COMBINED MMSE-ML DETECTION FOR WIRELESS MIMO-SDM COMMUNICATIONS

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1 Introduction

Wireless multiple input multiple output (MIMO) systems have been shown to promise significant performance improvement and bandwidth efficiency over the conventional single input single output (SISO) system [1],[2]. In wireless MIMO systems, multiple antennas are employed in both transmitter an receiver to combat fading and achieve spatial diversity. One typical MIMO system is the spatial division multiplexing (SDM) system, where multiple transmit data streams are simultaneously sent through different transmit antennas as illustrated in Fig. 1. With N transmit antennas, it is possible to increase the transmission rate of the system by N times. The main method to achieve this advantage is the detection or space-time processing technique used at the receiver since no processing is used at the transmitter.

Among detection techniques proposed for MIMO-SDM, the maximum likelihood (ML) detection [3] exhibits the best performance in terms of minimizing bit error rate (BER), but requires the largest computational complexity. The minimum mean square error (MMSE) method, on the other hand, requires less complexity but with poor performance. In order to balance performance of detector with its associated complexity, researchers at Bell Labs proposed to use the combined MMSE detection with successive interference cancellation (SIC), which is referred to as the V-BLAST (V-BLAST: Vertical-Bell Labs Layered Space-Time) detection [4]. The V-BLAST detection, originated from the detection technique used in the code division multiplexing access (CDMA) system [5], allows to achieve better performance while requires the same complexity order compared with the MMSE method.

In this paper, we also consider the problem of balancing performance of a detector with its associated complexity by proposing a combined MMSE and ML detector for MIMO-SDM systems. This combined MMSE-ML detector is based on our previous work for multiuser space-time block coded orthogonal frequency division multiplexing (STBC-OFDM) in [6]. In our proposed scheme, a maximum likelihood detector placed after the MMSE detector performs search to detect symbols with the highest probability of error, then applies the ML detection to these symbols to correct them. By controlling the number of corrected symbols we can balance the amount of improved performance and additional complexity of the detector. We show that with the same complexity order, our proposed detector outperforms the V-BLAST detection in terms of minimizing BER for MIMO-SDM systems with a small number of transmit antennas. For a system with large number of transmit antennas, the proposed detector provides better BER than the V-BLAST detector without ordering, and also better than the V-BLAST detector with ordering at low input signal to noise ration (SNR).

The remainder of this paper is organized as follows. We introduce the signal model for a MIMO-SDM system and briefly review the V-BLAST detection in Sect. 2. The proposed detector of the combined MMSE-ML is introduced in Sect. 3. Performance of the proposed detector is compared with that of the V-BLAST using the BER obtained via computer simulation in Sect. 4 and, finally, Sect. 5 concludes the paper.

2 MIMO Spatial Division Multiplexing

2.1 Signal Model

Consider a wireless communication system with N transmit and M receive antennas, which is often referred to as an $N \times M$ MIMO system, signalling through flat fading channels as shown in Fig. 1. The system equation describing this system is expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z}, \tag{1}$$

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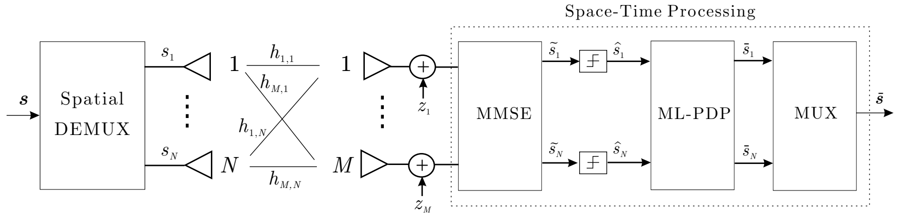

Figure 1: Proposed MMSE-ML detection for a MIMO-SDM system.

where $\boldsymbol{s} \triangleq [s_1, s_2, \ldots, s_N]^T$ is the $N \times 1$ transmit signal vector containing BPSK symbols; $\boldsymbol{y} \triangleq [y_1, y_2, \ldots, y_M]^T$ is the $M \times 1$ received signal vector; $\boldsymbol{z} \triangleq [z_1, z_2, \ldots, z_M]^T$ is the noise vector containing independent Gaussian noise samples with zero mean and variance σ_z, i.e., $z_m \sim \mathcal{N}_c(0, \sigma_z^2)$; and

$$\boldsymbol{H} \triangleq \begin{bmatrix} h_{1,1} & h_{1,2} & \ldots & h_{1,N} \\ h_{2,1} & h_{2,2} & \ldots & h_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1} & h_{M,2} & \ldots & h_{M,N} \end{bmatrix}. \tag{2}$$

is the $M \times N$ MIMO channel matrix with $h_{m,n}$ representing the complex path gain of the channel between the m-th receive and n-th transmit antenna, modeled by a complex Gaussian variable with zero mean and variance one per complex dimension, i.e., $h_{m,n} \sim \mathcal{N}_c(0, 1)$. In the above equations, the superscript $(\bullet)^T$ denotes the transpose operation.

2.2 MIMO-SDM Detection

It is noted that for MIMO-SDM systems, detection of one certain symbol s_n is performed under the presence of $(N-1)$ remaining symbols which is seen as interferences. A good detection technique should be able to effectively cancel these interferences. Among detection techniques, the ML method, such as proposed in [3], is optimum in terms of minimizing BER. However, its complexity is increased exponentially with N, thus use of the ML detection is limited and we shall skip discussion about it in this paper. The MMSE detection has low complexity, but with relatively poor performance. The most efficient method known so far is the V-BLAST detection, a combined MMSE detection with SIC, proposed in [4]. As the MMSE detection is the main component used in both the V-BLAST detection and our proposed MMSE-ML detection, in the sequel we shall present its principle first, followed by the V-BLAST detection.

2.2.1 MMSE Detection

Based on the MMSE method, the linear combining weight matrix, $\boldsymbol{W} \triangleq [\boldsymbol{w}_1,\ \boldsymbol{w}_2, \ldots, \boldsymbol{w}_N] \in \mathbb{C}^{M \times N}$, used to decouple transmit symbols s_n from the received signal vector $\boldsymbol{y}$ is derived from the following cost function $\boldsymbol{W} = \arg\min\limits_{\boldsymbol{W}} E\left\{\left\|\boldsymbol{x} - \boldsymbol{W}^H\boldsymbol{y}\right\|^2\right\}$, which gives us the solution expressed in well-known form of the Wiener-Hopf equation as

$$\boldsymbol{W} = \left(\boldsymbol{H}\boldsymbol{H}^H + \sigma_z^2\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}, \tag{3}$$

where $(\bullet)^H$ denotes the Hermitian transpose. The estimates of $\boldsymbol{s}$, defined as vector $\hat{\boldsymbol{s}}$, are then given via the linear combining and decision operation for BPSK signal as $\hat{\boldsymbol{s}} = \operatorname{sgn}\left\{\Re\{\boldsymbol{W}^H\boldsymbol{y}\}\right\}$, where $\Re\{\bullet\}$ and $\operatorname{sgn}\{\bullet\}$ represent the real operator and the signum function, respectively. It is noted that the complexity of the MMSE is $O(M^3)$.

2.2.2 V-BLAST Detection

In the V-BLAST detection, symbol from each transmit antenna is detected in one iteration by nulling out interferences from the remaining transmit antennas. This can be done by linear combining the received signal with nulling weight vector $\boldsymbol{w}_n$, corresponding to the symbol to be detected, obtained by using the zero-forcing or MMSE method. Since MMSE detector provides better BER performance than the zero-forcing, we shall focus our attention on this detector in this paper. The nulling weight vector using MMSE method is given in

(3). The detected signal is then fed back to the linear combining process and its contribution is canceled from the received signal in the next detection iteration. Using this *nulling and cancellation* detection, the signal to be detected in the next iteration is provided with larger diversity gain due to increased degree of freedom in the receive antenna array, thus has better BER performance. The average BER performance over N detection iterations of the V-BLAST detection was shown to be better than that of the MMSE detection [4].

Depending on the process of carrying out iteration, there are two versions of the V-BLAST detection: one with ordering and the other without ordering. In the *V-BLAST detection with ordering*, the received symbols from transmit antennas are detected in the decreasing order of SNR, i.e., a symbol with the highest SNR is detected first in each detection iteration. The *V-BLAST detection without ordering*, on the other hand, detects received symbols in an arbitrary order. The detection method with ordering provides better performance than the one without ordering, at a cost of small additional processing for finding a symbol with maximum SNR in each iteration. Detailed algorithm of the V-BLAST detection can be found in [4]. The complexity order of the V-BLAST detection is also the same with that of the MMSE, i.e., $O(M^3)$.

3 Combined MMSE-ML Detection

Configuration of the proposed MMSE-ML detector for MIMO-SDM is shown in the left side of Fig. 1. The principle of this combined scheme is to use an efficient ML post-detection processing (PDP) scheme with capability to find out a predefined number of estimates from MMSE detection with the highest probability of errors and then correct them to have better BER performance. Our proposed method of detecting symbols with the highest probability and correcting algorithm based on the ML method is presented below. For the sake of simplicity, we present the algorithm for the BPSK modulation, extension to other modulations is straightforward.

Let $\hat{\boldsymbol{s}} \triangleq [\hat{s}_1,\ \hat{s}_2,\ \ldots\ ,\hat{s}_N]^T$ and $\check{\boldsymbol{s}} \triangleq [\check{s}_1,\ \check{s}_2,\ \ldots,\ \check{s}_N]^T$ be two vectors containing the estimates from a MMSE and ML detector, respectively. Define a_i as the i-th element of vector $\boldsymbol{a}$. Now assume that $\hat{s}_i \neq \check{s}_i$ then it is clear that NOT$(\hat{s}_i) = \check{s}_i$, where NOT$(\bullet)$ is the logical negative operator. Define $\hat{\boldsymbol{s}}' \triangleq [\hat{s}_1,\ \ldots,\ \text{NOT}(\hat{s}_i),\ \ldots,\ \hat{s}_N]^T$. Since the estimated sequence from the ML detector is the one which give minimum ML metric, it is observed that Prob $\left(\|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}\|^2 > \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'\|^2\right) \approx 1$. Now let us consider 2 arbitrary estimated symbols i and j from the MMSE detector. Assume that $\hat{s}_i = \check{s}_i$ and $\hat{s}_j \neq \check{s}_j$. For two arbitrary symbols i and j we define 2 corresponding vectors

$$\hat{\boldsymbol{s}}'_i = [\hat{s}_1,\ \ldots,\ \text{NOT}(\hat{s}_i),\ \ldots,\ \ldots,\ \ldots,\ \hat{s}_N]^T \tag{4}$$

$$\hat{\boldsymbol{s}}'_j = [\hat{s}_1,\ \ldots,\ \ldots,\ \ldots,\ \text{NOT}(\hat{s}_j),\ \ldots,\ \hat{s}_N]^T\ . \tag{5}$$

Since $\hat{s}_i = \check{s}_i$ then NOT$(\hat{s}_i) \neq \check{s}_i$, and since $\hat{s}_j \neq \check{s}_j$, we have NOT$(\hat{s}_j) = \check{s}_j$. As observed above: $\text{Prob}\left(\|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_i\|^2 > \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_j\|^2\right) \approx 1$. As a result, if $\|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_i\|^2 > \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_j\|^2$ then we can conclude that the probability that the j-th symbol of $\hat{\boldsymbol{s}}$ is erroneous[1] is higher than that is the i-th symbol. Therefore, we can find a symbol with the highest probability of error by rotating the phase of the i-th symbol in $\hat{\boldsymbol{s}}$ to obtain $\hat{\boldsymbol{s}}'_i$, then using $\hat{\boldsymbol{s}}'_i$ to compute the Euclid metric $\gamma_i = \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_i\|^2$. The smaller γ_i is, the higher probability of error is for symbol i. Based on this simple method of detecting symbols with the highest probability of errors, the operational algorithm of the ML-PDP after the MMSE detector is developed for BPSK modulation as follows:

ML-PDP Algorithm

1) Input the estimates $\hat{\boldsymbol{s}}$ from N outputs of MMSE detector.

2) For $i = 1$ to N: rotate the phase of the i-th symbol in $\hat{\boldsymbol{s}}$ to have $\hat{\boldsymbol{s}}'_i$, then compute the Euclid metric $\gamma_i = \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_i\|^2$.

3) From N values of γ_i obtained in step 2, locate B symbols with the smallest γ_i.

4) Apply the ML detection to the B symbols located in step 3 while keeping the remaining symbols unchanged. This means that for 2^B possible states of the B bits we have 2^B combinations of $\hat{\boldsymbol{x}}$, defined as $\hat{\boldsymbol{s}}'_\ell$, $\ell = 1, 2, ..., B$, choose $\bar{\boldsymbol{s}} = \arg\min_{\hat{\boldsymbol{s}}'_\ell} \|\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{s}}'_\ell\|^2$ as the outputs of the detector.

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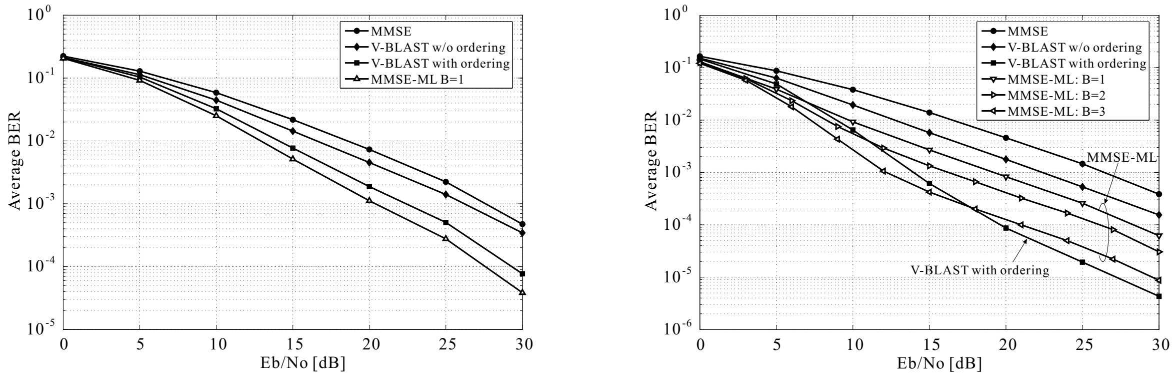
[1] Since estimates from a ML detector is an optimum sequence, here we refer MMSE estimated symbols different from the corresponding ones using the ML method as errors.



(a) 2×2 MIMO (b) 4×4 MIMO

Figure 2: Performance comparison of the proposed MMSE-ML with V-BLAST detection.

It is worth noting that the complexity of the proposed MMSE-ML detector is of order $O(\max[M^3, 2^B])$. Therefore, provided that $2^B \leqslant M^3$, the complexity order of the proposed MMSE-ML detector is also the same as that of the MMSE, i.e., $O(M^3)$.

4 Performance Evaluation

In order to evaluate performance of the proposed detector, we have set up computer simulations for 2×2 and 2×4 MIMO systems using BPSK modulation. Three detections, namely, V-BLAST without ordering, V-BLAST with ordering, and our proposed MMSE-ML, were adopted for estimating transmit symbols. In our proposed detector, the number of corrected symbols B were selected such that $2^B \leq M^3$, so as to keep the complexity order of the detector is the same as that of the MMSE and V-BLAST detectors. The obtained BER performances for the case of 2×2 and 4×4 MIMO are shown in Figs. 2(a) and (b), respectively. For reference, BER using the MMSE method is also shown in the figures. We notice from the figures that our proposed detector outperforms the MMSE and V-BLAST without ordering in both 2×2 and 4×4 MIMO systems. Compared with the V-BLAST detector with ordering, our proposed detector provides better BER in the 2×2 MIMO system. In the 4×4 MIMO system, the only condition helping the proposed detector to have better BER than the V-BLAST detector with ordering is with $B = 1$ and $E_b/N_0 < 16\mathrm{dB}$. The performance difference of the two detectors in the two MIMO systems is attributed to their processing methods. As more DOF is obtained with increased detection iteration, the V-BLAST detector inherits better diversity gain from a MIMO system with more transmit antennas, whereas BER performance of the MMSE-ML is independent of the number of transmit antennas but only on the number of corrected bits B. As a result, the V-BLAST with ordering outperforms the proposed detector in a system with more transmits antennas. Generally, we can conclude that our proposed detector is best suitable for a MIMO system with a small number of transmit antennas and low input SNR.

5 Conclusions

In this paper, we have proposed a combined MMSE-ML detector for wireless MIMO-SDM communications. The proposed combined MMSE-ML detector is particularly suitable for MIMO-SMD systems with a small number of transmit antennas and low input SNR.

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