Dielectric Rough Wall Parallel Plate Waveguide

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Abstract

This paper derives the modal fields propagating in a dielectric parallel plate waveguide with rough walls. An equivalent surface-impedance boundary condition is applied at two uniform planes near the rough walls. These two planes form the parallel plate waveguide. The roughness of the interfaces is accommodated by making the equivalent surface impedance a random function of position on the wall. The roughness is only considered along the propagation direction. The scalar potential functions from which the field components are derived satisfy an infinite set of stochastic integral equations, approximate solutions to which are obtained by iteration. It is found that (1) the "TE" and "TM" modes possess the similar propagation characteristics as propagating in the free space; (2) the power density carried by the incoherent part of the field depends on the operating frequency, the positions in the waveguide cross section, and roughness. Representative numerical results are presented to illustrate the analysis.

1. Introduction

Problems of wireless communications in urban canyons are of great interest from both theoretical and practical points of view. A first step in addressing such problems is to develop a characterization of the propagation channel, including the roughness of the buildings. In this paper we consider a representative problem, i.e. a dielectric rough wall parallel plate waveguide. We are going to examine the modal fields propagating in such environments.

We assume the parallel plate waveguide is uniform. Two planes near the rough wall buildings form the waveguide, as shown in Figure 1. For simplicity, we only consider the roughness along the propagation direction, i.e. y direction. We obviate the need to consider the electromagnetic fields in the exterior through the use of an equivalent surface-impedance boundary condition that is defined at two waveguide walls. The roughness effects of the building walls are accommodated by making the equivalent surface impedance a random function.

Following the procedure developed by Casey for a circular tunnel [1], we begin by presenting an expression for the equivalent surface impedance $Z_s(y)$ in terms of the signal frequency, the relative permittivity of the medium, and the roughness of the building wall. As shown in Figure 1, we assume that the building walls are located at $x = -\Delta(y)$ and $x = a + \Delta(y)$ in which

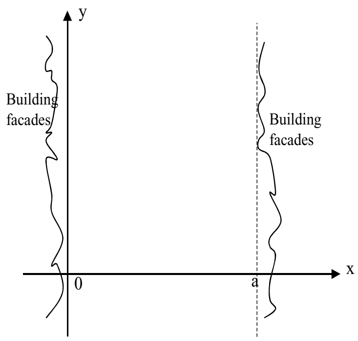

Fig. 1: Geometry and coordinate system for dielectric rough wall parallel plate waveguide

$\Delta(y)$ is a nonnegative homogeneous random function with prescribed (constant) mean $\overline{\Delta} = \mathcal{E}(\Delta)$ and autocovariance $C_\Delta(y - y') = \mathcal{E}\{\Delta(y)\Delta(y') - \overline{\Delta}^2\}$. We also assume no variation in z direction, $\partial/\partial z \equiv 0$.

We assume the time dependence $\exp(j\omega t)$ for all field quantities. The equivalent surface impedance at the reference planes is expressed in terms of $\Delta(y)$ by

$$Z_s(y) = \frac{Z_0}{\sqrt{\epsilon_r}} + jk_0 Z_0 \left(1 - \frac{1}{\epsilon_r}\right)\Delta(y) \qquad (1)$$

in which k_0 and Z_0 denote respectively the wavenumber and the intrinsic impedance of free space and ϵ_r is the (generally complex) relative permittivity of the buildings. It is assumed that the phase change $k_0\Delta$ associated with the distance between the reference planes and the local building walls is small compared to unity, and that the magnitude of the relative permittivity ϵ_r is large compared to unity.

The function $\Delta(y)$ is described by the Fourier-Stieltjes representation

$$\Delta(y) = \overline{\Delta} + \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jk_y y} d\nu(k_y) \qquad (2)$$

in which

$$\mathcal{E}\{d\nu(k_y)\} = 0 \qquad (3)$$

$$\mathcal{E}\{d\nu(k_y)d\nu^*(k'_y)\} = 2\pi\delta(k_y - k'_y)dk_y dk'_y S(k_y). \qquad (4)$$

The power spectral density function $S(k_y)$ is related to the autocovariance $C_\Delta(y-y')$ by

$$C_\Delta(\zeta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(k_y)e^{-jk_y\zeta}d\zeta \tag{5}$$

whence, by inverting the Fourier integral, we obtain

$$S(k_y) = \int_{-\infty}^{\infty} C_\Delta(y)e^{jk_y y}dy. \tag{6}$$

The equivalent surface impedance, normalized by the intrinsic impedance of free space, is therefore described by the Fourier-Stieltjes representation

$$\zeta_s(y) = \frac{Z_s(y)}{Z_0} = \overline{\zeta}_s + jk_0\left(1-\frac{1}{\epsilon_r}\right)\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jk_y y}d\nu(k_y) \tag{7}$$

in which

$$\overline{\zeta}_s = \mathcal{E}\{\zeta_s(y)\} = \frac{1}{\sqrt{\epsilon_r}} + jk_0\left(1-\frac{1}{\epsilon_r}\right)\overline{\Delta}. \tag{8}$$

2. Field Components and Boundary Conditions

The electromagnetic field within the waveguide is obtained from two scalar functions Φ and Ψ that satisfiy the Helmholtz equation

$$\nabla^2\begin{bmatrix}\Phi\\ \Psi\end{bmatrix} + k_0^2\begin{bmatrix}\Phi\\ \Psi\end{bmatrix} = 0. \tag{9}$$

The electric and magnetic fields are given in terms of Φ and Ψ by

$$\vec{E} = \frac{Z_0}{jk_0}\nabla\times\nabla\times\Psi\vec{a_x} - \nabla\times\Phi\vec{a_x}, \tag{10}$$

$$\vec{H} = \nabla\times\Psi\vec{a_x} + \frac{1}{jk_0Z_0}\nabla\times\nabla\times\Phi\vec{a_x}. \tag{11}$$

Here we use both TE and TM respect to x. We represent the scalar functions Ψ (which generates the TM field components) and Φ (which generates the TE field components) as

$$\begin{bmatrix} Z_0\Psi(x,y)\\ \Phi(x,y)\end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix} A(k_y) & B(k_y)\\ C(k_y) & D(k_y)\end{bmatrix} \times \begin{bmatrix}\sin k_x x\\ \cos k_x x\end{bmatrix} e^{-jk_y y}dk_y, \tag{12}$$

where $k_x = \sqrt{k_0^2-k_y^2}$, and the functions $A(k_y)$, $B(k_y)$, $C(k_y)$, and $D(k_y)$ are to be determined from the boundary conditions at the waveguide walls. The equivalent impedance boundary conditions are defined as $\vec{E}_t\times\vec{H}_t$ toward the boundary. These conditions are

$$\left.\begin{array}{l} E_y(0,y) = -Z_0\zeta_s(y)H_z(0,y)\\ E_y(a,y) = Z_0\zeta_s(y)H_z(a,y)\\ E_z(0,y) = Z_0\zeta_s(y)H_y(0,y)\\ E_y(a,y) = -Z_0\zeta_s(y)H_y(a,y)\end{array}\right\} \tag{13}$$

Substitution of the appropriate expressions obtained from equations (7), (10), and (11) yields an infinite set of coupled homogeneous stochastic integral equations for the functions $A(k_y)$, $B(k_y)$, $C(k_y)$, and $D(k_y)$. We define the column vector $\vec{A}(k_y)$ whose elements are $A(k_y)$, $B(k_y)$, $C(k_y)$, and $D(k_y)$ and the matrices $\mathbf{\Lambda}(k_y)$ and $\mathbf{\Gamma}(k_y)$ as follows:

$$\mathbf{\Lambda}(k_y) = \begin{bmatrix}\Lambda_{11}(k_y) & \Lambda_{12}(k_y) & 0 & 0\\ \Lambda_{21}(k_y) & \Lambda_{22}(k_y) & 0 & 0\\ 0 & 0 & \Lambda_{33}(k_y) & \Lambda_{34}(k_y)\\ 0 & 0 & \Lambda_{43}(k_y) & \Lambda_{44}(k_y)\end{bmatrix}, \tag{14}$$

and

$$\mathbf{\Gamma}(k_y) = \begin{bmatrix}0 & \Gamma_{12}(k_y) & 0 & 0\\ \Gamma_{21}(k_y) & \Gamma_{22}(k_y) & 0 & 0\\ 0 & 0 & \Gamma_{33}(k_y) & 0\\ 0 & 0 & \Gamma_{43}(k_y) & \Gamma_{44}(k_y)\end{bmatrix}, \tag{15}$$

in which

$$\left.\begin{array}{l}
\Lambda_{11}(k_y) = -\frac{k_xk_y}{k_0}\\
\Lambda_{12}(k_y) = j\overline{\zeta}_s k_y\\
\Lambda_{21}(k_y) = \frac{k_xk_y}{k_0}\cos k_xa + j\overline{\zeta}_s k_y\sin k_xa\\
\Lambda_{22}(k_y) = j\overline{\zeta}_s k_y\cos k_xa - \frac{k_xk_y}{k_0}\sin k_xa\\
\Lambda_{33}(k_y) = j\overline{\zeta}_s\frac{k_xk_y}{k_0}\\
\Lambda_{34}(k_y) = k_y\\
\Lambda_{43}(k_y) = k_y\sin k_xa - j\overline{\zeta}_s\frac{k_xk_y}{k_0}\cos k_xa\\
\Lambda_{44}(k_y) = k_y\cos k_xa + j\overline{\zeta}_s\frac{k_xk_y}{k_0}\sin k_xa\\
\Gamma_{12}(k_y) = k_0k_y\\
\Gamma_{21}(k_y) = k_0k_y\sin k_xa\\
\Gamma_{22}(k_y) = k_0k_y\cos k_xa\\
\Gamma_{33}(k_y) = k_xk_y\\
\Gamma_{43}(k_y) = -k_xk_y\cos k_xa\\
\Gamma_{44}(k_y) = k_xk_y\sin k_xa
\end{array}\right\}. \tag{16}$$

Then the vector $\vec{A}(k_y)$ satisfies the coupled homogeneous stochastic integral equations

$$\mathbf{\Lambda}(k_y)\cdot\vec{A}(k_y) = \left(1-\frac{1}{\epsilon_r}\right)\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{\Gamma}(k_y-k_y')\cdot\vec{A}(k_y-k_y')d\nu(k_y') \tag{17}$$

for $-\infty \le k_y \le \infty$.

3. Approximate Solution of the Coupled Equations

We develop an approximate solution to the coupled integral equations by iteration. We write

$$\vec{A}(k_y) = \vec{A}^{(0)}(k_y) + \vec{A}^{(1)}(k_y) + \cdots \tag{18}$$

The iteration scheme is

$$\mathbf{\Lambda}(k_y)\cdot\vec{A}^{(k)}(k_y) = \left(1-\frac{1}{\epsilon_r}\right)\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{\Gamma}(k_y-k_y')\cdot\vec{A}^{(k-1)}(k_y-k_y')d\nu(k_y') \tag{19}$$

for $k\ge 1$. The iteration is initialized using the condition

$$\mathbf{\Lambda}(k_y)\cdot\vec{A}^{(0)}(k_y) = 0. \tag{20}$$

In this work we consider the propagation of individual modes in the waveguide. The initialization of the iteration scheme is therefore selected so as to yield individual modes in the limit as the waveguide walls become smooth, that is, as the surface impedance becomes constant. The vanishing of the determinant of the matrix $\mathbf{\Lambda}(k_y)$ yields the dispersion relation

$$k_y^4\left[\left(\frac{k_x^2}{k_0^2}+\overline{\zeta}_s^{-2}\right)\sin k_xa-2j\frac{k_x}{k_0}\cos k_xa\right]\times$$
$$\left[2j\overline{\zeta}_s\frac{k_x}{k_0}\cos k_xa-\left(\overline{\zeta}_s^{-2}\frac{k_x^2}{k_0^2}+1\right)\sin k_xa\right]=0. \quad (21)$$

We denote the roots of this dispersion relation by k_{yl} for $l=0,1,2,\ldots$. We initialize the iteration at a specific root of the dispersion relation for $k_y=k_{yl}$

$$\vec{A}^{(0)}=2\pi\vec{A}_l\delta(k_y-k_{yl}) \quad (22)$$

in which $\vec{A}_l$ is a constant vector whose components are related to each other by the condition

$$\mathbf{\Lambda}(k_{yl})\cdot\vec{A}_l=0. \quad (23)$$

The initialization condition (22) satisfies equation (20) for all $k_y\neq k_{yl}$ by the virtue of the delta-function in the definition of $\vec{A}^{(0)}(k_y)$; and it satisfies this equation for $k_y=k_{yl}$ because the dispersion relation is satisfied there.

The first iteration yields

$$\vec{A}^{(1)}(k_y) = \left(1-\frac{1}{\epsilon_r}\right)\mathbf{\Lambda}^{-1}(k_y)\cdot\mathbf{\Gamma}(k_{yl})\cdot \vec{A}_l\int_{-\infty}^{\infty}\delta(k_y-k_y'-k_{yl})d\nu(k_y'). \quad (24)$$

Let

$$\boldsymbol{Q}(z)=\begin{bmatrix}\sin z & \cos z & 0 & 0\\ 0 & 0 & \sin z & \cos z\end{bmatrix}, \quad (25)$$

we thus obtain the following first-order expression for the lth order scalar functions Ψ_l and Φ_l:

$$\begin{bmatrix}Z_0\Psi_l(x,y)\\ \Phi_l(x,y)\end{bmatrix}=\boldsymbol{Q}\left(\sqrt{k_0^2-k_{yl}^2}x\right)\cdot\vec{A}_le^{-jk_{yl}y}$$
$$+\left(1-\frac{1}{\epsilon_r}\right)\frac{1}{2\pi}\int_{-\infty}^{\infty}\boldsymbol{Q}\left(\sqrt{k_0^2-(k_y'+k_{yl})^2}x\right)\cdot$$
$$\mathbf{\Lambda}^{-1}(k_y'+k_{yl})\cdot\mathbf{\Gamma}(k_{yl})\cdot\vec{A}_le^{-j(k_y'+k_{yl})y}d\nu(k_y'). \quad (26)$$

This completes the first-order approximate solution for the functions from which the electromagnetic field components are derived. These functions each comprise a deterministic or coherent part and a zero-mean random or incoherent part; and so therefore will also the field components derived from them.

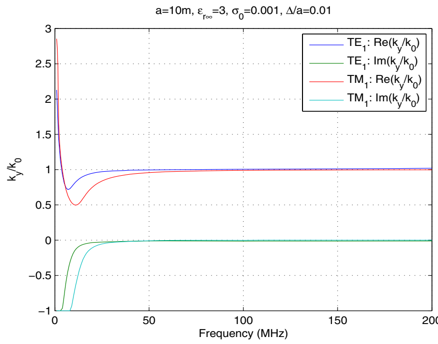

Fig. 2: Normalized propagation constant vs. frequency

4. Numerical Results

Representative numerical results of dispersion relation are shown in Figure 2. The relative permittivity of the building facades is given by [2]

$$\epsilon_r=\left(\sqrt{\epsilon_{r\infty}}+\sqrt{\sigma_0Z_0/(jk_0)}\right)^2, \quad (27)$$

in which $\epsilon_{r\infty}$ is the high-frequency relative permittivity and σ_0 is the low-frequency conductivity. We show the real and imaginary parts of the normalized complex propagation constant k_y/k_0 as functions of frequency for two lowest order modes in a parallel plate waveguide of width $a=10$ meters. The average roughness $\overline{\Delta}$ is equal to 0.01a; and $\epsilon_{r\infty}=3.0$ and $\sigma_0=0.001$S/m. The real parts of the propagation constant are positive and the imaginary parts are negative. The cutoff frequencies are approximately 18 MHz for "TE_1" mode and approximately 28 MHz for "TM_1" mode.

The power density $S_y(x,y)$ in the propagation direction is given in general by

$$S_y(x,y)=\frac{1}{2}\Re[E_z(x,y)H_x^*(x,y)-E_x(x,y)H_z^*(x,y)], \quad (28)$$

where $\Re(F)$ denotes the real part of F.

The expected value of the power density in propagation direction for a given mode is the sum of the coherent and incoherent power densities. The coherent part is given by

$$S_{yc}(x,y)=\frac{|k_{yl}|^2\Re(k_{yl})}{2k_0Z_0}[|A_l\sin k_{xl}x+B_l\cos k_{xl}x|^2$$
$$+\quad |C_l\sin k_{xl}x+D_l\cos k_{xl}x|^2]e^{2\Im(k_{yl})y}, \quad (29)$$

where $\Im(F)$ denotes the imaginary part of F, $k_{xl}^2=k_0^2-k_{yl}^2$. The incoherent power density for lth mode in propagation direction is

$$S_{yi}(x,y)=\left|1-\frac{1}{\epsilon_r}\right|^2\frac{e^{2\Im(k_{yl})y}}{4\pi k_0Z_0}\int_{-\infty}^{\infty}S(k_y')\Re[(k_y'+k_{yl})$$
$$(k_y'+k_{yl}^*)^2]e^{2\Im(k_y')y}\{\Re[S_1(x,y,k_y')S_1^*(x',y',k_y')]+$$
$$+\Re[S_2(x,y,k_y')S_2^*(x',y',k_y')]\}dk_y', \quad (30)$$

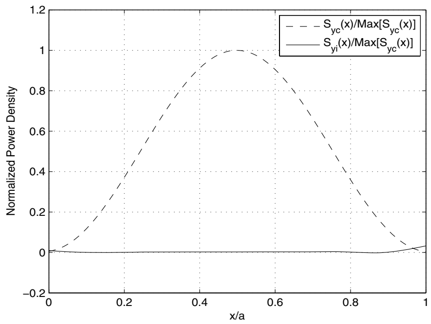

Fig. 3: Normalized coherent and incoherent power density in propagation direction vs. x/a: $a = 10$ meters, $\overline{\Delta} = 0.01a$, $f = 100$MHz, "TE_1" mode.

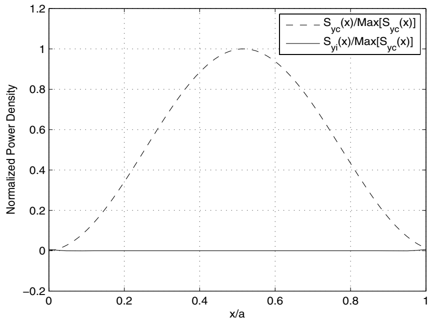

Fig. 4: Normalized coherent and incoherent power density in propagation direction vs. x/a: $a = 10$ meters, $\overline{\Delta} = 0.01a$, $f = 450$MHz, "TE_1" mode.

where

$$\left.\begin{aligned} S_1(x,y,k'_y) &= A_l^{(1)}(k'_y)\sin k_{xl+}x + B_l^{(1)}(k'_y)\cos k_{xl+}x \\ S_2(x,y,k'_y) &= C_l^{(1)}(k'_y)\sin k_{xl+}x + D_l^{(1)}(k'_y)\cos k_{xl+}x \end{aligned}\right\} \tag{31}$$

$k_{xl+}^2 = k_0^2 - (k'_y + k_{yl})^2$, and $*$ denotes the complex conjugate. We assume herein that the autocorrelation function C_Δ is a zero mean Gaussian for the building wall roughness

$$C_\Delta(y) = \sigma^2 e^{-\frac{y^2}{d^2}} \tag{32}$$

where σ and d are the standard deviation and correlation length of the roughness, respectively. The associated power spectral density [3, page 205] is also Gaussian

$$S(k_y) = \sigma^2\sqrt{\pi} d e^{-\left(\frac{k_y d}{2}\right)^2}. \tag{33}$$

We show two examples in Figure 3 and Figure 4, in which we plot the coherent and incoherent power densities in propagation direction for the first "TE" mode at any y position in the waveguide, normalized to their maximum values , vs. normalized position, x/a. The parameters we used are: $a = 10$ meters, $\Delta = 0.01a$, $d = 0.3a$, $\epsilon_{r\infty} = 3$, $\sigma_0 = 0.001$S/m, frequencies are 100 MHz and 450 MHz, respectively.

From these two figures, we noted that the incoherent power density is small compared to the coherent power density. We also noted that the incoherent power density is confined to the regions near the walls, which was observed in [1]. We discover that the incoherent power density is the function of the frequency, roughness, and position in waveguide cross section.

5. Conclusion

We have formulated the problem of electromagnetic wave propagation in a dielectric rough wall parallel waveguide, using an equivalent surface-impedance boundary condition. Using an iterative technique, we have obtained an approximate solution to the coupled homogeneous stochastic integral equations that describe the propagation. Under an appropriate initialization, our approximate solution represents single leaky waveguide modes. We have shown that the incoherent power density is varying with frequency, roughness and positions.

We can now develop the covariance functions of the electromagnetic field in the dielectric rough wall parallel plate waveguide. Arguments based on the central limit theorem can be used to show that the field is Gaussian; the covariance functions will therefore complete the description of the probabilistic character of the field of which the wireless communications community will be greatly benefited [4] [5].

Acknowledgments

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