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Nonlinear Dynamics of a Flexible Rotor in Active Magnetic Bearings

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Abstract– This work reports on a numerical investigation on the nonlinear dynamics of a flexible rotor in active magnetic bearings. The mathematical model of the flexible rotor - active magnetic bearing system used in this study incorporates nonlinearity arising from the magnetic actuator forces that are nonlinear functions of the coil current and the air gap between the rotor and the stator, and from the geometric coupling of the magnetic actuators. The response of the rotor, with the variation of the speed parameter, Ω, displayed a rich variety of nonlinear dynamical behaviour including sub-synchronous vibrations of periods-3, -6 and -9, quasi-periodicity and chaos. Such vibrations in the operation of rotating machinery is undesirable and should be avoided as they introduce cyclic stresses in the rotor, which in turn may rapidly induce fatigue failure.

1. Introduction

The utilization of active magnetic bearings is found in a wide class of rotating machinery. The main advantage of this type of bearings over the conventional bearings is their higher mechanical efficiency due to reduced friction losses since no contact occurs between the rotor and the bearing stator during operation of the machine. The magnetic bearings are, however, highly nonlinear and their interaction with the rotor that they support can lead to various nonlinear phenomena in the rotor's response. The most prominent source of nonlinearity in active magnetic bearings is the relationship between the forces generated in the electromagnetic actuator and the coil current and the air gap between the rotor and the stator. The force is proportional to the current squared and inversely proportional to the gap squared. Cross-coupling between the electromagnetic forces acting in two orthogonal directions is also a source of nonlinearity in a magnetic bearing system. One of the main causes of the cross-coupling effect is attributed to the geometry of the actuators. The air gap at a point on a magnetic pole is actually not constant over the entire pole area due to the geometrical curvature of the pole. This results in a normal force, which is perpendicular to the principal force, which in turn causes geometric coupling between these forces.

There have been quite a number of published articles on the nonlinear dynamics of rotors supported in active magnetic bearings in recent years. The effect of nonlinearity arising from cross-coupling due to gyroscopic motion on the response of a rigid rotor mounted in magnetic bearings examined in [1] showed the occurrence of Hopf bifurcation at certain values of operating speed. Multiple co-existing solutions were found at primary resonance of a rigid rotor response in magnetic bearings incorporating nonlinearity due to geometric coupling of the actuators [2]. The effects of geometric coupling on the nonlinear response of a magnetic bearing supported rotor were also investigated in [3] and [4], revealing the occurrences of quasi-periodic and period-2 vibrations, and jump phenomena in the response of the rotor. Numerical integration and numerical continuation methods were used to investigate the unbalance response of a rigid rotor in magnetic bearings [5]. This work showed the occurrence of symmetry-breaking and period-doubling bifurcations. The nonlinear response of a flexible rotor supported by magnetic and auxiliary bearings was numerically investigated in [6]. The results revealed sub-synchronous vibrations of periods-2, -4 and -8, and quasi-periodic and chaotic vibrations in the response of the rotor. The stability and bifurcations of a flexible rotor supported by radial and thrust magnetic bearings were examined using the Floquet theory [7]. This work showed the importance of incorporating thrust magnetic bearings into the mathematical model of the rotor-bearing system, as they significantly influence the nonlinear dynamics of the system.

In the work presented herein, the influence of operating speed on the nonlinear response of a flexible rotor in radial active magnetic bearings is numerically investigated. Nonlinearity arising from the magnetic actuator forces that are nonlinear functions of the coil current and the air gap between the rotor and the stator, and from geometrical coupling of the magnetic actuators is incorporated into the mathematical model of the rotor-bearing system.

2. Mathematical Model

The derivation of the governing equations of the flexible rotor in active magnetic bearings is undertaken with the following assumptions being valid: (i) rotor is symmetric with part of its mass lumped at the rotor mid-span with the remainder at the bearing stations, (ii) rotor speed is constant, (iii) rotor and support stiffness are radially symmetric, (iv) damping force acting on the disc at rotor mid-span due to air dynamics is viscous, (v) rotor imbalance is defined in a single plane on the disc at the rotor mid-span, (vi) rotor motion in the axial direction is neglected, (vii) gyroscopic effect is neglected, (viii) leakage of magnetic flux is neglected, (ix) fringing effect, i.e., the spreading of magnetic flux in the air gap is neglected and (x) the magnetic iron is operating below saturation level. Accounting for the external forces acting on the rotor mid-span and bearing journal that include the rotor imbalance force, shaft elastic force, viscous damping force, magnetic bearing forces, and gravity, the governing equations can be expressed by Eq. (1).

$$x''_D = -\frac{2\zeta}{f}x'_D - \frac{1}{f^2}(x_D - x_J) + U\Omega^2\cos\Omega\tau$$

$$y''_D = -\frac{2\zeta}{f}y'_D - \frac{1}{f^2}(y_D - y_J) - \frac{W}{f^2} + U\Omega^2\sin\Omega\tau$$

$$x''_J = F_{x+} - F_{x-} + \alpha x_J(F_{y+} + F_{y-}) - \frac{1}{f^2\gamma}(x_J - x_D)$$

$$y''_J = F_{y+} - F_{y-} + \alpha y_J(F_{x+} + F_{x-}) - \frac{1}{f^2\gamma}(y_J - y_D) - \frac{W}{f^2} \quad (1)$$

where

$$F_{x+} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 - Px_J - Dx'_J)}{(1 - x_J)}\right]^2$$

$$F_{x-} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 + Px_J + Dx'_J)}{(1 + x_J)}\right]^2$$

$$F_{y+} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 - Py_J - Dy'_J)}{(1 - y_J)}\right]^2$$

$$F_{y-} = \left(\frac{1}{4(P-1)}\right)\left[\frac{(1 + Py_J + Dy'_J)}{(1 + y_J)}\right]^2$$

The response of the system can be described by the non-dimensional displacements x_D and y_D of the geometric center of the rotor mid-span, and the non-dimensional displacements x_J and y_J of the geometric center of the journal. ζ is half the viscous damping ratio on the disc at the rotor mid-span. f, the frequency ratio, is the ratio of the linear natural frequency of the magnetic bearing system, ω_n, to the pin-pin natural frequency of the flexible rotor, ω_r. U, the unbalance parameter, which is a measure of the rotor imbalance, is defined as the ratio of the eccentricity of the rotor center of mass from its geometric center of rotation, to the nominal air gap of the magnetic bearing. Ω, the speed parameter, is the ratio of the rotor operating speed, ω, to the linear natural frequency of the magnetic bearing system, ω_n. W, the gravity parameter, represents the unidirectional static force acting on the disc at the rotor mid-span, and at the bearing stations. γ, the mass ratio, is the ratio of the journal mass, m_J, to half-mass of the disc at the rotor mid-span, m_D. α is the geometric coupling parameter, which is the ratio of the attractive, on-axis force between each magnet and the shaft to the normal, off-axis force. P and D are respectively the non-dimensional proportional and derivative feedback gains of the controller. τ is the non-dimensional time.

3. Results and Discussion

Numerical integration of Eq. (1) was undertaken using the MATLAB software package, which utilizes a variable-step continuous solver based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. Values of design and operating parameters of U=0.1, W=0.0, ζ=0.001, P=1.1, D=0.03, γ=0.2, f=1.5 and α=0.28 used in the numerical simulation are representative of practical rotor-bearing systems. Ω was varied from 0.05 to 5.0 at intervals of 0.01 in order to investigate the effect of increasing the speed parameter on the response of the magnetically supported flexible rotor.

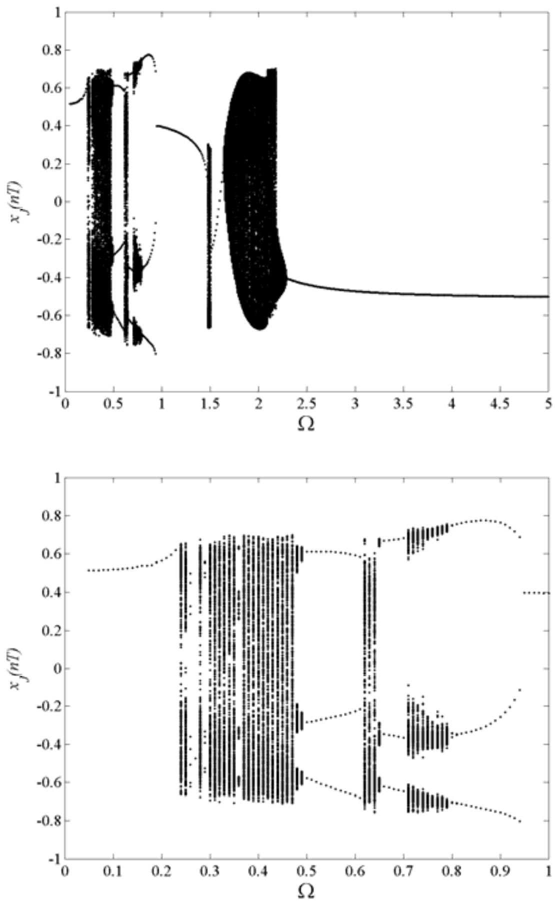

Fig.1 Bifurcation diagram of the rotor response.

The results of the numerical simulation are illustrated using bifurcation diagrams, time waveforms, Poincaré maps and power spectrum plots. The time waveform represents the instantaneous position of the journal in the X-direction plotted against non-dimensional time. Poincaré map is obtained by sampling the trajectory of the rotor whirl orbit at constant interval of the forcing period of $T = 2\pi / \Omega$ and projecting the outcome on the $x_J(nT)$ versus $y_J(nT)$ plane. The variation of $x_J(nT)$ in the Poincaré map with Ω is then plotted to form the bifurcation diagram. The power spectrum, which exhibits the frequency contents of the rotor response at the bearing station, is determined from the Fourier transformation of the time series of the journal response in the X-direction.

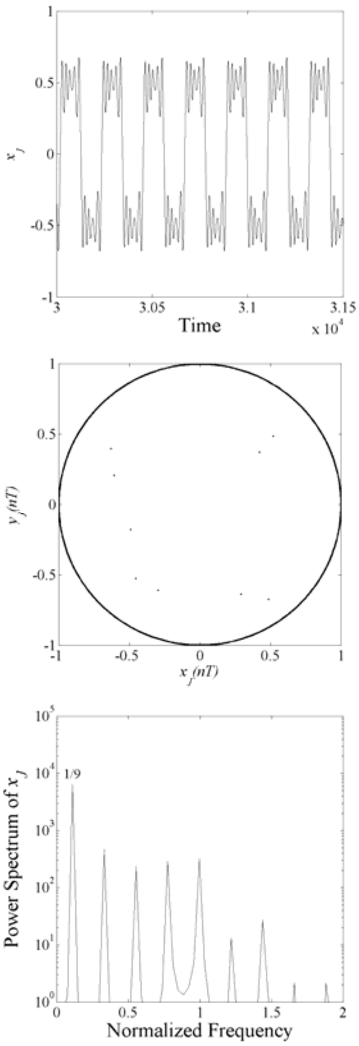

Fig.2 Rotor response for Ω =0.26.

The bifurcation diagram of the rotor response for the range $0.05 \le \Omega \le 5.0$, and its enlargement for the range $0.05 \le \Omega \le 1.0$, are shown in Fig. 1. For the range $0.05 \le \Omega \le 0.23$, the response of the rotor was synchronous, i.e., period-1. Chaotic motion of the rotor was observed in the range $0.24 \le \Omega \le 0.49$, except for Ω =0.26 and 0.29, where a sub-synchronous response of period-9 was seen, and for Ω =0.27 where the response was synchronous. The time waveform, Poincaré map and power spectrum plot of the period-9 response of the rotor for Ω =0.26 are shown in Fig. 2.

With further increase of Ω, sub-synchronous response of period-3 and chaotic vibrations of the rotor were observed alternately. Period-3 response was observed to exist in the range $0.50 \le \Omega \le 0.61$, $0.66 \le \Omega \le 0.70$ and $0.82 \le \Omega \le 0.94$. Chaotic response of the rotor was seen in the range $0.62 \le \Omega \le 0.65$ and $0.71 \le \Omega \le 0.80$. The period-3 response of the rotor for Ω =0.55 is shown in Fig. 3 using time waveform, Poincaré map and power spectrum plot. The Poincaré map and power spectrum plot of the chaotic rotor response for Ω =0.75 are illustrated in Fig. 4. For Ω =0.81, sub-synchronous rotor response of period-6 is seen to exist.

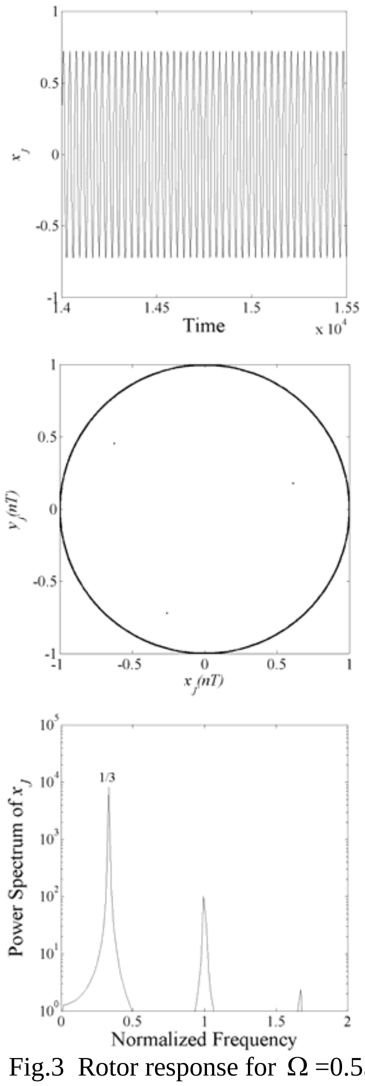

Fig.3 Rotor response for Ω =0.55.

The response of the rotor was found to be synchronous for $0.95 \le \Omega \le 1.64$, except for a small range $1.48 \le \Omega \le 1.50$ where quasi-periodic vibrations were seen. For $1.65 \le \Omega \le 2.29$, the response of the rotor was quasi-periodic except for values of Ω between 2.10 and

2.19, where chaos was observed. The Poincaré map and power spectrum plot of the quasi-periodic response of the rotor for $\Omega = 2.09$ are shown in Fig. 5. For the range $2.30 \le \Omega \le 5.0$, the response of the rotor was synchronous.

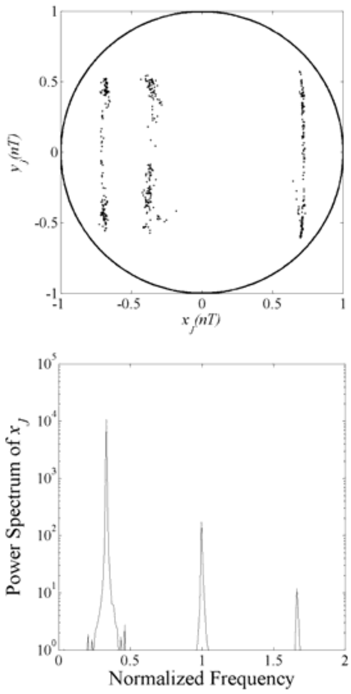

Fig.4 Rotor response for $\Omega = 0.75$.

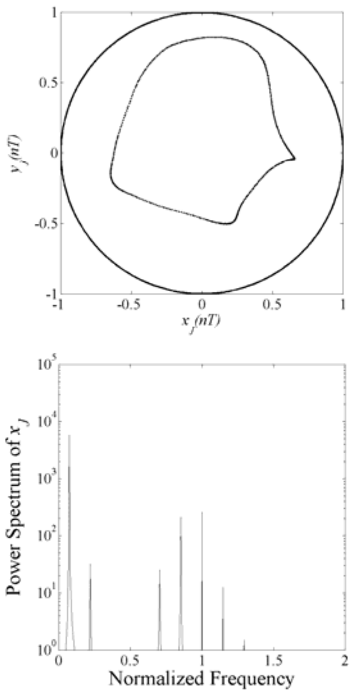

Fig.5 Rotor response for $\Omega = 2.09$.

4. Summary

The nonlinear response of a flexible rotor in active magnetic bearings has been numerically investigated in this work. Nonlinearity arising from the magnetic actuator forces that are nonlinear functions of the coil current and the air gap between the rotor and the stator, and from the geometric coupling of the magnetic actuators were incorporated into the mathematical model of the flexible rotor - active magnetic bearing system. With the variation of the speed parameter, Ω, the rotor response displayed a rich variety of nonlinear dynamical behaviour including sub-synchronous vibrations of periods-3, -6 and -9, and quasi-periodic and chaotic vibrations. Such vibrations should be avoided in the operation of rotating machinery as they introduce cyclic stresses, which in turn may rapidly induce fatigue failure of the main components of these machines.

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