries of the covariance matrix

$$R'_{ij} = \sum_{k=1}^{D} P_k \exp [\, jk_0(l_j-l_i)\,\frac{\lambda}{2}\,\sin\theta_k]\,\xi_k^2 \qquad i\neq j$$

$$= \sum_{k=1}^{D} P_k + \sigma_n^2 \qquad i=j \qquad (1)$$

where ξ_k represents the different kinds of perturbation :

$$1. \quad \eta_k = \frac{\sin(k_0\, \delta \sin \theta_k)}{k_0\, \delta \sin \theta_k} \tag{2}$$

for one dimensional uniform distribution ($-\delta,\delta$).

$$2. \quad \eta_k' = \frac{\sin(k_0\, \delta \sin\theta_k)}{k_0\, \delta \sin \theta_k} \times \frac{\sin(k_0\, \delta \cos\theta_k)}{k_0\, \delta \cos \theta_k} \tag{3}$$

For two dimensional uniform distribution ($-\delta,\delta$),

$$3. \quad \eta_k'' = e^{-\sigma^2 k_0^2 /2} \tag{4}$$

for two dimensional Gaussian distribution $\mathcal{N}(0,\sigma^2)$.Now let us define

$$a(\theta_k) = [\, 1,\ \exp(-jk_0 l_2 \frac{\lambda}{2} \sin \theta_k), \ldots ,\exp(-jk_0 l_M \frac{\lambda}{2} \sin\theta_k)] \tag{5}$$

when we took $l_1 = 0$; that is the reference point is at the first element.From (1) with $l_1 = 0$ and (5) we write for the perturbed covariance matrix

$$R' = \sum_{k=1}^{D} \xi_k^2 P_k\ a(\theta)\ a(\theta)^\dagger + \sum_{k=1}^{M} P_k(1-\xi_k^2)I + \sigma_n^2 I \tag{6}$$

$$= S' + N' + N$$

where

$$S' = \sum_{k=1}^{D} \xi_k^2 P_k\ a(\theta_k)\ a(\theta_k)^\dagger \tag{7}$$

$$N' = \sum_{k=1}^{D} P_k\ (1 - \xi_k^2)I \tag{8}$$

$$N = \sigma_n^2 I \tag{9}$$

S',N' and N are the signal, the perturbation and the additive noise covariance matrices, respectively. For the unperturbed array the covariance matrix is given by

$$R = \sum_{k=1}^{D} P_k \, a(\theta_k) \, a(\theta_k)^{\dagger} + \sigma_n^2 \, I \qquad (10)$$

$$= S + N$$

where

$$S = \sum_{k=1}^{D} P_k \, a(\theta_k) \, a(\theta_k)^{\dagger} \qquad (11)$$

$$N = \sigma_n^2 \, I \qquad (12)$$

$$R' - R = S' - S + N'$$

$$= \sum_{k=1}^{D} (\xi_k^2 - 1) P_k \, a(\theta_k) \, a(\theta_k)^{\dagger} + \sum_{k=1}^{D} (1-\xi_k^2) P_k I \qquad (13)$$

$\xi_k^2 \leq 1$ hence (6) means that due to perturbation the effective powers of the different signal decrease by a factor = ξ_k^2 and an extra noise N' is generated which is equivalent to an additive noise whose variance

$$\sigma_{n'}^2 = (1 - \xi_k^2) \, P_k \qquad (14)$$

This is to say that as an effect of perturbation the resultant signal-to-noise ratio reduced.If a superresolution method is used for example then such perturbation has no effect asymptotically.

3. Simulation Results

With 5 element URA and three sources impinging on the array from 10°,20° and 30° off broadside,we depict in Fig 3-1 the average error in direction(i.e sum of errors divided by number of sources) versus the number of snapshots. As expected errors are larger when we use two dimensional, instead of one dimensional perturbation or when we increase the maximum value of perturbation δ. Also it is clear from this curve as well as from others that errors go to zero asymptotically as was concluded in the text. Particularly worth noticing, that when we include two dimensional perturbation the direction finding estimator becomes very poor when we use a small number of snapshots. Not as bad, when only one dimensional perturbations are taken into account,Fig 3-2 is similar to Fig 3-1 except the comparison is now done for two dimensional uniform to two dimensional Gaussian perturbation.

References

[1]M.S. Bartlett, "Periodogram analysis and continuous spectra," biometrika, vol. 37, pp 1-16 june 1950

[2] M. Morf,B.Dickenson,T.Kailathand a. Vieria,"Efficient solution of Covariance equation for Linear prediction, IEEE Trans, Acoust. speech signal processing., vol. ASSP-25, pp. 429-433, Oct. 1977

[3] J. Capon,"High-resolution frequency-wavenumber spectrum analysis," in Proc. IEEE, vol. 57 pp 1408-1418, Aug. 1969

[4] R.O. Schmidt,"Multiple emitter location and signal parameter estimation," in Proc. RADC spectral Est. Workshop, Griffs AFBS, NY 1979

[5] S. Pillai, Y. Bar-ness and F. Haber,"A new Approach to Array Geometry for improved Spatial Spectrum estimation," Proceeding of the IEEE vol 23 No 10 oct. 1985

Fig 3-1

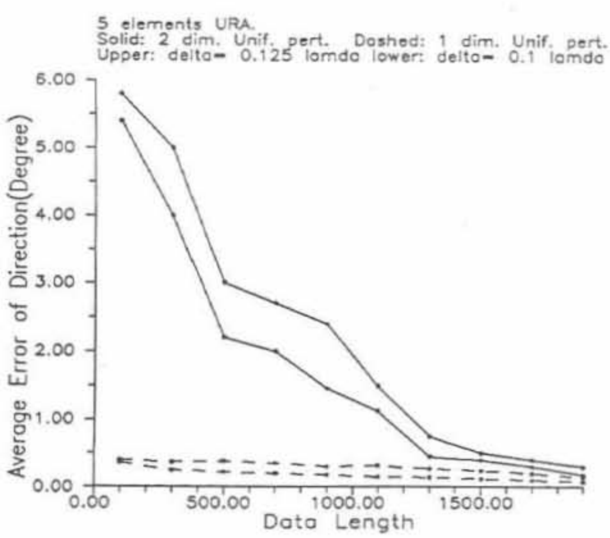

Fig 3-2