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Bifurcation analysis for a simple chaotic circuit and its coupled systems

Akihisa Tamura
Graduate School of Advanced Technology and Science
The University of Tokushima
Email: tamura@is.tokushima-u.ac.jp

Tetsushi Ueta
Center for Advanced Information Technology
The University of Tokushima
tetsushi@is.tokushima-u.ac.jp

***Abstract*— Tamaševičius et. al proposed a very simple chaotic oscillator. Some experimental results, origin of the mathematical model, and numerical simulations are reported. However, bifurcation analysis has not been investigated in details. In this paper, we compute bifurcation sets of this model by using bifurcation theory and discuss its dynamical properties. The bifurcation structures are identified in various parameter planes. Moreover we investigate chaos synchronization in its coupled system. Synchronization of periodic and chaotic states are depicted in the bifurcation diagram, and it is clarified that the period-doubling bifurcation is deeply related to desynchronization.**

I. Introduction

Tamaševičius et. al proposed a very simple chaotic oscillator. Some experimental results, origin of the mathematical model, and numerical simulations are reported. However, bifurcation analysis has not been investigated in details. In this paper, first of all, we compute bifurcation sets observed in this model by using bifurcation theory, understand parameter dependency, and clarify bifurcation structure. Next, we investigate chaos synchronization of its coupled system. The chaos synchronous region is concretely delimited. Finally, we confirm the phenomenon by implementing the circuit.

II. Bifurcations in Single Circuit

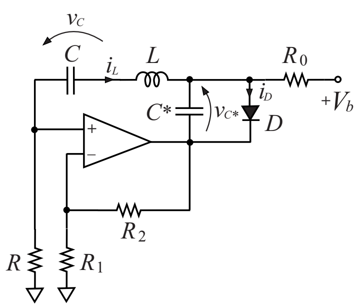

Fig. 1. Circuit model

Figure 1 shows a simple circuit proposed by Tamaševičius [1]. The normalized equation set is as follows:

$$
\begin{aligned}
\dot{x} &= y \\
\dot{y} &= ay - x - z \\
\epsilon\dot{z} &= b + y - c(\exp z - 1)
\end{aligned}
$$

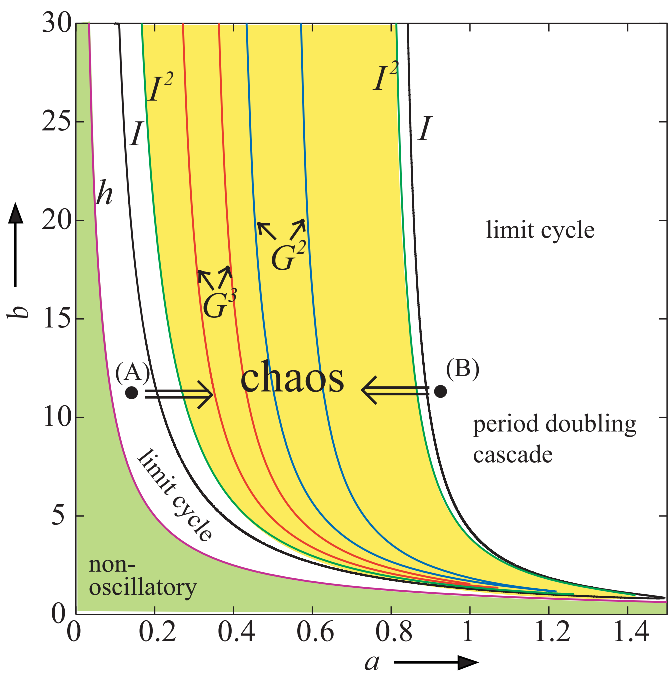

Fig. 2. Bifurcation diagram in the a-b plane

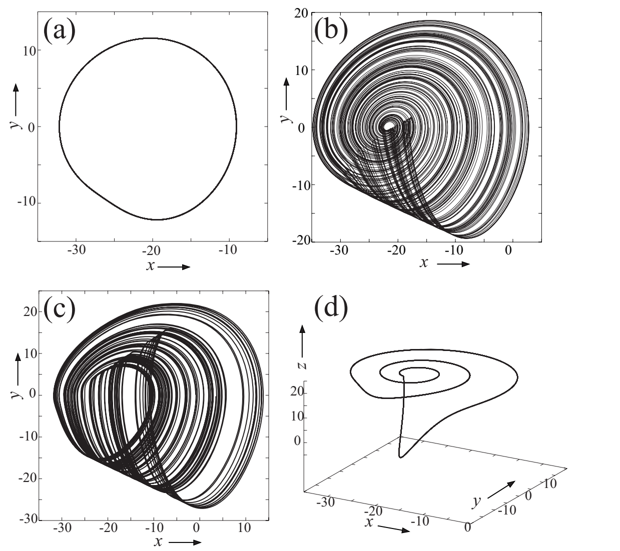

Fig. 3. Phase portraits with $b = 11.4$. (a) $a = 0.19$. (b) $a = 0.45$. (c) $a = 0.8$. (d) $a = 0.35$.

A. Bifurcation phenomena in the a-b plane

Figure 2 shows a bifurcation diagram of the system in the a-b parameters plane. a corresponds to R, R_1, R_2 and b corresponds to R_0. The parameter values $c = 4 \times 10^{-9}$ and $\epsilon = 0.13$ illustrated in Ref. [1] are related to the capacitors and the diode. We fix them as constants in this paper. We set the Poincaré section at $y = 0$. By altering four resistors, we can scan and confirm all phenomena in this diagram, both theoretically and numerically.

As for our notation, a Hopf bifurcation curve for an equilibrium is labeled h. Symbols I and G refer to period-doubling (PD), and tangent (T) bifurcations of periodic solutions, respectively. Superscripts of these symbols show degrees of periodicity. The parameter plane is split into oscillatory and non-oscillatory regions by the Hopf bifurcation curve. Figure 3(a) shows a limit cycle in the x-y plane. In Fig. 2, there exists an island surrounded by PD bifurcation curves. Inside this, since there exist PD cascades (Fig. 2(A), (B)), chaotic states are easily observed (Fig. 3(b), (c)). In the chaotic region, there exist period locking areas delimited by the T bifurcation curves (Fig. 3(d)). Inside this area, we observe an $I^{3 \cdot 2k}$ PD cascade. From this bifurcation structure, and observation of chaotic attractors, this system can be classified as of Rössler type. For $b > 15$, bifurcation structure is not sensitive to variations of b.

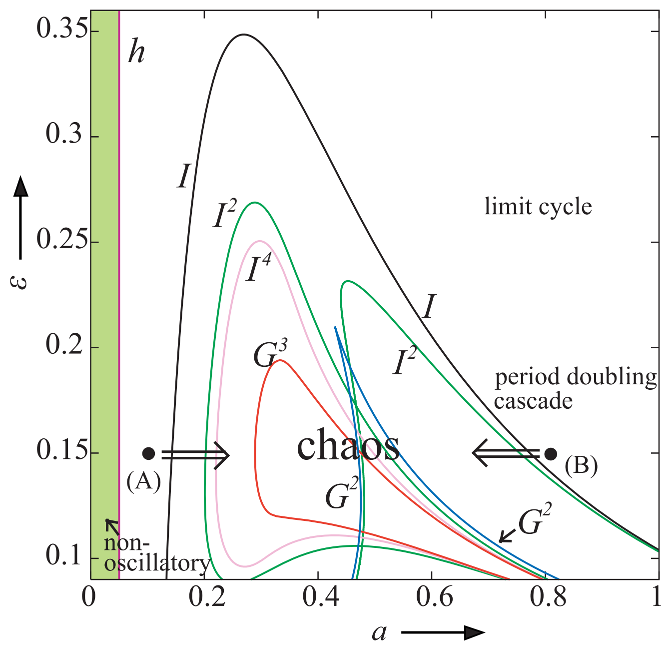

Fig. 4. Bifurcation diagram in the a-ϵ plane

B. Bifurcation phenomena in the a-ϵ plane

Figure 4 shows a bifurcation diagram of the system in the a-ϵ plane ($b = 15$, $c = 4 \times 10^{-9}$). By altering three resistors and two capacitors, we can scan and confirm all phenomena in this diagram.

From the Hopf bifurcation set, we confirmed oscillatory behavior in the right-hand side region. Since there exist PD cascades along the arrows ($\Rightarrow$) in the Fig. 4(A), (B), chaotic states are expected. In fact we observe them, see Fig. 5(a), (b). One can recognize the fish-hook structure composed by I, I^2 and G^2.

The Hopf bifurcation does not undergo the influence of the parameter ϵ from Fig. 4 because it forms a straight line. In other words, it implies that the two capacitors do not influence the oscillations of the circuit. When ϵ increases, the chaotic region is lost. Note that the solution diverges for $\epsilon < 0.1$.

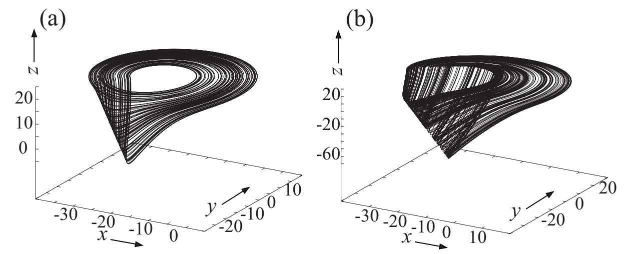

Fig. 5. Phase portraits with $\epsilon = 0.15$. (a) $a = 0..285$. (b) $a = 0.705$.

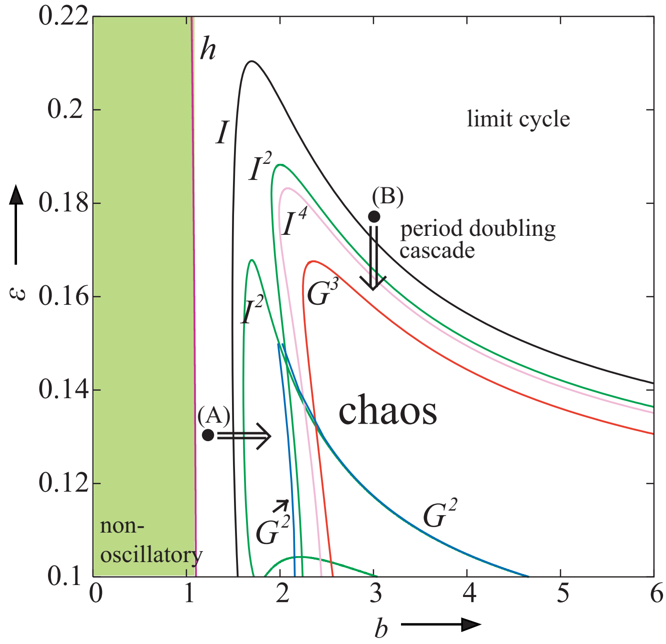

Fig. 6. Bifurcation diagram in the b-ϵ plane

C. Bifurcation phenomena in the b-ϵ plane

Figure 6 shows a bifurcation diagram of the system in the b-ϵ plane ($a = 0.9$, $c = 4 \times 10^{-9}$).

Basically, Fig. 6 can be split into two parts by the Hopf bifurcation: oscillatory and non-oscillatory regions. Since there exist PD cascades along arrows ($\Rightarrow$) in the Fig. 6(A), (B), chaotic states are observed. One can recognize the fish-hook structure composed by I, I^2 and G^2 like it appears in the a-ϵ plane.

III. Synchronization in Coupled Circuits

Inphase synchronization of periodic solutions is a common phenomenon when two identical oscillators are coupled by a register. In this section, we investigate synchronizations in coupled oscillators as a bifurcation problem, and show a synchronization region including chaos synchronization in a bifurcation diagram.

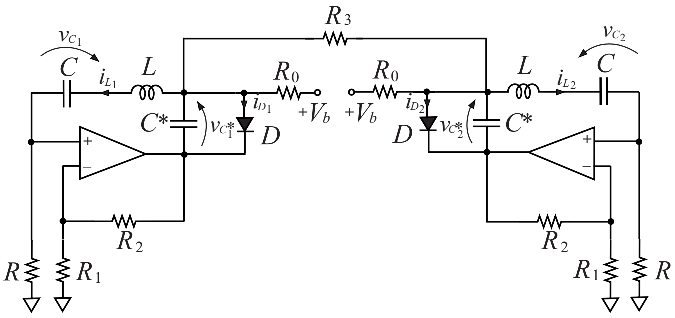

Fig. 7. Circuit model

From Fig. 7, we obtain a normalized equation set such as:

$$
\begin{aligned}
\dot{x_1} &= y_1 \\
\dot{y_1} &= ay_1 - x_1 - z_1 \\
\epsilon\dot{z_1} &= b + y_1 - c(\exp z_1 - 1) \\
&\quad -\delta(z_1 + (a+\gamma)y_1 - z_2 - (a+\gamma)y_2) \\
\dot{x_2} &= y_2 \\
\dot{y_2} &= ay_2 - x_2 - z_2 \\
\epsilon\dot{z_2} &= b + y_2 - c(\exp z_2 - 1) \\
&\quad +\delta(z_1 + (a+\gamma)y_1 - z_2 - (a+\gamma)y_2)
\end{aligned}
$$

A. Bifurcation phenomena in the a-δ plane

Figure 8 shows a bifurcation diagram of the system in the a-δ plane ($b = 11.4$, $c = 4\times 10^{-9}$, $\epsilon = 0.13$, $\gamma = 1.0$). The Hopf bifurcation line splits the whole diagram into the oscillatory and non-oscillatory regions. In the oscillatory region, since there exist PD cascades, chaotic states are easily observed. While the synchronous region is shown in the Fig. 8. Here we define that synchronization is the vanishing of differences between the corresponding variables, e.g, x_1 and x_2. We confirm the synchronization by observation of phase portraits. An automatic detection method will be examined in the future. From Fig. 8(A), the synchronizing limit cycle can be observed, and where PD cascades exist, chaos synchronization states are observed. The attractors are shown Fig. 9. The synchronization is also confirmed by the circuit implementation (Fig. 10). By PD bifurcation I, it becomes an asynchronous period-2 state from the synchronous limit cycle state of the Fig. 8(B). When the parameter is changed, it becomes asynchronous chaos (Fig. 11). In the Fig. 8(C), asynchronous chaos is confirmed. The degree of coupling strengthens when δ increases (Fig. 12).

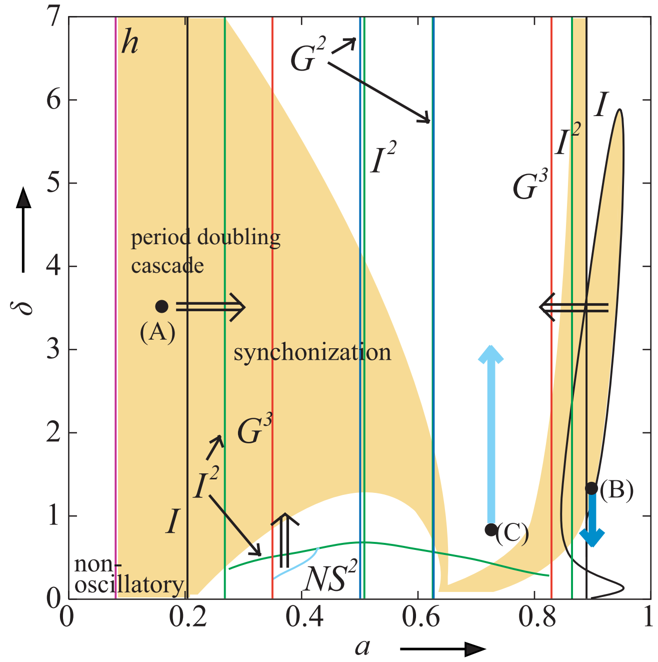

Fig. 8. Bifurcation diagram in the a-δ plane

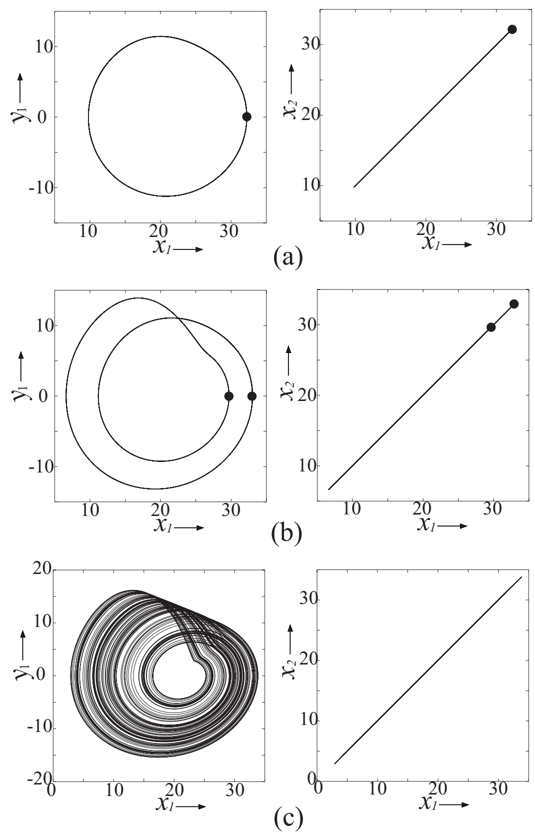

Fig. 9. Phase portraits with $\delta = 3.51$. (a) $a = 0.15$. (b) $a = 0.25$. (c) $a = 0.34$.

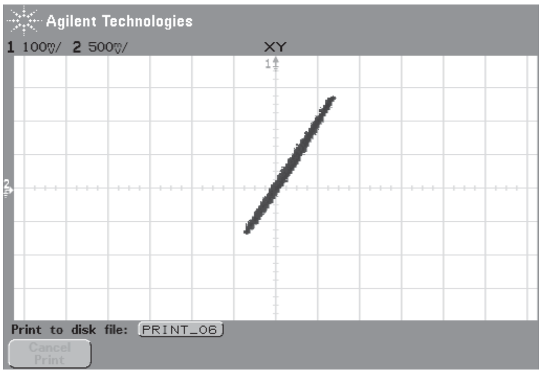

Fig. 10. Circuit implementation results. The circuit parameters were the following : $L = 100\text{mH}$, $C = 100\text{nF}$, $C^* = 22\text{nF}$, $R = 1\text{k}\Omega$, $R_0 = 20\text{k}\Omega$, $R_1 = 10\text{k}\Omega$, $R_2 = 4\text{k}\Omega$, $R_3 = 330\text{k}\Omega$.

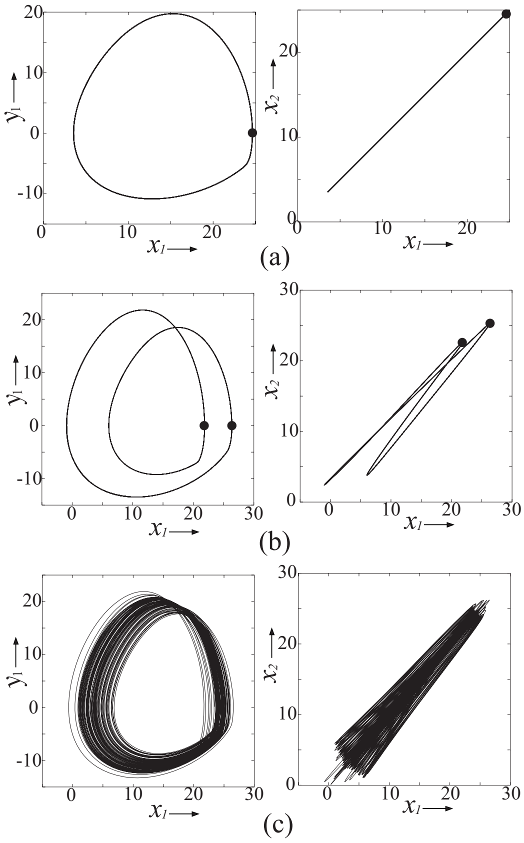

Fig. 11. Phase portraits with $a = 0.9$. (a) $\delta = 1.2$. (b) $\delta = 1.1$. (c) $\delta = 1.0$

IV. Conclusions

In this paper, the bifurcation phenomenon of the simple system and the coupled system was investigated in detail.

First of all, the phenomenon and the bifurcation occuring in the simple system were analyzed. As a result, oscillatory, non-oscillatory and chaotic regions were clarified by showing the hopf bifurcation, PD bifurcations and T bifurcations in the parameters plane. The concrete attractors were shown. The influence that each element such as resistors and capacitors have on the circuit was confirmed.

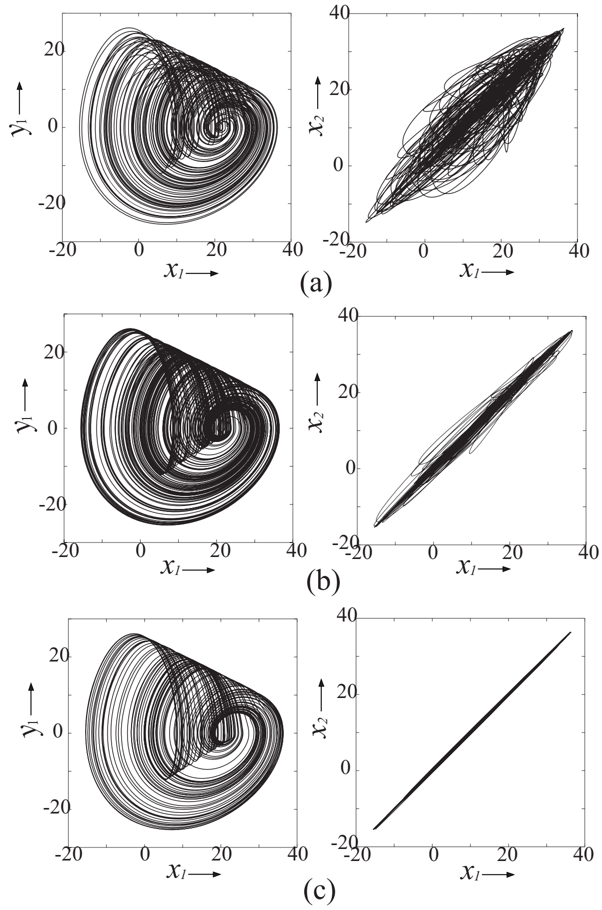

Fig. 12. Phase portraits with $a = 0.7$. (a) $\delta = 1.0$. (b) $\delta = 1.5$. (c) $\delta = 0.3$.

Next, the coupled system was analyzed. we investigated chaos synchronization in the coupled system. As a result, we were able to show synchronous regions and confirmed the chaos sync. phenomenon. Moreover, synchronizations of periodic and chaotic states are depicted in the bifurcation diagram, and it is clarified that the period-doubling bifurcation is deeply related with the loss of synchronization.

Finally, the chaos synchronization observed in theory was confirmed practically in the circuit implementation.

References

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