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STATISTICAL ANTENNA THEORY

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Abstract

The present paper gives a short characteristics of the statistical antenna theory (SAT) nature and current state.This theory allows to study to the full extent the properties of the real antennas being in principle radiating systems with random sources. In brief form the paper presents: the SAT contents, methods of this theory direct and inverse problems solution, the main effects, characterizing the influence of "randomness" in antennas on their parameters.

1. Introduction

The statistical antenna theory is a theory of antennas with random sources. Amplitude,phase, sources polarization, their number, orientation etc. can be random. In one case the fluctuations (random errors) emerge in antenna itself due to the antenna elements manufacture inaccuracies,their parameters instability, failures, operation (weight, wind, thermal) deformations etc. In other cases fluctuations are caused by propagation conditions of the wave falling on the antenna. The presence of the randomness in the antenna deteriorates its characteristics, limits their maximum attainable values. This is of particular importance for large reflector antennas and phased antenna arrays. As the large antennas are extremely expensive it is very important to know the reasons of the randomness origin and their nature, to be able to compute their influence on the antenna parameters, to synthesize antennas with allowance for the randomness occurring in them,to know the possibilities to weaken their influence. Solution of all this questions constitutes the subject of the SAT appeared at the turn of the 60's.

As the practice confirms at present the large expensive antennas design, computation of their real and maximum attainable characteristics with allowance for random design factors, different operation conditions, and also the medium influence are impossible without use of the SAT to a certain extent, its approaches and results. The reasonable requirements to antenna production, tolerances, antenna elements parameters stability, their reliability are formed on the SAT basis.

From the above said it follows that the range of problems solved by the SAT is rather wide.

For the first time the SAT as a new scientific direction was thoroughly formulated in the monograph [1]. For the time passed after appearance of [1] many papers directed to the development of the general theory, to the generalization of the results presented in [1], various application problems solution are published. The aim of the given survey consists in presentation in a short form of the SAT modern concept - its contents, the degree of studying of the theory various sections, the most important for practice results. The greater part of results given in this survey is based on the previously published results of the author, this is marked in the text by the corresponding references.

2. The SAT Contents

The basic relations of the general antenna theory combined with probability theory and random functions theory form the basis of the SAT apparatus.

As in the common (deterministic) antenna theory, it is possible to single out two large sections in the SAT devoted to the solution of direct and inverse problems, respectively (Fig.1).

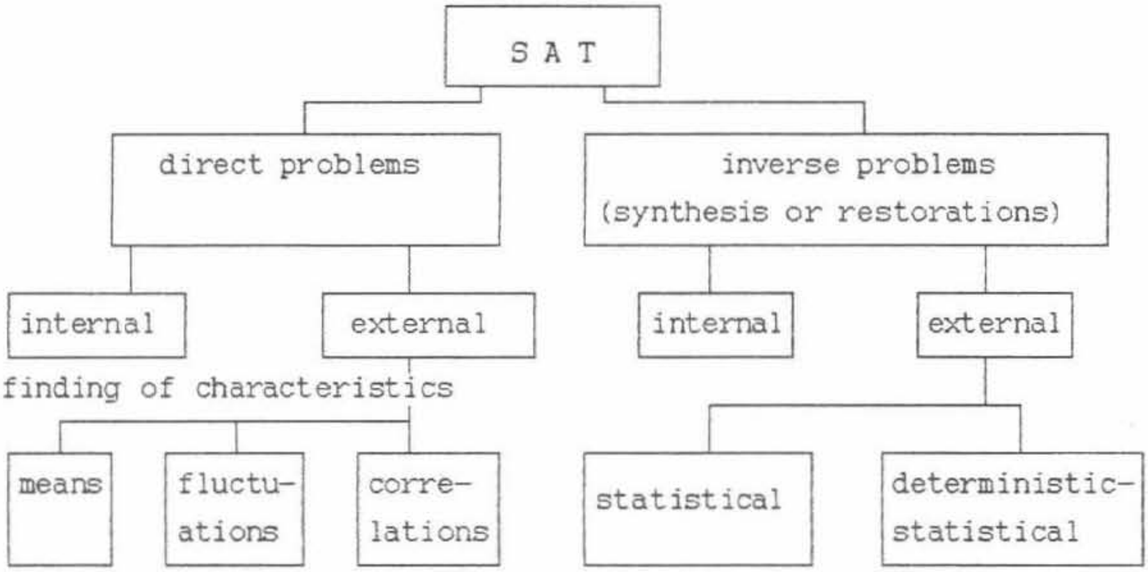

Fig.1 Contents of SAT

The direct problem consist in determination of the antenna radiation field statistics with either randomness origin mechanism. The problem is divided, as a rule, into two independent problems-internal and external ones. The aim of the internal problem solution consists in finding of statistics of source distribution in antenna. The external problem consist in finding of the antenna field statistics by the found source statistics. As a rule three groups of statistical characteristics are studied: antenna parameters mean values, their fluctuations and antenna field correlation properties. The latter characterize the coupling of the antenna field fluctuations in the "adjacent" points of space.

The SAT inverse problem is intended to define the antenna "statistical" structure by its radiation field given statistics. The inverse problem (similarly the direct one) is also divided into two problems: the internal and external, but the sequence of their solution is another one. At first the external problem is solved. The aim of its solution consists in finding the source distribution statistics in the antenna by the given radiation field statistics.

Here there may be two types of such problems: the synthesis problems when it is necessary to find the statistics of source distribution in the antenna which provides for the antenna field statistical characteristics optimum in any sense, and the restoration (reconstructions) problems when it is needed to determine by the known radiation field statistics what the statistics of the source distribution in the antenna is. After solution of the external inverse problem the internal problem should be solved. In the synthesis problems the aim of its solution is to define the antenna structure which will guarantee the source distribution in the antenna found in the course of the external problem solution. In the restoration problems the internal problem has for an object to determine what state of the antenna (or what values of the wave propagation medium parameters) answers to the statistics of sources in the antenna found in the course of the external problem solution.

The inverse deterministic-statistical problems connected with finding of such sources regular distribution, which in presence of fluctuation with known statistics of the antenna provides optimal (or measured) antenna statistical characteristics are very important.

Let us consider in short the methods and the most important results of the direct and inverse SAT problems solution.

3. Direct problems

3.1. External problems. The direct external problems is most developed and most commonly used in practice section of the SAT. The foundations of this theory and its application to antennas of the main types (continuous and discrete) are treated in [1]. The expression for separate realization of radiation field of the antenna belonging to this or that class taking into account the presence of fluctuations in it is the initial one. According to this expression using the antenna theory general formulae one can estimate its various characteristics such as radiation pattern (RP), its width, directivity gain (DG), side lobe level (SLL) etc. Now using the apparatus of the random functions theory and data about the source statistics the mean values of antennas parameters, their fluctuations and field correlation characteristic are found by a large number of realizations averaging.

The main in all these researches is to establish the basic effects characterizing the antenna parameters change in the presence of "randomness" in them, to ascertain these effects dependence on the sources probabilistic properties.

The most complete computations in [1] are carried out for linear continuous synphasal system with a uniform amplitude distribution A_o and random phase fluctuations $\varphi(x)$.

For such a system the expression for a separate field realization can be written as

$$f(\psi)= A_o \int_{-1}^{1} \exp\{j[\varphi(x)+\psi x]\}dx \qquad (1)$$

where $\psi=(\pi L/\lambda)\sin\theta$, L - is the antenna dimension, λ - is a wave length, θ - is an angle measured from the normal to the system axis, $x=2z/L$ is a relative coordinate, z - is a coordinate along the antenna. It is convenient to assume $A_o=1/2$. Then for the system without errors $f_o(\psi) = (\sin\psi)/\psi$.

The function $\varphi(x)$ is considered to be a normal homogeneous random function with a mean value $\overline{\varphi(x)}=0$, dispersion $\overline{\varphi^2(x)} = \sigma^2(x) = \alpha$ and correlation coefficient $r = \exp\{-(x-x_1)^2/c^2\}$. Here c - is a correlation radius in relative units connected with the correlation radius of phase fluctuations along the antenna ρ by the relation $c = 2\rho/L$.

The expression (1) with the accepted assumptions about $\varphi(x)$ is taken as a initial one for finding the linear antenna field statistics. The general formulas are obtained for the main antenna statistical characteristics, their various particular cases are regarded. The numerous plots characterizing the dependence of statistical parameters on dispersion and correlation radius of phase fluctuation are cited.

Later the developed in [1] theory for the far zone was generalized also to the Fresnel zone [2]. Its practical significance has essentially increased of late due to the modern antennas dimension increase (and, respectively, to the far zone boundary moving away) and also due to the EMC problem intensification.

For the Fresnel zone the following expression will be the initial instead of [1]

$$f(\psi,\chi)=\frac{1}{2}\int_{-1}^{1}\exp\{j[\varphi(x)+\psi x-\chi x^2]\}dx \qquad (2)$$

where $\chi = kL^2\cos^2\theta/8R$, $k = 2\pi/\lambda$ is a wave number, R is a distance from the antenna center. The rest symbols have the same sense as those in (1). As the value χ contains R, then the field angular distribution (or antenna's RP, if to use conventionally this notion in the Fresnel zone) will be dependent on the distance. That is why one has to study additionally in the Fresnel zone a number of regularities and effects having no analogs in the far zone. As such we can indicate all regularities defining the evolution of the field statistical characteristics in the longitudal direction [2].

The relation (2) can be formally used for the analysis of the antenna field statistics focused to the point in the Fresnel zone. In this case it is necessary to carry out the substitution of χ by

$$\chi_F=\frac{\pi L^2}{4\lambda}\left(\frac{1}{R}-\frac{1}{R_F}\right)$$

where R_F is a distance to the focusing point. Appearance of R_F in the expression for the field of the focused antenna leads to emergence of a number of new (when compared with a unfocused system) statistical effects subject to analysis. Among them there are: the influence of the source fluctuations on the focal spot structure (its expansion, the symmetry degree), the spot fluctuations etc.

Generally of the theory developed in [1, 2] is defined by that it is, in principle suitable for antennas of various types, different probabilistic properties of sources, various mechanisms of fluctuations origin. The latter is reflected in particular in that the developed apparatus is equally suitable both for studying the ensemble statistics when investigating the scatter of the same type antenna parameters with respect to antenna ensemble or time statistics when investigating the scatter of the antenna parameters of one and the same antenna in time.

As it has been shown in [1,2] the presence of randomness in the antennas results in all its characteristics change. The nature of some of statistical effects, their dependence on the fluctuation parameter α and c one can see in Fig.2,3. Fig.2 shows the mean RP by power of the linear antenna $|f(\psi)|^2$. The curve corresponding to $\alpha = 0$ defines the RP in the absence of errors: $f_0^2(\psi)=(\sin^2\psi)/\psi^2$. As can be seen from Fig.2 the random errors in the source distribution result in smoothing and smearing of RP - decrease of the field in the main direction, the "null" filling, the side lobe radiation increase, the main lobe broadening.

Smoothing of the field interference picture takes place in the Fresnel zone as well (Fig.3).But here it is expressed less boldly. It is associated with that in the Fresnel zone the initial distribution of the field intensity (distribution without errors, $\alpha = 0$) is already smoothed due to the influence of the phase errors caused by the finiteness of distance to the observation point.

The important aspect of the cited investigations is determination of the range of fluctuations parameters (for example their correlation radii) by which these or those effects manifest themselves to the full extent. Thus, for example, for the broad-side antennas with small phase fluctuations the main influence of SLL increase and DG decrease is affected by the fluctuations with small correlation radii ($\rho \ll L$). Most unpleasant are the fluctuations for which $\rho \sim \lambda$. As to the principle maximum direction deviation and broadening of mean RP the main role here belongs to the fluctuations whose correlation radii are comparable with antenna dimensions.

The maximum of these effects take place when ρ~0.4L.

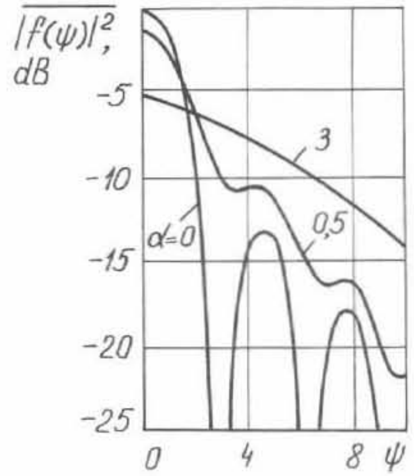

Fig.2 Mean linear antenna pattern for different α (c=0.5).

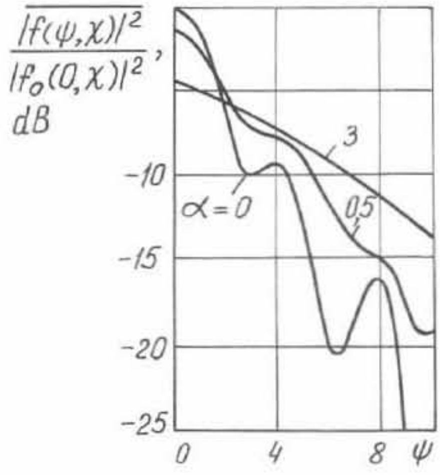

Fig.3 Distribution of the linear antenna mean field intensity in the Fresnel zone ($R=L^2/2\lambda$) for different α (c=0.5).

Rather important for practice is the fact that the randomness in the antenna imposes serious restrictions on the attainable values of the antenna parameters. This limitation are connected with such concepts and effects as the maximum DG [1]. the DG saturation [1]. the DG maximum attainable value in synthesis [3]. the maximum attainable SLL [4.5]. Serious restrictions imposed on the SLL appreciably hamper the EMC and noise immunity problems of radio systems.

Field fluctuations in the antenna aperture can be rather significant in some cases (in the transhorizon propagation lines. long short-wave lines. the presence of plasma formations on the wave path). This leads to the RP essential distortion and sometimes the RP "scatters" altogether. Such effects was observed by us on the lines of tropospheric scatter propagation [1].

Striving to overcome the indicated restrictions on the antenna characteristics. has stimulated. in particular. development of new antenna construction principles including the transition to the adaptive systems [5.6]. Among them we have antennas "adapting" to the incident wave from distorted by the media influence of many types of self-focusing antenna responding to the fluctuations originating in the antenna itself.

In conclusion it should be noted on circumstance more. emerging in the course of the SAT direct external problems studying. Its essence consists in that a whole number of habitual concepts of the general antenna theory must be corrected or re-defined in the statistical approach to the antenna analysis. Thus. for example. in the antenna theory there exist the notions of the normalized RP. DG. the RP width etc. But by the statistical approach one should distinguish the mean normalized RP and the normalized mean RP. the mean DG can be defined in two ways (and the result will be different). the mean value of the maximum DG and the maximum value of the mean DG are different values. the width of the mean RP and the mean width of RP also differ etc. Depending on the concrete situation it is reasonable to use either this or that definition of the DG. the RP width etc.

3.2. Internal problems. The aim of the direct internal problems solution. as has been noted already. is find the source statistics in the antenna (dispersions and correlation radii of fluctuations. in particular). These data are the input one computing the antenna radiation field statistics by the formulas of the external problems theory. The internal problems in every case are solved in their own way depending on the fluctuations origin mechanism, the type and structure of the antennas. This explains the fact that the number of works devoted to the internal problems were relatively small. Recently owing to the forced development of large expensive antennas the progress is outlined in the field of study of the random errors inherent to these antennas.

For the reflector antennas the internal problem solution means above all estimation of the reflector random deformations resulting from the errors in manufacturing, erection and operation factors as well. So far it has been developed a number of a new rather precision methods of the mirror deformations determination such as radars, laser phase ranges, holographic etc. ensuring an accuracy of a microns. Special measuring complexes and chambers for full-scale measurements of the large reflector antennas operation deformations are built. Algorithms and programs are developed for computing mechanical and thermal deformations of mirrors, simulation of possible conditions of operation of the satellite antennas.

For the antenna arrays the internal problem solution consists in definition of the error statistics of array radiators exciting currents. Inaccuracies in the array elements fabrication, instability of generators, amplifiers, phaser etc. can be the error sources. As a rule the errors of separate array elements are defined by experiment. The deficiency of such approach consists in that it does not take into account the source statistic variation with beam swinging resulting from the interaction of radiators. This has stimulated the appearance of a number of theoretical works devoted to the investigations of the voltage fluctuations influence on the arrays input and fluctuations on its element parameters on the array field statistics, its DG and RP. Here the interaction of the radiators is fully taken into account. It should be noted the presence of works in which at the beginning the strict solution of the boundary problem for separate arrays realization is found with the consequent application of the statistical probing method (Monte-Carlo method) [7]. In this case there are no division of the SAT direct problem into internal and external ones.

If the field fluctuations in the aperture are introduced by the incident wave then the internal problem represents in principle the wave propagation problem in either of its statement. The possibilities to solve such problems recently broaden essentially due to successes of the theory of wave propagation in random media and new wide experimental data in this field.

More complete information concerning the SAT direct internal problems can be found in [5,6].

4. SAT inverse problems

So far the antenna arrays deterministic - statistical synthesis were given the most considerations to among the whole complex of the inverse SAT problems (Fig.1). Most commonly synthesis was carried out proceeding from certain requirements to the integral parameters. Energy functional use formed the basis of it

$$\bar{æ} = \int_{4\pi} \overline{|f(\vec{u})|^2}\, g_1(\vec{u})d\Omega \Big/ \int_{4\pi} \overline{|f(\vec{u})|^2}\, g_2(\vec{u})d\Omega. \qquad (3)$$

Here $\overline{|f(\vec{u})|^2}$ is the mean RP by power of the array, $\vec{u}$ is unit vector per an observation point, $g_{1,2}$ are weighting functions. Depending on their

form the functional $\bar{æ}$ represents this or that energy factor of the antenna quality i.e. the mean DG, mean power scattering coefficient, the antenna mean noise temperature etc.

In the absence of errors $\varkappa$ defines the antenna quality factors used in the deterministic synthesis theory. The use of the mean RP in (3) and respectively passing from $\varkappa$ to $\bar{\varkappa}$ denotes an account of fluctuations in antennas in the course of their integral parameters optimization.

In the case when the unknown in the optimization problem is the regular amplitude and phase distribution (APD), the problem solution method is as in [3]. The relation for $\bar{\varkappa}$ is recorded as two Hermitian forms relation with respect to the unknown APD column-vector. Then the maximum eigenvalue of the regular bunch of matrix, presenting the numerator and the denominator of the relation (3) and the eigenvector corresponding to it which define the maximum $\bar{\varkappa}$ value and the optimal APD, is estimated with the linear algebra methods. In a number of cases (e.g. with fluctuations small or they are independent in different array elements) one succeeds in obtaining a solution in an explicit form. In the general case it is necessary to use the numerical methods. The rate of success here is defined by the efficiency of large matrix inversion methods.

If the amplitude distribution is the sought one in the optimization problem then the method, in general, remains the same as in the APD synthesis. The more complicated prove to be the phase optimization problems presenting the peculiar interest (as the phase regulation is more simple and economic then the amplitude regulation) and the problems of the radiators optimum placement estimation.

Besides the antenna integral parameters statistical optimization problem the problems of their statistical synthesis by the given RP were also regarded.

Here as the functional being optimized the following value was chosen

$$\overline{\delta^2} = \int_{4\pi} \overline{|f(\vec{u})-f_g(\vec{u})|^2}\; g_1(\vec{u})d\Omega \qquad (4)$$

characterizing the degree of scattering of the synthesized RP $f(\vec{u})$ about the given $f_g(\vec{u})$ due to the source fluctuations. Weighting function $g(\vec{u})$ in (4) allows to regulate the given RP approximation accuracy in definite angular sector.

The value $\overline{\delta^2}$ represents generalization of the root-mean-square criterion δ^2 (expression (4) without averaging sign) used in deterministic synthesis theory.

The synthesis problem consist in finding the regular source distribution minimizing the value $\overline{\delta^2}$ in presence of "specified" fluctuations in this distribution. If the APD is unknown, then $\overline{\delta^2}$ is a sum of Hermitian matrix and linear form with respect to the APD. The problem solution resolves itself into the positive definite matrix inversion with subsequent multiplication by the linear form vector. The solution is obtained in the explicit form for any antenna arrays and arbitrary random errors.

The above mentioned methods of the antenna statistical synthesis solution by the integral parameters or the given RP are similar to the methods of the analogous problems solution in the deterministic synthesis theory. But the results of their solution in a number of cases as with small distances between radiators (less than $\lambda/2$) can significantly differ. This is especially appreciable for the end-fire antenna arrays.

Statistical approach to the synthesis problem allows to receive gain (sometimes rather appreciable one) in the antenna parameters being optimized and it also allows to estimate practically attainable values of the antenna quality factors.

In principle, rather valuable feature of the statistical synthesis is that the account of fluctuations at the stage of the synthesis problem statement leads to the natural regularization of this problem, suppressing the superdirectivity effects.

Conclusion

A short study of the SAT present state carried out in this survey shows that it has been done much in this field until now. The direct external problems theory turns to be more developed. It has been shown that the source fluctuations in antennas significantly influence the antenna radiation field structure in the far and Fresnel zones, impose limitations on the attainable values of the main antenna parameters, dictates in a number of cases the choice of new antenna design principles.

The framework of the survey has defined rather shortened presentation even of the most important results of the SAT. Some interesting questions have been omitted at all. Among them are, for example, the statistic of antenna integral characteristics [2], the theory of random arrays [8], the statistical restoration problem [3,5], statistical aspect of the antenna measurement theory [9], etc. (see reviews [5,6]). Now the degree of studying of the SAT various directions is rather different. Some of them (such as the inverse problem theory) need further development. Steadily relative to practical requirements new problems emerge as well. The noted circumstances make further investigations in the SAT field rather actual.

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