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COMPROMISE OPTIMIZATION IN THE THEORY OF ARRAY SYNTHESIS

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Functional approach to formulating and solving problems of compromise synthesis is developed. This approach enables to obtain simple formulas for finding global optimum by means of variational methods in a number of cases important for applications. The approach is demonstrated for the functionals corresponding to the three types of arrays which are widely used, i.e.systems with summary and difference radiation patterns, and arrays with reactively loaded elements.

I.Summary radiation patterns. A typical problem generating such a pattern is that of maximization of realized gain with constraints on the level of side lobes in a discrete set of directions $\vec{u}_1, \dots, \vec{u}_M$ or in the set of solid angles $\Omega_1, \dots, \Omega_L$ $(\Omega_i \cap \Omega_j = 0, i \neq j)$. In the first case the problem is mathematically reduced [I,2] to maximizing the functional

$$max \to G = \frac{|(J, e_0)|^2}{(HJ, J)} \qquad (I)$$

under the conditions

$$\frac{(J, e_m)}{(J, e_0)} = \alpha_m \qquad (m = 1, \dots, M) \qquad (2)$$

and the solution yields the optimal current

$$J = \sum_1^M Z_m H^{-1} e_m + e_0 ,$$

where Z_m is found from the simultaneous linear equations

$$\sum_{m=1}^M Z_m (H^{-1} e_m, e_n - \alpha_n^* e_0) = (H^{-1} e_0, \alpha_n^* e_0 - e_n) \quad (n = 1, \dots M)$$

Here $J = (J_1, \dots, J_N)$ is a current vector; $e = (e^{jk\vec{r}_1 \vec{u}_0}, \dots, e^{-jk\vec{r}_n \vec{u}_0})$; $\vec{r}_n$ is a radius-vector showing the location of elements, $k = \frac{2\pi}{\lambda}$ is a wave number; $\vec{u}$ is a unit vector of direction, H is a Hermitian matrix with elements

$$h_{m,n} = \frac{1}{4\pi} \int_\Omega (e^{jk(\vec{r}_n - \vec{r}_m)\vec{u}} + \frac{R_n}{R_\Sigma} \delta_{m,n}) d\Omega .$$

Here R_Σ and R_n stand for radiation resistance and loss resistance in elements, respectively.

In the second case the constraints are formulated by means of the relation

$$\frac{\int_{\bigcup_1^L \Omega_j} |(J, e)|^2 d\Omega}{(HJ, J)} = \frac{(AJ, J)}{(HJ, J)} \leq \alpha \qquad (3)$$

The solution of the problem (1), (3), because of spectral properties of Hermitian matrices, has the form

$$J = Z(E_\lambda + \rho E)^{-1}\mathcal{E}$$

Here Z is a matrix constructed from the columns of principal vectors Z_m of the bundle of Hermitian matrices $HZ_m = \lambda_m AZ_m$ $(m = 1, \dots, N)$;
E_λ is a diagonal matrix with elements $\lambda_1, \dots, \lambda_N$; $\mathcal{E} = (Z^*)'e$;
ρ is the least-positive root of the polynomial

$$\sum_1^N (1 - \alpha\lambda_n)|\mathcal{E}_n|^2 \prod_{\substack{m=1 \\ m \neq n}}^N (\lambda_m + \rho)^2 = 0$$

2. Difference radiation patterns. Such patterns are characteristic for minimal signal pelengation systems, which form patterns equal to zero in the main direction (θ_0, φ_0). Introducing the measure of steepness of directivity function in the zero point by means of the functional

$$\max \longrightarrow \mathcal{K} = \Big[\lim_{\substack{\theta \to \theta_0 \\ \varphi \to \varphi_0}} \frac{d|F|}{d\theta}\Big]^2 = \Big|\frac{d}{d\theta} F(\theta_0, \varphi_0)\Big|^2 = |(J, \tilde{e})|^2,$$

where $F(\theta, \varphi)$ is the array pattern with respect to the field, one can show [3] that the determination of pelengation accuracy in the available arrays is reduced to maximising the functional $\mathcal{K}$ with the constraints

$$F(\theta_0, \varphi_0) = (J, e_0) \ ; \ (H_1 J, J)\big/(C_1 + (H_2 J, J)) = s,$$

where the numerator of the latter expression is the average value of signal in the chosen zone of the main zero while the denominator is the sum of internal and external noise power. In this case the optimum is reached on the currents

$$J = \frac{1}{\lambda_2}(H_1 - sH_2)^{-1}(\lambda_1 e_0 + \tilde{e}); \quad \lambda_1 = -\frac{((H_1 - sH_2)^{-1}\tilde{e}, e_0))}{((H_1 - sH_2)^{-1}e_0, e_0)};$$

$$\lambda_2 = \sqrt{C_1 s/(\lambda_1 e_0 + \tilde{e}, (H_1 - sH_2)^{-1}(\lambda_1 e_0 + \tilde{e}))}$$

3. Synthesis of arrays with reactive loads. This problem, particularly arising when optimising arrays with parasitic elements, is reduced to the best approximation of the given diagram F_0 on the set of feasible currents determined by the condition of load reactivity

$$(HJ, J) = Re(U, J),$$

where U is a vector of normalized strains on the element terminals, H is a matrix of normalized resistances. One can show that the global minimum is reached on the currents

$$J = \frac{1}{1+\lambda} H^{-1}(C + \frac{\lambda}{2} U)$$

Here

$$C = (C_1, \dots, C_N) \quad ; \quad C_m = \frac{1}{4\pi} \int_{\Omega} F_0 \, e^{-jk \vec{r}_m \vec{u}} \, d\Omega \, ,$$

$$\lambda = -1 + \sqrt{1 + \frac{(H^{-1}C, C) - Re(U, H^{-1}C)}{\frac{1}{4}(U, H^{-1}U)}}$$

The above-mentioned results were used to investigate the possibility of optimising radiotechnical and constructive parameters of arrays taking into account the particularities of superconductive state and regime of higher directivity [1, 2].

References

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