Effect of Rainfall Rate Inhomogeneity on Relations among Propagation Properties

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Abstract

Several approximated theoretical formulae among propagation properties, such as rainfall rate, attenuation, depolarization, and so on, have been proposed. These relations are based on the assumption of homogeneous rainfall rate model. In this paper, effects of rainfall rate inhomogeneity on these relations are discussed. In deriving the effect, from empirical knowledge, probability distribution function of rainfall rate is approximated by log-normal one, and spatial correlation properties of rainfall rate, R, and the n-th power of that, R^n, are approximated by exponential-type functions. It is observed that in some relations the effect is not significant from the viewpoint of practical link design.

1. INTRODUCTION

It is convenient in designing a microwave and millimeter wave dual-polarised out-door radio link performance to have simple mutual relationships among propagation properties, such as rainfall rate, rain-induced attenuation, depolarization(XPD), and phase of cross-polarized signal relative to co-polarized signal. For this purpose, several relationships are proposed[1-5]. In deriving these relationships, homogeneous rainfall rate along the rain path is assumed. But in reality, especially for intense rainfall rate and/or long rain path length cases, rainfall rate are not constant along the rain path as shown in Fig.1.

In this paper, effects of rainfall rate inhomogeneity on these relations are discussed. As regards the relation between rainfall rate and rain-induced attenuation, especially the method to derive attenuation statistics from rainfall rate one, many discussions have been done taking the inhomogeneity of rainfall rate into consideration[6,7]. In the Morita and Higuti(M-H) attenuation prediction method[6], firstly rainfall rate probability distribution is approximated by log-normal distribution based on long-term observation of measured rainfall rates over Japan, and secondly the spatial correlation function of specific attenuation is derived from that of rainfall rate. This M-H prediction method was extended by Ihara, et al.[8] for improved applicability of the method to millimeter wave bands, in which spatial correlation of n-th power of rainfall rate should be considered.

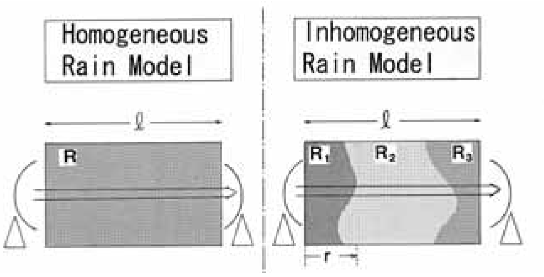

Fig. 1: Homogeneous and Inhomogeneous rain models

In the following sections, by using the above knowledge, effect of rainfall rate inhomogeneity on;

- XPD derivation using rainfall rate
- XPD derivation using attenuation
- XPD derivation using XPD frequency scaling
- Cross-polarized phase derivation using rainfall rate
- Cross-polarized phase derivation using attenuation

are discussed quantitatively.

2. INHOMOGENEITY EXPRESSION

A. Propagation Properties as a Function of Rainfall Rate

In Table 1, propagation properties (attenuation, XPD and depolarization phase) are expressed as a function of rainfall rate in both homogeneous and inhomogeneous rain model cases. In this table, a, b, c, d, w, x, y and z are parameters for approximation, and R, r and l are rainfall rate, distance along the path and the path length, respectively. And X_0 and ${}_0$ are additional terms which are not influenced by homogeneity discussion. As mentioned previously, attenuation statistics for inhomogeneous rain model can be predicted by M-H

TABLE 1: COMPARISON OF PROPAGATION PROPERTY EXPRESSION FOR HOMOGENEOUS AND INHOMOGENEOUS RAINFALL MODEL

Propagation Characteristics	Approximated Equation	
	Homogeneous Rain	Inhomogeneous Rain
Rain Attenuation	$aR^b l$	$a\int_0^l R(r)^b\,dr$
Depolarization (XPD)	$-20\log(c R^d l)+X_0$	$-20\log(c \int_0^l R(r)^d\,dr)+X_0$
Depolarization Phase	$\tan^{-1}(\frac{xR^y}{wR^z})+\psi_0$	$\tan^{-1}(\frac{x\int_0^l R(r)^y dr}{w\int_0^l R(r)^z dr})+\psi_0$

method using spatial correlation function of R. In this paper the M-H procedure is applied for prediction of XPD and depolarization phase in case of inhomogeneous rain. For this purpose, spatial correlation function of R^n, where n is positive number, should be derived from that of R.

B. Spatial Correlation Property of R^n [8]

When spatial correlation function of R, $_1$ is expressed as (1), which may be approved by experimentally, that of R^n, $_2$ is approximated by (2).

$$\rho_1 = \exp(-\alpha_1 \sqrt{D}) \tag{1}$$

$$\rho_2(n) = \exp(-\alpha_2(n)\sqrt{D}) \tag{2}$$

$$\alpha_2(n) = \alpha_1 n^2 \exp\left[\frac{(n^2-1)s^2}{K^2}\right]\left[\exp(\frac{s^2}{K^2})-1\right]\left[\exp(\frac{s^2 n^2}{K^2})-1\right]^{-1} \tag{3}$$

where, is correlation parameters and D is distance(km), and s is one of rainfall rate probability distribution function approximated by log-normal distribution, that is a standard deviation of log R. K is constant, log e 0.4343.

In the following section, the three properties, attenuation, XPD and depolarization phase, for inhomogeneous rain case are derived using the M-H method together with equations (2) and (3). Although in the following calculation, raindrop size distribution(DSD) of Marshall and Palmer type is adopted, qualitative features derived in the following discussion may be applied even in the case of other DSDs.

3. Inhomogeneity Effect

Fukuchi, et al. proposed approximated theoretical relationships for derivation of XPD and depolarization phase from rainfall rate or attenuation[3,5], and also proposed XPD frequency scaling relation[4]. These relationships are summarized in Table 2. In these references, the parameters of the relationships derived from the homogeneous rainfall model are given.

A. Effect on Derivation of XPD

Fig.2 shows XPD correction value due to rainfall inhomogeneity as functions of path length and cumulative time percentage, P, at frequency of 19.45GHz, when the XPD is derived from R-XPD relationship. More specifically, if rainfall rate inhomogeneity should be considered, this correction value should be added to the XPD value derived from rainfall rate value based on the homogeneous rainfall model. This result is derived under the assumption of rainfall rate probability distribution parameters of m=-0.305 and s=0.655, and parameter of rain spatial correlation, $_1$=0.35. The probability distribution of rainfall rate is approximated to log-normal distribution and parameters m and s are mean and standard deviation of logR, respectively. As shown in this figure, R-XPD relation is affected by rainfall rate inhomogeneity directly, and then the correction values

TABLE 2: APPROXIMATED THEORETICAL FORMULAE FOR DERIVATION OF XPD AND CROSS-POLARIZED PHASE

Input Output	Approximated Relations where, f:frequency(GHz), R:Rainfall Rate(mm/h) L:Rain Path Length(km), A:Attenuation(dB) u, U, V, , , :Parameters XPD, Correction Terms	References
RainRate XPD	$XPD \cong u_1 - u_2 \log f - (u_3 - u_4 f)\log R$ $-20\log L + \Delta XPD_1$	[3]
Attenuation XPD	$XPD \cong U_1 \log f + U_2 - V \log A$ $+(V-20)\log L + \Delta XPD_2$	[3]
XPD2 XPD1	[Frequency Scaling of XPD] $XPD_1 \cong XPD_2 - W$ $W = u_2 \log(\frac{f_1}{f_2}) - u_4(f_1 - f_2)\log R + \Delta XPD_3$	[4]
RainRate Depolarization Phase	$\psi \cong \tan^{-1}(\xi R^{\delta}) + \Delta\psi$	[5]
Attenuation Depolarization Phase	$\psi \cong \tan^{-1}\left[\zeta (\frac{A}{L})^{\eta}\right] + \Delta\psi$	[5]

increase as path length increases and/or cumulative time percentage decreases which corresponds to the large rainfall rate situation.

Fig.3 shows XPD correction value as functions of path length and the parameter $_1$ at frequency of 19.45GHz. The shaded area includes the parameter ranges -0.5 < m < 0.4, 0.55 < s < 0.67, and R < 60mm/h. These ranges correspond to probable rainfall events all over Japan including high rainfall rates and long path lengths. According to this figure, absolute value of XPD correction will be less than 0.5dB when XPD is derived from Attenuation-XPD relationship. This is because that the rainfall rate inhomogeneity affects both attenuation and XPD nearly the same manner, then the effect become insignificant for Attenuation-XPD relationship.

Fig.4 shows frequency dependence of the XPD correction value in deriving XPD from Attenuation-XPD relationship with the same parameters in Fig.3 except path length that is assumed to be 10km. Also it is observed that the rainfall rate inhomogeneity effect is not significant for Attenuation-XPD relationship.

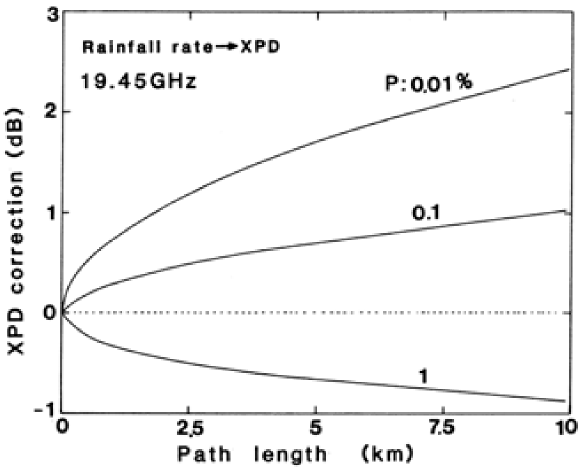

Fig. 2: XPD Correction Value for R-XPD Relationship

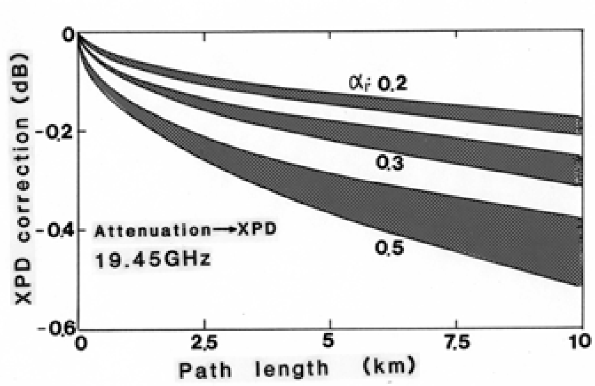

Fig. 3: XPD Correction Value for Attenuation-XPD Relationship

Fig.5 shows XPD correction value as a contour map when XPD at frequency f_2 is derived from XPD at f_1 using XPD frequency scaling relationship. In deriving these results, the parameters m=-0.305, s=0.655, P=0.1%, $_1$=0.35 and L=10km are assumed. According to this figure, absolute value of XPD correction increases as frequency difference increases, however, the XPD correction values are within a range of ¶0.2dB. Then also the effect of rainfall rate inhomogeneity is not significant for XPD frequency scaling relationship.

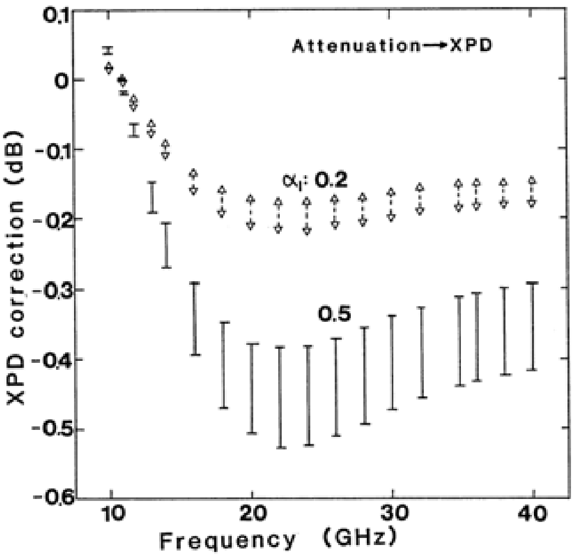

Fig. 4: Frequency dependence of XPD Correction Value for Attenuation-XPD Relationship

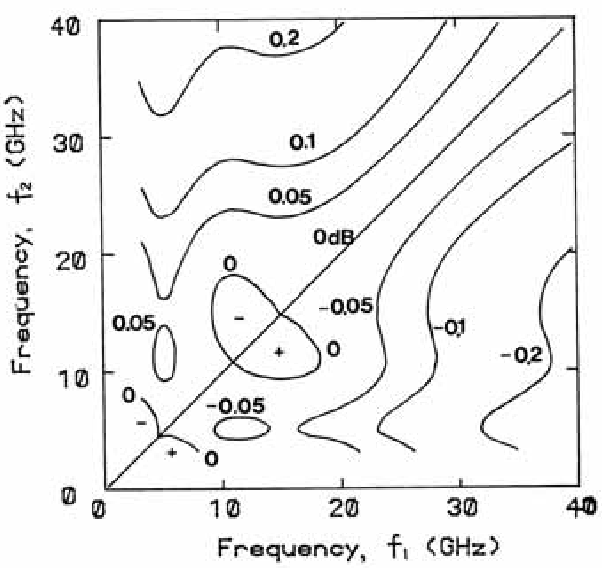

Fig. 5: XPD Correction Value for XPD Frequency Scaling Relationship (XPD_2 is derived from XPD_1)

B. Effect on Derivation of Depolarization Phase

Fig.6 shows phase correction value in degrees as a function of frequency. In this figure, correction values for both cases that depolarization phases are derived from rainfall rate(solid lines) and attenuation(dashed lines) are shown. In deriving these results, parameters m=-0.305, s=0.655, $_1$=0.35, L=5km and 0.01<P<1% are assumed. It is observed that the absolute value of the phase correction value tends to increase as frequency increases. However, the absolute value may be less than 3 degrees, or 4 degrees if L=10km, then the rainfall rate inhomogeneity has not significant effect both on the R-depolarization phase relationship and Attenuation-depolarization phase one.

The above results can be understood in such a way that the inhomogeneity affects nearly the same manner in both cross-polarized and co-polarized components and depolarization phase relative to the co-polarized one becomes insensitive to the inhomogeneity as shown in Table 1.

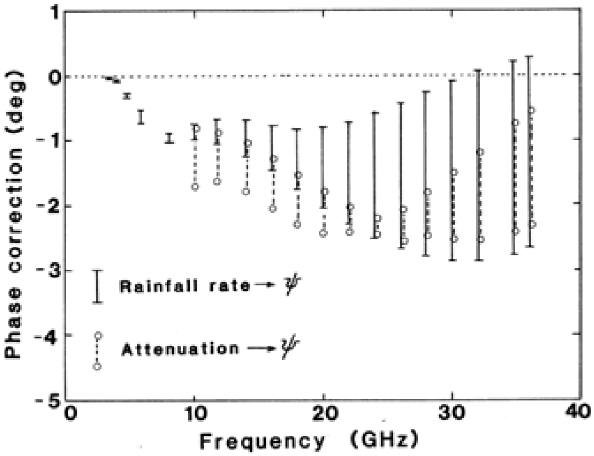

Fig. 6: Depolarization Phase Correction Value for both R-Depolarization Phase(solid lines) and Attenuation-Depolarization Phase(dashed lines) Relationships

4. CONCLUSIONS

Effects of rainfall rate inhomogeneity on relations for deriving XPD and depolarization phase are discussed quantitatively. The inhomogeneity is expressed by spatial correlation functions of rainfall rate, R, and R^n. It is concluded that the inhomogeneity effect is not significant in these relations, Attenuation-XPD, R-depolarization phase, Attenuation-depolarization phase, and XPD frequency scaling. This feature can be interpreted by compensation manner when the inhomogeneity affects propagation properties. The above relations are practically applicable in the propagation case in which large rainfall rate inhomogeneity is expected, such as large rainfall rate and/or large path length cases.

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