ANALYSIS OF TWO-PORT CIRCULAR MICROSTRIP PATCH ANTENNAS USING MULTIPORT NETWORK MODEL

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1. Introduction:

In addition to the rectangular patch, the circular patch is one of the widely used microstrip antenna configurations. Two-port circular patches (*Figure* 1) form elements of series-fed arrays. An approximate analysis of this configuration was reported in [1], wherein only the dominant mode of the cavity is considered and the radiated power and losses are accounted for by considering an effective loss tangent for the substrate dielectric medium. The purpose of the present paper is to report a method for more accurate analysis of two-port (and multiple port) circular microstrip patches by extending the multiport network model (MNM) [2]. The method consists of modeling the external fields as a multiport segment and incorporating the effect of microstrip feed junctions reactances by modeling sections of feed lines as rectangular planar segments. Results computed by this approach have been verified by an experiment. It is shown that, unlike in the case of a rectangular patch, the edge-admittance network (EAN) for the circular patch must include a fringing inductance also.

2. Method of Analysis:

An equivalent multiport network for a two-port circular patch (fed with microstrip lines) is shown in *Figure* 1. Each segment in the *Figure* (namely PATCH, two FLNs and EAN) is characterized in terms of impedance-matrix or admittance-matrix. Segmentation method [3] is used to combine these segments to yield the scattering parameters of the two-port patch and the values of the voltages at the NC ports at the edge of the patch. This voltage distribution is used to compute the radiation characteristics of the patch. Descriptions of these segments of MNM are given in this Section.

2.1 <u>Fields inside the patch</u>: The fields underneath the patch are computed by considering the patch as a two-dimensional resonator. The Z-matrix characterization of this circular segment (with NC+2×ND ports), denoted by 'PATCH' in *Figure* 1, is obtained by using the Green's function formulation discussed in [2]. Expression for Green's function [4], computed using eigenfunction expansion technique, involves a double summation. The computation of the elements of Z-matrix based on this expansion is very slow. An alternative expression for Green's function based on mode-matching technique is given by

$$G(\rho, \phi | \rho_0, \phi_0) = \frac{i\omega\mu h}{4} \sum_{n=0}^{\infty} \sigma_n \cos n(\phi - \phi_0) \frac{J_n(k\rho_<)}{J_n'(ka)} \cdot$$

$$\left[J_n(k\rho_>) Y_n'(ka) - Y_n(k\rho_>) J_n'(ka) \right] \quad (1)$$

where J_n and Y_n are Bessel functions of 1st and 2nd kind. $\rho_> = Max(\rho, \rho_0)$ and $\rho_< = Min(\rho, \rho_0)$. h is the thickness of the substrate, a is the radius of the circular patch and σ_n equals 1 for n=0 and 2 otherwise.

2.2 Microstrip feed reactances: The effects of the input and output microstrip feed junction reactances are included by considering sections of the feed lines as rectangular planar segments (denoted by FLNs in *Figure* 1) and connected to the patch at ND (typically 5 or higher) number of ports. The fringing fields along the transmission line are taken into account by using an effective dielectric constant and effective width for the line. Elements of Z-matrix of a rectangular segment are given in [2].

2.3 Fields outside the patch: The power radiated from the patch and the effect of fringing fields at the edges are included in the MNM analysis by connecting an equivalent edge admittance network (EAN) to the antenna's periphery at NC number of ports (as shown in *Figure* 1). The EAN consists of a pair of equivalent radiation conductance G_p and capacitance C_p (connected to each of the ports), and a fringing inductance L_p (connected in between the adjacent ports as shown in *Figure* 1 inset).

Edge conductance : The radiation conductance per unit length along the edge of a circular patch is related to the power radiated from the circular patch and to the voltage distribution at the edge by

$$G_r = 2P_r / \int_c V^2(\phi)(ad\phi) \tag{2}$$

where the integration in (2) is along the periphery c of the patch (of radius a). For a circular patch operating in the dominant mode (1,1), the voltage distribution at the periphery is $V(\phi) = V_m \cos(\phi - \phi_m)$, where ϕ_m is the angle at which the voltage is maximum. So, the integration in the denominator of equation (2) is equal to $(\pi a V_m^2)$. The radiated power from the patch is computed by using the equivalence theorem. This is done by replacing the fringing field at the periphery by an equivalent magnetic current loop of radius a. Expression for radiated power based on this approach is given in [5]. The radiation conductance connected to the ith port (of width W_i) is $G_p = G_r \times W_i$.

Edge capacitance: The effects of fringing electric fields at the patch periphery are included by using an edge capacitance (C_p) connected to the NC ports at the open edge in parallel with G_p. A formula for the fringing edge capacitance of a circular resonator derived from the total static capacitance given by Kirchhoff [6] has been used.

Edge inductance: The fringing edge inductance L_e accounts for the energy stored in the fringing magnetic fields. These fringing magnetic fields are associated with the tangential current flowing at the edge of the patch. Since the widths of the NC ports along the edge are small, the fringing inductance per unit length (L) along the edge is approximately related to the edge capacitance per unit length as:

$$L = \frac{\mu_0 \epsilon_0}{C(\epsilon_r = 1)} \tag{3}$$

where $C(\epsilon_r = 1)$ is the edge capacitance per unit length when dielectric is replaced by air. This formulation can be justified by using similarity between current distribution of a circular patch and that of a rectangular patch operating in the dominant mode.

The fringing inductance connected between ports i and i+1 along the edge is $L_p = L \times (W_i + W_{i+1})/2$.

The fringing inductance is not needed for EAN's at a radiating edge of a rectangular patch. This is because, the voltage is uniform along a radiating edge and no current flows through the inductances if connected. Inductances are required for a rectangular patch when EAN are connected along a non-radiating edge where the voltage has a cosine variation. It can be shown, in this case, that the fringing inductance per unit length is related to the fringing capacitance per unit length by (6). However, when a planar waveguide model (using effective width and effective dielectric constant) is used [7], the fringing inductance is included in this planar model itself.

3. Comparison with Experiment:

A two-port circular microstrip patch (a=0.7825 cm, ϵ_r=2.2, d=1/32 inch, ϕ_{12} = 156.5°) fed with microstrip lines located at the periphery was fabricated for verifying the MNM model proposed above. The characteristic impedance of the input and output lines is chosen to be 50 Ω. *Figure* 2 shows the variation of theoretical and experimental results for input VSWR versus frequency from 7 to 8 GHz. The computed resonance frequency using MNM is 7.6 GHz and the measured resonance frequency is 7.65 GHz. The bandwidth of the two-port patch for VSWR≤ 1.5 is of the order of 10 %. *Figure* 3 shows a comparison between theoretical and measured magnitudes of the transmission coefficient (S_{21}) of the two-port patch. Good agreement between theory and experiment is obtained over the whole range of frequency considered. Theoretical values of $|S_{21}|$ are slightly smaller than the measured values. This is may be due to the extra losses in the connectors.

4. Concluding Remarks:

The MNM model [2] has been extended for the analysis of two-port circular microstrip patches. Fringing inductance needs to be included in the edge admittance networks. Results based on MNM show an excellent agreement with experiment. The method presented is useful for the design of series-fed arrays using circular patches. The method is applicable to circular patches with more than two ports also.

References:
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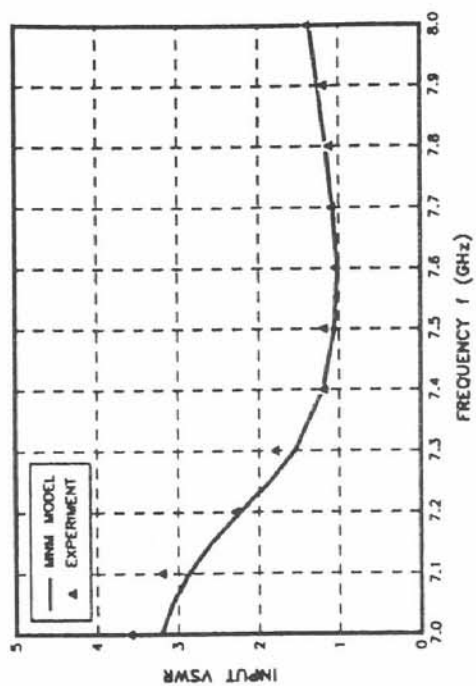[7] I. Wolff and N. Knoppik, IEEE Trans. Microwave Theory Tech., 1974, MTT-22, pp. 857-864.

Fig. 2 Comparison of experimental and theoretical values of input VSWR for the two-port circular patch.
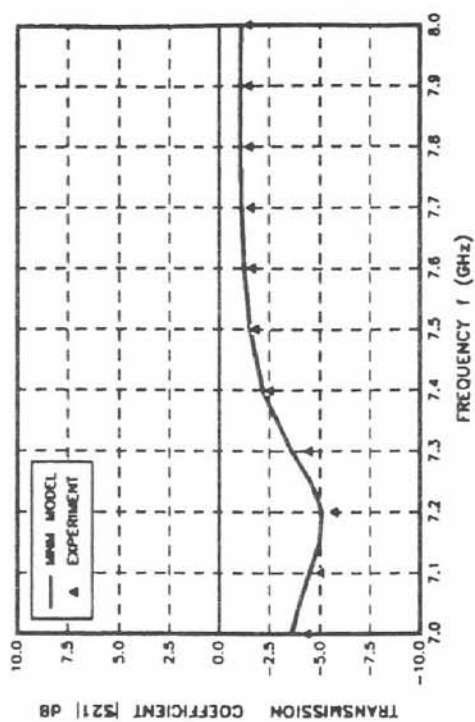

Fig. 3 Comparison of experimental and theoretical values of $|S_{21}|$ for the two-port circular patch.
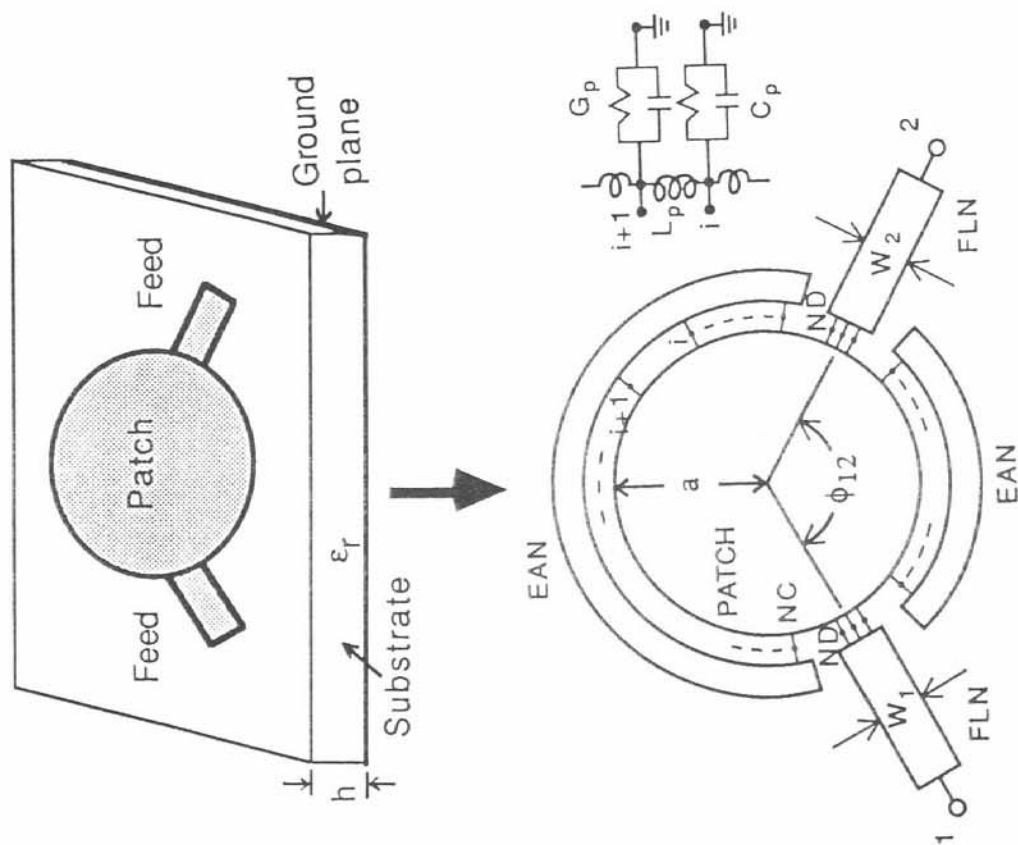

Fig 1 Multiport network model of a two-port circular microstrip patch.