PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

# ARRAY PATTERN SYNTHESIS IN THE PRESENCE OF ELEMENT FAILURE

M H ER

Centre for Signal Processing
School of Electrical & Electronic Engineering
Nanyang Technological University
Nanyang Avenue, Singapore 2263

## 1. INTRODUCTION

Arrays have been used in many signal acquisition applications [1] such as sonar, radar, communications and seismology. One of the practical problems in array systems design is how to cope with element failure as there is a high probability that one or more elements will fail (no output) for every array used in the real world. Ramsdale and Howerton [2] had shown that element failure produces an elevated sidelobe level whose peak value can be estimated and in many cases bounded above by the ratio of the sum of the amplitude weights of the faulty elements to the sum of the amplitude weights of the functioning elements. Processing techniques had been proposed in the literature to regain the sidelobe suppression lost due to inoperative elements. Among such techniques are algorithms for adjusting the amplitude weights [3, 4], cross-sensor beamforming [5], subaperture processing [6] and estimating the signal at inoperative elements by using those from neighbouring elements [2].

In this paper, a simple and effective algorithm is presented to regain sidelobe suppression lost due to element failure with narrowest possible beamwidth. The basic idea is to match the array response in the presence of faulty elements to a desired response over the mainlobe width while minimizing the mean-square value of the array response over the sidelobe regions. This formulation results in a constrained optimization problem involving a quadratic constraint and a set of linear constraints on the weights. The paper presents an efficient numerical technique to obtain the optimal weights. Numerical results are presented to illustrate the effectiveness of the approach.

## 2. PROBLEM FORMULATION

Without loss of generality, consider a linear array of N isotropic antenna elements with uniform spacing. The antenna far-field pattern is given by $G(f_0,\theta) = \underline{W}^H\underline{S}(f_0, \theta)$, where $f_0$ is the frequency of interest, superscript H denotes complex conjugate transpose, and $\underline{W}$ is the N-dimensional complex weight vector given by $\underline{W} = [w_1, w_2, \ldots, w_N]^T$, and $\underline{S}(f_0,\theta)$ is the N-dimensional steering vector given by $\underline{S}(f_0,\theta) = [\exp(j2\pi f_0\tau_1) \ \ldots \ \exp(j2\pi f_0\tau_N)]^T$, where $\{\tau_i, \ i = 1, 2, \ldots, N\}$ are the propagation delays between the plane wavefront and the antenna elements.

The mean-square error between the desired response and the response of the array system over the mainlobe width is given by

$$e^2 = \frac{1}{\beta}\int_{\theta_0-\Delta\theta/2}^{\theta_0+\Delta\theta/2} \left| A(f_0,\theta) - \underline{W}^H \underline{S}(f_0,\theta) \right|^2 d\theta$$
$$= \underline{W}^H Q_1 \underline{W} - \underline{W}^H \underline{P} - \underline{P}^H \underline{W} + 1 \tag{1}$$

where $A(f_0,\theta)$ is the desired response, $\theta_0$ the look direction, $\Delta\theta$ the mainlobe of interest and $Q_1$ and $\underline{P}$ are a N x N dimensional Hermitian matrix and N-dimensional vector respectively.

The mean-square value of the array response over the sidelobe regions is given by

$$\rho = \frac{1}{\Delta\theta_1}\left[\int_{-\pi/2}^{-\theta_1}\left|G(f_0,\theta)\right|^2 d\theta + \int_{\theta_1}^{\pi/2}\left|G(f_0,\theta)\right|^2 d\theta\right] = \underline{W}^H Q_2 \underline{W} \tag{2}$$

where $\Delta\theta_1 = [\pi/2 - \theta_1]$, and $Q_2$ is a N x N dimensional Hermitian matrix.

The optimal weight vector in the presence of element failure is the solution to the following constrained optimization problem:

$$\underset{\underline{W}}{\text{minimize}} \quad (\underline{W}^H Q_1 \underline{W} - \underline{W}^H \underline{P} - \underline{P}^H \underline{W} + 1) \tag{3a}$$

$$\text{subject to} \quad \underline{W}^H Q_2 \underline{W} \le \xi \tag{3b}$$

$$C^H \underline{W} = \underline{0} \tag{3c}$$

where $\xi$ defines the mean-square sidelobe level and C is the N x n dimensional matrix given

$$C^T = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \updownarrow n \tag{4}$$

The linear constraints given by (3c) force the weights of the inoperative elements to zero. The dimension n specifies the number of faulty elements.

## 3. PROBLEM SOLUTION

Using the method of Lagrange Multipliers [7], it can be shown that the optimal weight vector $\hat{\underline{W}}$ which solves the (3) is given by

$$\hat{\underline{W}} = (Q_1+\hat{\alpha}_0 Q_2)^{-1}\left\{I - C\left[C^H(Q_1+\hat{\alpha}_0 Q_2)^{-1}C\right]^{-1} C^H(Q_1+\hat{\alpha}_0 Q_2)^{-1}\right\}\underline{P} \tag{5}$$

where $\hat{\alpha}_0$ is determined such that

$$\hat{\underline{W}}^H Q_2 \hat{\underline{W}} = \xi \tag{6}$$

The computational complexity of determining $\hat{\alpha}_0$ can be very much reduced by using the matrix factorization method as follows:

Since $Q_2$ and $Q_1$ are Hermitian matrices, with $Q_2$ positive definite, there exists a nonsingular matrix $\Gamma$ [8] such that

$$Q_2 = \Gamma \, \Gamma^H \tag{7}$$

$$Q_1 = \Gamma \, \Lambda \, \Gamma^H \tag{8}$$

Substituting (7) and (8) into (5) and in turn into (6) and after some manipulation, one obtains

$$\| \, (\Lambda + \hat{\alpha}_0 I)^{-1} \Big\{ I - \Gamma^{-1} C \Big[ C^H (\Gamma^H)^{-1} \, (\Lambda + \hat{\alpha}_0 I)^{-1} \, \Gamma^{-1} C \Big]^{-1} C^H (\Gamma^H)^{-1} \, (\Lambda + \hat{\alpha}_0 I)^{-1} \Big\} \, \Gamma^{-1} \, \underline{P} \, \|^2 = \xi \tag{9}$$

Any root finding method can be used to solve for $\hat{\lambda}_0$ which satisfies the equation defined by (9). It was found that the problem of determining $\hat{\alpha}_0$ which satisfies (9) can be made very efficient by using the half-interval method followed by Regular Falsi algorithm [9] as it has shown excellent convergence properties.

Note that in (9), $\Gamma^{-1}$, $\Lambda$ and $\underline{P}$ can be pre-computed and stored. Hence in the presence of element failure, one needs to speficy the matrix C and solve for $\hat{\alpha}_0$. The computation load to solve for $\hat{\alpha}_0$ is very cheap using (9).

## 4. NUMERICAL RESULTS

To demonstrate the performance achievable with the proposed approach, computer studies involving a linear array of 32 elements have been carried out. The inter-element spacing was set at $0.5\lambda_0$.

Figure 1 shows the directional pattern of a conventional beamformer using uniform taper weights without (solid line) and with $2^{nd}$ and $9^{th}$ elements faulty (dash line). It can be seen that in the presence of faulty elements, the sidelobe level increases.

Figure 2 shows the directional pattern in the presence of elements failure after optimization. $Q_1$ is integrated over the range from $-4°$ to $4°$ and $Q_2$ is integrated over $[-90°, -6°]$ and $[6°, 90°]$. Two values of $\xi$ were used in the design, $\xi = 10^{-3}$ (solid line) and $\xi = 10^{-5}$ (dash line). It can be seen that the optimized patterns achieve low sidelobe with marginal increase in the mainlobe.

## 5. CONCLUSIONS

The paper has presented a simple and effective algorithm to regain sidelobe suppression lost due to element failure with narrowest possible beamwidth. The problem is formulated as a constrained optimization problem involving a quadratic constraint and a set of linear constraints on the weights. An efficient numerical technique based on matrix factorization has been proposed to solve for the optimal weights. Numerical results showed that the proposed technique is very useful in optimizing the beam pattern in the presence of element failure.

## REFERENCES


[1] The Handbook of Antenna Design, Volume 1 and Volume 2, Editors, A.W. Rudge, K. Milne, A.D. Olver and P. Knight, Peter Peregrinus Ltd., London, U.K.

[2] D.J. Ramsdale and R.A. Howerton, "Effect of element failure and random errors in amplitude and phase on the sidelobe level attainable with a linear array", J. Acoust. Soc. Am., Volume 68, No. 3, pp. 901-906, September 1980.

[3] J.K. Hsiao, "A review of thinning techniques for linear arrays", NRL Memo Report 3495, May 1977.

[4] H.S.C. Wang, "Amplitude shading of sonar transducer arrays", J. Acoust. Soc. Am., Volume 57, pp. 1076-1084, 1975.

[5] H.P. Bucker, "Cross-sensor beamforming with a sparse linear array", J. Acoust. Soc. Am., Volume 61, pp. 494-498, 1977.

[6] M.J. Hinich, "Bearing estimation using a perturbed linear array", J. Acoust. Soc. Am., Volume 61, pp. 1540-1544, 1977.

[7] A.E. Bryson, Jr. and Y.C. Ho, Applied Optimal Control, Waltham, Massachusetts, Blaisdell, 1969.

[8] R. Bellman, Introduction to Matrix Analysis, McGraw-Hill, New York, 1960.

[9] B. Carnahan, H.A. Luthar and J.O. Wikas, Applied Numerical Methods, Wiley, New York, 1969.


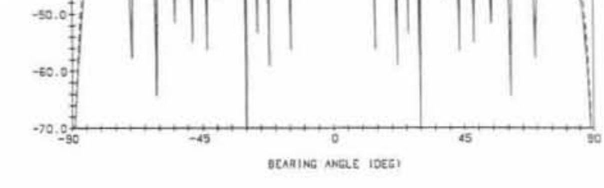


Figure 1

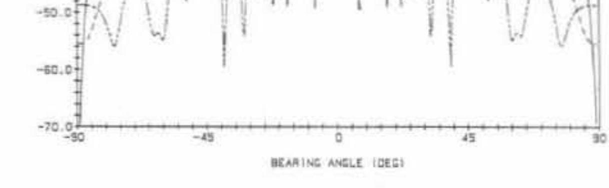


Figure 2