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Chaotic Complex-Valued Associative Memory

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Abstract—In this paper, we propose a chaotic complex-valued associative memory which can realize a dynamic association of multi-valued patterns. The proposed model is based on a complex-valued associative memory and a chaotic associative memory. The complex-valued associative memory can treat multi-valued patterns, and the chaotic associative memory can recall stored patterns dynamically. The proposed model utilizes the properties of these conventional models to realize the dynamic association of multi-valued patterns. We carried out a series of computer experiments and confirmed that the proposed model realizes the dynamic association. And, we show the influence of some parameters and state number for the dynamic association.

1. Introduction

Recently, neural networks are drawing much attention as a method to realize flexible information processing. The associative memories such as the Hopfield network[1], the Bidirectional Associative Memory[2], the Episodic Associative Memory[3] have been proposed. However, these models can not deal with multi-valued patterns. Although the associative memory using Self-Organizing Feature Map[4] which can deal with real-valued patterns has been proposed, it is not robust for damage of neurons because it is based on the local representation. The complex-valued associative memory whose input, output and internal state have the complex-value has been proposed[5]. It can deal with multi-valued patterns and complex-valued signals.

On the other hand, a number of studies on chaos have been conducted. Chaos is the unpredictable phenomenon which occurs in nonlinear dynamical systems. And, the chaotic neuron model which considers the spatio-temporal summation, refractoriness and continuous output has been proposed[6]. It is known that dynamic associations can be realized in the associative memories composed of the chaotic neurons.

In this paper, we propose the chaotic complex-valued associative memory which is introduced to the complex-valued associative memory with chaos. The proposed model realizes the association of multi-valued patterns by using the complex-valued neuron models, and realizes the dynamic associative of patterns by chaos.

2. Complex-Valued Associative Memory

In the complex-valued associative memory[5], the input, the output, the internal state and the weight have complex-value. The dynamics of the nth neuron in the complex-valued associative memory is given by the following equation:

$$x_n(t+1) = f\left(\sum_{j=1}^{N} w_{nj} x_j(t)\right) \tag{1}$$
$$x_n(t), w_{nj} \in \mathrm{C}$$

where $x_n(t)$ is the output of the nth neuron at the time t, N is number of the neuron and w_{nj} is the weight from the jth neuron to the nth neuron. In the auto-associative type complex-valued associative memory, the weight matrix $\boldsymbol{w}$ is given by the following equation:

$$\boldsymbol{w} = \sum_{p=1}^{P} \boldsymbol{X}^{(p)} \boldsymbol{X}^{(p)*} - P\boldsymbol{I}_N \tag{2}$$

where P is the number of learning patterns, $\boldsymbol{X}^{(p)}(p = 1, 2, \cdots, P)$ is the pth learning pattern, $\boldsymbol{I}_N$ is the N-dimensional identity matrix, * is a conjugate transpose matrix. The output function $f(\cdot)$ is given by the following equation:

$$f(u) = \frac{\eta u}{\eta - 1.0 + |u|} \qquad \eta \in \mathrm{R} \tag{3}$$

where η is the constant ($\eta > 1$).

3. Chaotic Neural Network

The chaotic neuron model[6] which considers the spatio-temporal summation, refractoriness and continuous output introduces chaos into the conventional neuron model. The dynamics of the chaotic neuron model is given by the following equation:

$$x(t+1) = g\left(A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta\right) \tag{4}$$

where $x(t)$ is the output of the neuron at the time t, $A(t)$ is the external input at the time t, α is the scaling factor of refractoriness ($\alpha > 0$), k is the dumping factor ($0 \leq k < 1$), θ is the threshold of the neuron. The output function $g(\cdot)$ is given by the following sigmoidal function:

$$g(u) = \frac{1}{1 + \exp(-u/\varepsilon)} \tag{5}$$

where ε is the steepness parameter. The chaotic neuron model can make a chaotic response using appropriate k and α. The network which is composed of chaotic neurons is called the chaotic neural network. The dynamics of the nth chaotic neuron which has the M external inputs and the N neurons is given by the following equation:

$$\begin{aligned} x_n(t+1) = g\Bigg(&\sum_{m=1}^{M} v_{nm} \sum_{d=0}^{t} k_s^d A_m(t-d) \\ &+ \sum_{j=1}^{N} w_{nj} \sum_{d=0}^{t} k_m^d x_j(t-d) \\ &-\alpha \sum_{d=0}^{t} k_r^d x_n(t-d) - \theta_n \Bigg) \end{aligned} \tag{6}$$

where $x_n(t)$ is the output of the nth neuron at the time t, v_{nm} is the weight from the external input $A_m(t)$ to the nth neuron, $A_m(t)$ is the mth external input at the time t, w_{nj} is the weight between the jth neuron and the nth neuron, k_s, k_m and k_r are the damping factors, α is the scaling factor of the refractoriness and θ_n is the threshold of the nth neuron. Each term in $g(\cdot)$ is the external input, interconnections, refractoriness. The chaotic associative memory is known that is able to recall the stoard patterns dynamically.

4. Chaotic Complex-Valued Neuron Model

The proposed chaotic complex-valued neuron model is introduced to the complex-valued neuron model with chaos. The dynamics of the chaotic complex-valued neuron is given by the following equation:

$$x(t+1) = f\left(A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta\right) \tag{7}$$

$$A(t), x(t), \theta \in \mathrm{C} \qquad k, \alpha \in \mathrm{R}$$

In Eq.(7), the output function of Eq.(5) is used to instead of Eq.(3), and $A(t)$, $x(t)$ and θ are complex-value. If the external input $A(t)$ in Eq.(7) is constant($A(t) = A$), Eq.(7) is modified as the following equation:

$$\begin{aligned} x(t+1) &= f\left(A - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta\right) \\ &= f(ku(t) - \alpha f(u(t)) + (A-\theta)(1-k)) \\ &= f(ku(t) - \alpha f(u(t)) + a) \end{aligned} \tag{8}$$

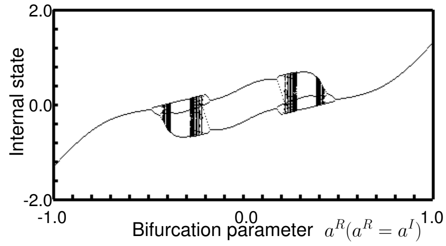


Figure 1: Bifurcation diagram of internal state (real part).

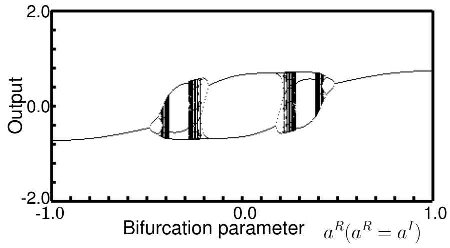


Figure 2: Bifurcation diagram of output (real part).

where $u(t)$ is the internal state at the time t, $a(= (A-\theta)(1-k))$ is the bifurcation parameter. Figure 1,2 show the bifurcation diagram of real part of internal state and output at $\alpha = 1.0$, $k = 0.8$.

5. Chaotic Complex-Valued Associative Memory

Here, we explain the proposed chaotic complex-valued associative memory. In the proposed model, dynamic association of multi-valued patterns is realized using the chaotic complex-valued neuron model explained in **4**.

5.1. Structure

The proposed chaotic complex-valued associative memory has N chaotic complex-valued neurons and N external inputs. In the proposed model, the neurons are connected each other. Figure 3 shows the structure of the proposed model.

5.2. Learning of Proposed Model

In the proposed model, the weights are trained by the correlation learning given by Eq.(2).

5.3. Dynamics

The dynamics of the nth neuron in the proposed model is given by the following equation:

$$x_n(t+1) = f\left(\sum_{d=0}^{t} k_s^d A_n(t-d)\right. \tag{9}$$

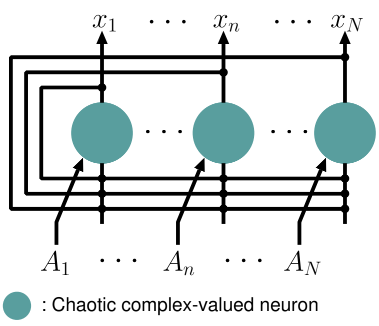


Figure 3: Structure of proposed model.

$$+\sum_{j=1}^{N} w_{nj} \sum_{d=0}^{t} k_m^d x_j(t-d)$$

$$\left. - \alpha \sum_{d=0}^{t} k_r^d x_n(t-d) \right)$$

$$w_{nj}, x_n(t), A_n(t) \in \mathrm{C} \quad k_s, k_m, k_r, \alpha \in \mathrm{R}$$

where $x_n(t)$ is the output of the nth neuron at the time t, $A_n(t)$ is the external input of the nth neuron at the time t, w_{nj} is the weight between the jth neuron and the nth neuron. $x_n(t)$, $A_n(t)$ and w_{nj} are complex value. k_s, k_m, k_r are the dumpling factors, α is the scaling factor of refractoriness. k_s, k_m, k_r and α are real value.

6. Computer Experiment Results

Here, we show the computer experiment results to demonstrate the effectiveness of the proposed model.

6.1. Dynamic Asscotiation

The four-valued patterns shown in Fig.4 were memorized in the chaotic complex-valued associative memory. In this experiment, the parameters were set as follows: $N = 400$, $k_s = 0.0$, $k_m = 0.1$, $k_r = 0.9$, $\alpha = 70$, $\eta = 1.1$, and the external input was set to 0. Figure 5 shows the association result of the chaotic complex-valued associative memory when the pattern 2 was given. The outputs of the chaotic complex-valued associative memory satisfy $|x_n| < \eta$, but the outputs are quantized in Fig.5. The outputs are quantized as follows:

$$\hat{x}_n = \underset{\omega^s}{\mathrm{argmin}} \, (\omega^s - x_n)^*(\omega^s - x_n) \qquad (10)$$

$$s = 0, 1, \ldots, S-1$$

where $\hat{x}_n$ is the quantized output of the nth neuron, S is the number of the output states when the outputs are quantized, and S was set to 4 in this explement. And, ω is given by the following equation:

$$\omega = \exp\left(i\frac{2\pi}{S}\right) \qquad (11)$$

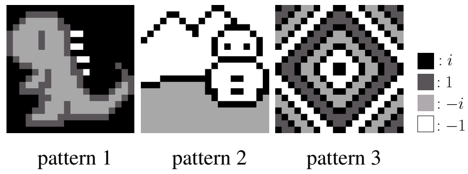


Figure 4: Learning patterns.

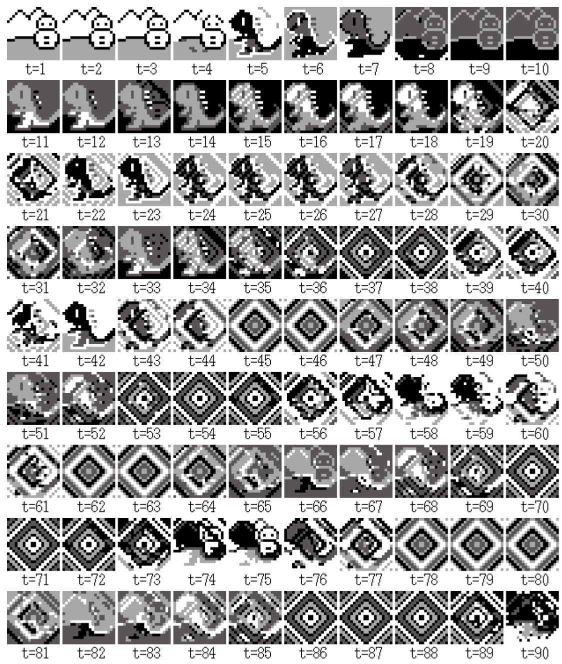


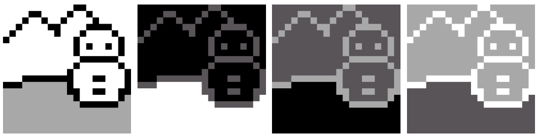
Figure 5: Association result when pattern 2 was given.

where i is the imaginary unit. In the complex-valued associative memory, the learning patterns are memorized as the equilibrium set not the equilibrium point. It means that the relation between the internal state and the output of the complex-valued associative memory is given by Eq.(12).

$$\exp(i\beta)x_n = g\left(\sum_{k=1}^{N} w_{nk} \exp(i\beta)x_k\right) \qquad (12)$$

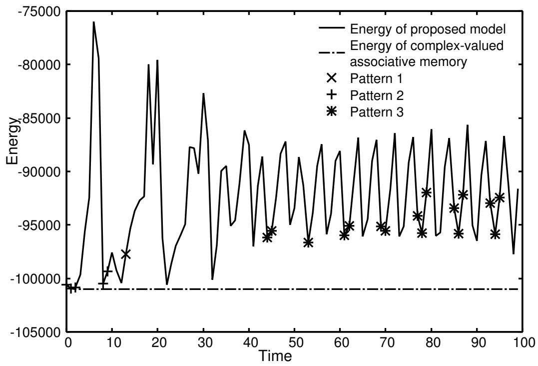
In this experiment, β is constrained at $\beta = \frac{2\pi}{S}s$ ($s = 0, 1, \ldots, S-1$), because the outputs are quantized. There are S patterns which satisfy the constraint, and the pattern is same as the learning pattern if s is 0. And, the patterns of $s = 1 \sim S-1$ are included in the equilibrium set which contains the learning patterns. In this paper, we call the patterns of $s = 1 \sim S-1$ the "rotated pattern" to distinguish those from the learning pattern. Figure 6 shows the rotated patterns of the learning pattern 2 in Fig.4. In Fig.6, (a) is the learning pattern, and (b), (c), (d) are the rotated patterns.

In Fig.5, when the pattern 2 was given at $t = 0$, the pattern 2 was recalled during $t = 1 \sim 3$. After that, the superimposed pattern of the pattern 1 and the pattern 2 was recalled. At $t = 9$, the rotated pattern of the pattern 2 was

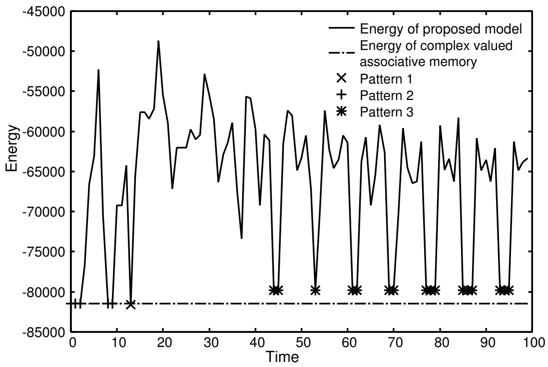


(a) pattern 2-1 (b) pattern 2-2 (c) pattern 2-3 (d) pattern 2-4

Figure 6: Example of rotated patterns.



(a) Energy transition of output $\boldsymbol{x}$



(b) Energy transition of quantized output $\hat{\boldsymbol{x}}$

Figure 7: Energy transition of chaotic complex-valued associative memory.

recalled. Then, at $t = 46$, the pattern 3 was recalled after the superimposed patterns of the pattern 1 and the pattern 3 were recalled. Figure 5 shows that the proposed model whose parameters are set appropriately is able to recall the stored patterns dynamically.

6.2. Energy Transition

The energy function of the proposed model is given by the following equation:

$$E(\boldsymbol{x}) = -\frac{1}{2}\boldsymbol{x}^{*}\boldsymbol{w}\boldsymbol{x} \tag{13}$$

where $\boldsymbol{x}$ is the output of the network, $\boldsymbol{w}$ is the weight matrix. The energy $E(\boldsymbol{x})$ is given by the real value, because $\boldsymbol{w}$ is the Hermitian matrix. Figure 7 shows the energy transition of the output in Fig.5. Figure 7(a) shows the energy transition of the output $\boldsymbol{x}$ in Fig.5. Figure 7(b) shows the energy transition of the output when the output of $\boldsymbol{x}$ in Eq.(13) is replaced by the quantized output $\hat{\boldsymbol{x}}$. Figure 7(a) shows that the energy increases and decreases randomly. In the complex-valued associative memory, the recalled patterns correspond to the minimums of the energy. In contrast, in the chaotic complex-valued associative memory, the recalled patterns do not correspond to the minimums of the energy. And the energy increases and decreases randomly, because the refractoriness in Eq.(9) makes the output depart from the last output.

7. Conclusion

In this paper, we have proposed the chaotic complex-valued associative memory which is able to recall the multi-value patterns dynamically. The proposed model is based on the complex-valued associative memory and the chaotic associative memory, and the proposed model is constructed of the chaotic complex-valued neuron models. We carried out a series of computer experiments and confirmed that α and the state number influence dynamic associations.

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