A-1-1

A RAPID SYNTHESIS ALGORITHM FOR UNIFORMLY SPACED MICROWAVE ANTENNA ARRAYS

A. Van de Capelle (+), E. Van Lil (++)
K.U.leuven, Dept. of Electrical Engineering, Div. M.I.L.
B-3030 Heverlee, Belgium

Introduction

Assuming that an array can be represented by a discrete number of samples of a continuous amplitude distribution, a very rapid algorithm, in fact a generalization of the method of Lanczos (°) will be derived here for synthesizing two-dimensional arrays. Unfortunately the validity of this method is limited to equally spaced arrays, of which the array factor is given as a complex number, together with the frequency and the distances between two elements in the two used directions.

The generalized Lanczos method

Once the pattern we want to realize has been divided by the pattern of one element (of course no finite radiation may be asked in points where the basic radiator has a null), and a choice of the phase and eventually extensions of the array function outside of the visible range has been assumed (so reducing the power pattern to an array factor defined in the range $-\pi \rightarrow +\pi$), we can deduce from the definition of the array factor, namely :

$$f_2'(\tau_x, \tau_y) = \sum_{l=0}^{n_x} \sum_{m=0}^{n_y} a_{l,m}.\exp(j(\tau_x.l+\tau_y.m))$$

with $\tau_x=\xi.\Delta x$, $N_x=n_x + 1$ the number of elements in the x direction, and analogue formulas for y, that :

$$a_{l,m}=(1/2\pi)^2.\int_0^{2\pi}\int_0^{2\pi} f_2'(\tau_x,\tau_y).\exp(-j(\tau_x l+\tau_y m))/d\tau_x d\tau_y$$

where l and m are limited by the number of spatial harmonics

(+) A. Van de Capelle is a qualified researcher of the National Foundation for Scientific Research of Belgium.
(++) E. Van Lil is a research assistant of the National Foundation for Scientific research of Belgium.

necessary to describe the radiation pattern (otherwise the integral is zero), or by the greatest number of array elements allowed (in general a N dimensional array can represent N spatial harmonics with the zero order term included).

If we suppose the untill now unknown array periodically extended M_x and M_y times (a three dimensional illustration is given in fig.1), the array factor is found to be :

$$f_2''(\tau_x,\tau_y)=f_2'(\tau_x,\tau_y).\ \frac{\sin(M_xN_x\tau_x/2)}{\sin(N_x\tau_x/2)}\cdot\frac{\sin(M_yN_y\tau_y/2)}{\sin(N_y\tau_y/2)}$$

when both the basic and the extended array are symmetric around the origin (not shown in the figure).

When M_x and M_y are getting infinite, the supplementary factors becom equivalent to the following Dirac impulses : $2.\pi/N_x.\delta(\tau_x-2\pi i/N_x)$ for the x-direction.
So the integral is reduced to a double sum :

$$a_{l,m}=(1/N_xN_y)\sum_{i=0}^{n_x}\sum_{j=0}^{n_y} f_2'(2\pi i/N_x,2\pi j/N_y).e^{-2\pi j(il/N_x+jm/N_y)}$$

Inside the visible range the change of the variables τ_x and τ_y to θ and ϕ is easily made, while the used values outside the visible region will depend on the assumed choice. The interpretation of the basic formula is very simple : of the total number of harmonics (infinite) representing the infinitely extended array (a series of Dirac impulses at the places $x=l.\Delta x$), only the first N_x and N_y are retained, giving a continuous function (fig.2) ; it can be noted that the function is narrower sampled when the number of array elements, and thus the needed information increases. Here we see also that the first factor of the terms of the sum is independent of l and m ; thus, only $N_x.N_y$ ($=N^2$ if $N_x=N_y$) values are to be computed before starting the summation.
The calculations of the second factor can also be made before the summation, because the periodicity of the exponential function allows to reduce il to $il-N_xk$, what can always be made less than N_x. The exponentials require also no more than N_x+N_y evaluations.

Conclusion

The least square property of the Fourier transform allows to approximate as nearly as wanted any continuous radiation pattern (any discontinuity will cause Gibbs phenomena, not occuring when the constraints are kept realistic). Its rapidity makes of this method a very powerful tool in synthesizing arrays.

References

(°) M.T. Ma, "Theory and application of antenna arrays" John Wiley and Sons, Inc., 1974, pp. 120-121

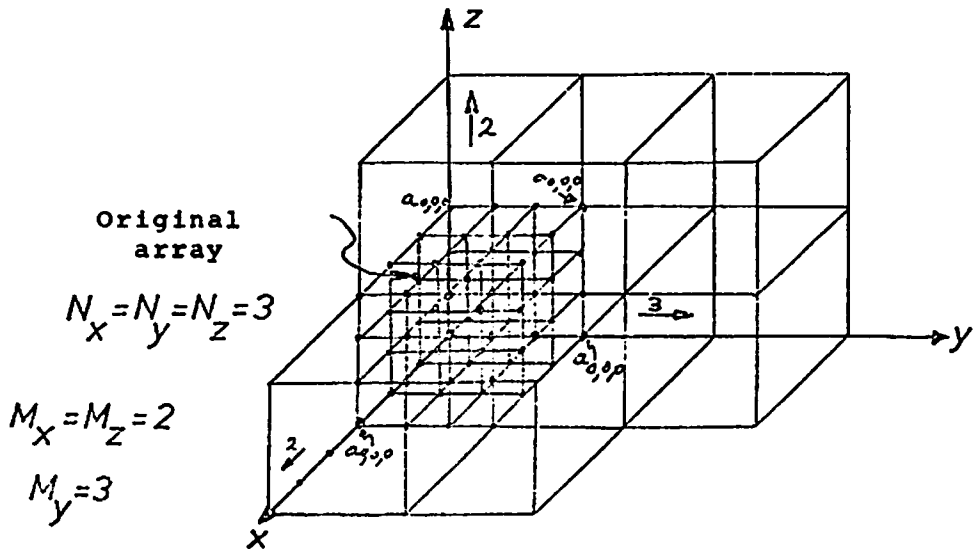

Fig.1 : The extended array

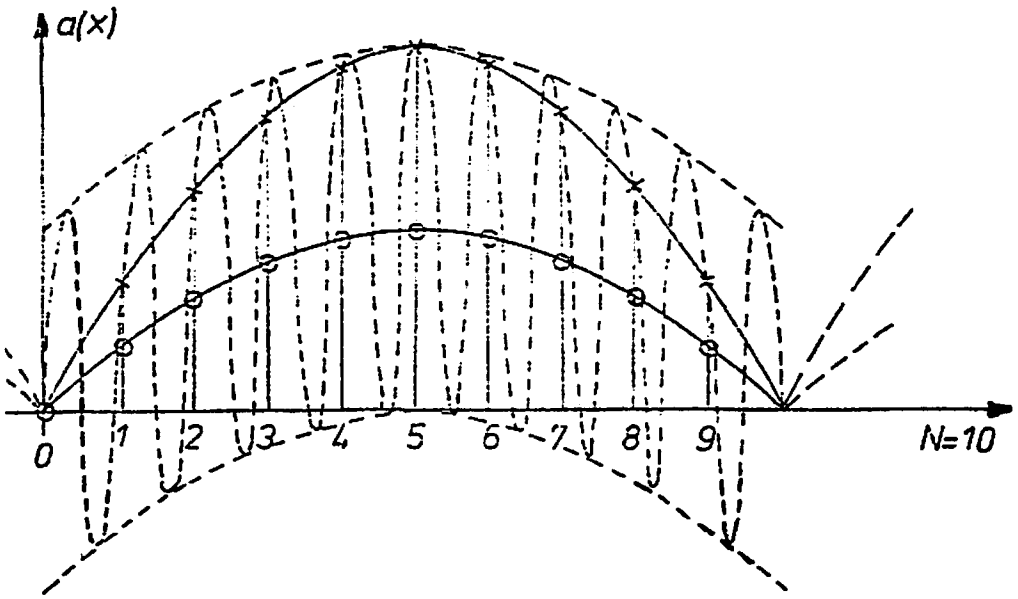

Fig.2 : The influence of the higher harmonics.