SURFACE CURRENT AND APERTURE FIELD INTEGRATIONS FOR REFLECTOR ANTENNAS

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ABSTRACT

The fields of reflector antennas determined by integration of the current on the illuminated surface of the reflector are identical to the fields determined by aperture field integration with the Kottler-Franz formulas over any surface S_a that caps the reflector. As a corollary to this equivalence theorem the fields predicted by integration of the physical optics (PO) surface currents and the Kottler-Franz integration of the geometrical optics (GO) aperture fields on S_a agree to within the locally plane-wave approximation inherent in PO and GO. Moreover, within the region of accuracy of the fields predicted by PO current or GO aperture field integration, the far fields predicted by the Kottler-Franz aperture integration are closely approximated by the far fields obtained from aperture integration of the tangential electric or magnetic field alone. In particular, discrepancies in symmetry between the far fields of offset reflector antennas obtained from PO current and GO aperture field integrations disappear when the aperture of integration is chosen to cap (or nearly cap) the reflector.

ANALYSIS

Consider the illuminated surface S_r of a perfectly conducting reflector capped in free space by the imaginary aperture surface S_a. The surface $S_r + S_a$ encloses the free-space volume V. The feed sources of radiation illuminating the reflector are assumed to lie outside V. Applying the Kottler-Franz formulas [1], [2] to the total electric and magnetic fields over the closed surface $S_r + S_a$, we get the following expressions for the electric and magnetic fields $(\overline{E}_r, \overline{H}_r)$ radiated by the current $\overline{K}$ on the illuminated side of the reflector:

$$\overline{E}_r(\overline{r}) = \frac{i}{\omega\epsilon_0} \nabla \times \nabla \times \int_{S_r} \overline{K}g \, dS' = \nabla \times \int_{S_a} \hat{n}' \times \overline{E}g \, dS' + \frac{i}{\omega\epsilon_0} \nabla \times \nabla \times \int_{S_a} \hat{n}' \times \overline{H}g \, dS' \tag{1a}$$

$$\overline{H}_r(\overline{r}) = \nabla \times \int_{S_r} \overline{K}g \, dS' = \nabla \times \int_{S_a} \hat{n}' \times \overline{H}g \, dS' - \frac{i}{\omega\epsilon_0} \nabla \times \nabla \times \int_{S_a} \hat{n}' \times \overline{E}g \, dS' \,. \tag{1b}$$

<u>Equations (1a) and (1b) state that the electric and magnetic fields determined by integration of the current on the illuminated surface S_r of the reflector are identical to the fields determined by an aperture integration with the Kottler-Franz formulas of the total tangential fields over any surface S_a that caps the reflector.</u>

The far fields, which are found from (1) by letting $\overline{r}$ approach infinity, can be computed by either approximating the current over the surface S_r or the fields over the aperture S_a.

THE PO CURRENT AND GO APERTURE FIELD APPROXIMATION

Substitution of the physical optics (PO) current and geometrical optics (GO) aperture fields into (2a) produces the following alternative approximate expressions for the far fields of the reflector

$$\overline{E}_r(\overline{r}\to\infty) \approx \frac{ikZ_o e^{ikr}}{2\pi r}\, \hat{e}_r \times \hat{e}_r \times \int_{S_r} \hat{n}'\times\overline{H}_{inc} e^{-ik\hat{e}_r\cdot\overline{r}'} dS'$$

$$\approx \frac{-ikZ_o e^{ikr}}{4\pi r}\, \hat{e}_r \times \int_{S_a} [\hat{n}'\times(\hat{n}_o\times\overline{H}_{go})$$

$$+\hat{e}_r\times(\hat{n}'\times\overline{H}_{go})] e^{-ik\hat{e}_r\cdot\overline{r}'} dS'. \qquad (2)$$

Within an angle θ of about 15° or 20° from boresight, $\hat{e}_r$ and $\hat{n}_o$ are nearly parallel and the integrand in S_a of (2) reduces further to the approximation

$$\overline{E}_r(\overline{r}\to\infty) \approx \frac{ikZ_o e^{ikr}}{2\pi r}\, \hat{e}_r \times \hat{e}_r \times \int_{S_r} \hat{n}'\times\overline{H}_{inc} e^{-ik\hat{e}_r\cdot\overline{r}'} dS'$$

$$\approx \frac{ik e^{ikr}}{2\pi r}\, \hat{e}_r \times \int_{S_a} \hat{n}'\times\overline{E}_{go} e^{-ik\hat{e}_r\cdot\overline{r}'} dS' \qquad (3)$$

Equation (2) states that to within the locally plane-wave approximation used to estimate the surface currents and the aperture fields, the far fields determined by the PO current integration and GO aperture integration with the Kottler-Franz formulas are equal, provided the aperture S_a caps the reflector surface S_r. Moreover, (3) shows that the GO aperture inte- with the Kottler-Franz formulas is closely approximated by the familiar integration of the tangential electric (or magnetic) field for angles θ within about 20° of the main beam direction. If S_a is chosen to be a plane, then the aperture integration in (3) is identical to that obtained by applying the Smythe formulas [3] to $\overline{E}_{go}$ within S_a and ignoring the contribution from the fields outside S_a on the infinite plane of integration required by the Smythe formulas. Equations (1),(2), and (3) obtained from the Kottler-Franz formulas show that integration outside S_a is not required to evaluate fields produced by the current on the front surface of the reflector as long as the surface of integration S_a caps the reflector.

EXAMPLES

For conventional (nonoffset) reflectors both PO current and GO aperture integration have been applied with equal success [4]. Typically, both methods predict far fields that agree closely with measured data within the main beam and first few sidelobes. (Beyond the first few sidelobes, the fields diffracted by the antenna struts, the feed, and the rest of the supporting structure preclude usually a meaningful comparison between measured data and the results of PO current or GO aperture field integration.)

For offset reflectors PO current integration predicts a slightly asymmetric far-field pattern in the plane of the offset that agrees slightly better with measured data than GO aperture field integration, when the aperture chosen for the field integration is the projected aperture plane normal to the electrical boresight direction of the offset reflector, rather than the plane that caps the reflector surface [5], [6]. Specifically, consider the offset reflector studied by Rudge [5] and described in Fig. 1. Rudge integrated the GO electric field over the projected aperture plane S_a^p and got the symmetric copolar far-field pattern shown in [5, Fig.2(b)]. However, the same figure shows that the measured pattern in this plane of offset (xz plane) is slightly asymmetric. Recently Rahmat-Samii has shown that this slight asymmetry is correctly predicted by PO current integration over the surface S_r of the reflector [6]. In accordance with the theoretical results of the present communication, the same asymmetry would be predicted by the aperture field integration merely by choosing the aperture plane S_a that caps (or nearly caps) the reflector instead of the projected aperture S_a^p.

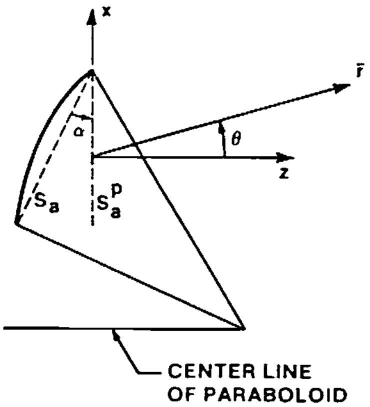

Fig. 1. Offset reflector from [9] (α = 23.6°).

TABLE I
SHIFT IN FAR-FIELD PATTERN VERSUS OBSERVATION ANGLE θ FROM (11)
($\tan\alpha = \tan 23.6° = 0.44$)

θ (DEGREES)	$\Delta\theta$ (DEGREES)
0	0.0
± 1	.004
± 2	.015
± 3	.034
± 4	.061
± 5	.095
± 6	.138
± 7	.188
± 8	.246
± 9	.311
± 10	.384

To verify this we could integrate numerically the GO electric field in (3) over S_a. However, a much simpler way is to note that the integral over S_a is related to the integral over S_a^p by

$$\hat{e}_r \times \int_{S_a} \hat{n}' \times \bar{E}_{go} e^{-ik\hat{e}_r \cdot \bar{r}'} dS' \approx \hat{e}_r \times \int_{S_a^p} \hat{n}' \times \bar{E}_{go} e^{-ik\hat{e}_r \cdot \bar{r}'} e^{ikx' \tan\alpha(1-\cos\theta)} dS' \quad (4)$$

The extra phase factor $\exp(ikx' \tan\alpha(1-\cos\theta))$ in the S_a^p integral of (4) merely accounts for the difference in path length between the S_a and

S_a^p planes and the far field point. Since this integral over S_a^p in (4) is the same one that Rudge [5] computed except for the extra phase factor, the far field pattern in the offset plane is equal to Rudge's pattern shifted by

$$\Delta\theta \approx (1-\cos\theta)\tan\alpha \approx \frac{\theta^2}{2}\tan\alpha \qquad (5)$$

This $\Delta\theta$ shift in pattern, which depends on the angle θ to the far field, produces an asymmetric far-field pattern in the plane of the offset. Table I lists the $\Delta\theta$ shift given in (5) as a function of angles θ. Comparing Table I with [6, Fig. 4(a)], one sees that the GO aperture field integration over the capping plane S_a predicts the same asymmetric pattern as the PO current integration. Moreover, the aperture field integration over S_a is no more complicated than the aperture field integration over the projected aperture S_a^p with the integrand multiplied by a slowly varying phase factor to account for the path length difference between S_a and S_a^p. (This simple conversion can be made whenever the feed lies at the focal point of a parabolic reflector.)

CONCLUDING REMARKS

The PO current integration has the advantage of using the simple $2\hat{n} \times \bar{H}_{inc}$ approximation for the surface current, but the disadvantage of having to evaluate rather complicated integrals over the reflector surface. Conversely, the GO aperture field integration has the advantage of an extremely simple double Fourier transform that can be evaluated efficiently with fast Fourier transform subroutines, but the disadvantage of having to ray trace from the reflector surface to the aperture plane (or vice versa) in order to get the GO aperture fields.

Which method should one use, the PO current integration or the GO aperture field integration to estimate the far fields of reflector antennas? The answer to this question depends more upon the predisposition of the user and the particular antenna in question than a definite advantage of one method over the other. However, the theoretical and numerical results of this communication indicate that the accuracy obtainable by both methods is comparable and thus accuracy need not be a consideration in deciding between the two methods. Ideally, one would use both methods to gain confidence in their mutual applicability and to confirm both computer programs.

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