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Phase synchronization control in partially unlocking oscillator arrays

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Abstract—A phase synchronization method is proposed for beam-scanning control, utilizing a newly identified phase synchronization pattern. The proposed method provides more flexible control compared to conventional methods with mutually locked oscillator arrays, both for sinusoidal, and for impulsive injection signals, as confirmed by systematic simulations and mathematical analysis.

1. Introduction

Coupled oscillator arrays emerge in a wide range of engineering issues. Examples include millimeter-wave power-combining and beam-scanning control systems, central pattern generators (CPG) in robotics, and Josephson junction arrays. In contrast with these examples that utilize the mutually locked (synchronized) state, little attention has been paid to unlocking states. Presumably, this is because few applications have been sought in such unlocking situations.

In beam-scanning control systems using coupled oscillator arrays, a linear phase progression must be maintained across the array for beam forming (see Fig. 1), and the radiated beam is steered by controlling the phase difference between the adjacent oscillators. Methods of controlling such phase differences have been proposed and demonstrated, respectively, by Stephan [2] and by Liao and York [3], where oscillators at both ends of the array (oscillators 1 and N in Fig. 1(a)) are controlled to steer the beam. In Liao and York [3], asymmetrical frequency detuning is applied to the oscillators at the ends of the array. In such coupled oscillator-based beam steering, the coupling is loose (weak) when the oscillators are coupled radiatively for instance. Accordingly, as shown in [3] when a particular scan angle, say +12.5°, is required, the frequencies of the end oscillators must be 9.985 GHz and 10.015 GHz, respectively, for instance, while the other oscillators have a frequency of 10.0 GHz. This suggests that the end oscillators require careful frequency control because the frequency control resolution for the end oscillators must be within the order of a few kHz.

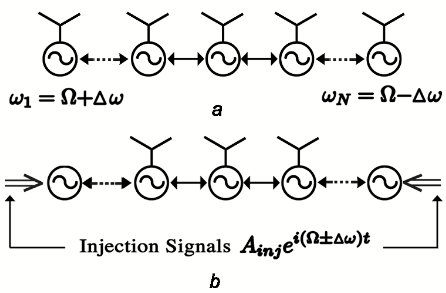

Fig. 1 Coupled oscillator arrays for beam-scanning
(**a**) Conventional system
(**b**) Proposed, partially unlocking system

We herein consider a counterpart of the injection-locked state, i.e., unlocking states, which emerge quite naturally. In the above example, if the end oscillators have frequencies of approximately 9.0 GHz and 11.0 GHz, respectively, for instance, then they are unlocking with respect to the other oscillators. We have analyzed such unlocking states, and found that a robust, exactly linear phase progression is still obtained in the array, except for a few oscillators near both ends.

In the case of weakly coupled, weakly nonlinear oscillators, this somewhat counterintuitive phenomena is systematically analyzed both numerically and theoretically, leading to a closed formula of the phase progression for given frequencies and amplitudes of the end oscillators (or sinusoidal injection signals). In addition to the case of sinusoidal injection signals, we have numerically analized oscillator arrays with impulsive injection signals at the both ends, and confirmed that a certain amount of linear phase progression is obtained even when relatively small impulses are injected to the array, as opposed to the sinusoidal injection case.

Based on this robust phase progression in the unlocking state, a method of beam-scanning control is proposed, which does not require high frequency control resolution.

2. Phase Synchronization in Partially Unlocking Oscillator Arrays

For weakly coupled quasi-optical (van der Pol-like) oscillators, York [1] developed a systematic reduction of model equations. In this section, we follow this reduction and explain how the linear phase progression is realized in the coupled oscillator array. Contrary to previous studies, we consider herein an injection-unlocking state, and analyze the phase relationship in the oscillator array. We also assume a weak coupling between adjacent oscillators, as well as sufficiently uniform oscillator characteristics. Under such conditions, a systematic derivation of the phase equation for oscillators can be constructed [1], which eventually takes the following form:

$$\dot{\theta}_i = \omega_i + \kappa \sum_j \frac{A_j}{A_i} \sin(\Phi + \theta_j - \theta_i). \tag{1}$$

In Eq. (1), θ_i, A_i, and ω_i represent the oscillation phase, amplitude, and free-running frequency of the ith oscillator, respectively. Based on the above assumptions, $A_{i,j} \sim 1$ holds and $\kappa A_j/A_i \equiv \kappa$ is denoted by $\Delta\omega_m$. This $\Delta\omega_m$ is interpreted as the locking range of each oscillator, which is assumed to be small as well as identical for all oscillators. The phase lag Φ reflects the signal delay, which cannot be neglected for the case of radiative coupling. However, if the coupling is constructed of one-wavelength waveguides, Φ is assumed to be 0, and we focus on this case. Similar to Liao and York [3], we consider herein the frequency distribution as: $\omega_1 = \Omega + \Delta\omega$, $\omega_N = \Omega - \Delta\omega$, $\omega_2, ..., \omega_{N-1} = \Omega$, where Ω and $\Delta\omega$ denote the common locked frequency and the frequency detuning, respectively. Hereinafter, $\Delta\omega_m$ is set to 1 without loss of generality, and all numerical integrations are carried out by the fourth-order Runge-Kutta method with a step size of 0.001.

In contrast to the locked state of linear phase progression, we consider an unlocking state which is obtained when $|\Delta\omega| > \Delta\omega_m$. We systematically changed $\Delta\omega/\Delta\omega_m$ from 1.0 to 20.0 and observed the phase relationships. Figure 2 shows a typical unlocking phase relationship numerically observed in Eq. (1), where $\Delta\omega/\Delta\omega_m$ is set to 5.0, and a snapshot of $\theta_{i+1} - \theta_i$ is taken at $t = 5,000$. This example exhibits the following three characteristics observed in these unlocking cases:

(1) Oscillators near both ends of the array oscillate at common angular frequencies close to $\Omega \pm \Delta\omega$, respectively, and the other oscillators oscillate at Ω.

(2) Inside the array, a linear phase progression appears, in which the phase difference $\theta_{i+1} - \theta_i$ $(i = 4, ..., 21)$ becomes time-constant and its fluctuation is always negligibly small. In contrast to the inside of the array, the oscillators near both ends exhibit a certain amount of fluctuation in phase difference $\theta_{i+1} - \theta_i$. The ranges for these fluctuations are shown by as the 'minimum and maximum phase differences' in Fig. 2.

(3) In such a phase progression, the phase difference $\phi_i - \phi_{i+1} (\equiv \Delta\phi)$ is measured as $\Delta\phi \sim A_{inj}^2/2\Delta\omega$ if the end oscillators have the same oscillation amplitude $A_1 = A_N \equiv A_{inj}$.

3. Phase Control by Unlocking Oscillators

Based on these characteristics, an application of the unlocking array is suggested to control the linear phase

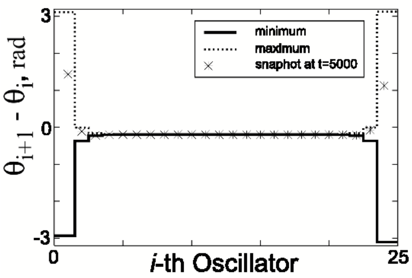

Fig. 2 Phase progression pattern observed in the unlocking system (Eq. (1))

progression both by the amplitude A_{inj} and the detuning $\Delta\omega$ at the end oscillators. First, we consider weakly coupled, weakly nonlinear oscillator arrays, namely the case for sinusoidal injection, theoretically (in 3.1) and numerically (in 3.2). Then, the case for impulsive injection is numerically analized, and the result is compared to the case for sinusoidal injection (in 3.3).

3.1. Theoretical Results

To consider these observations theoretically, we introduce a slightly modified version of Eq. (1):

$$\begin{aligned} \dot{\theta}_1 &= \Omega + A_{inj}\sin((\Omega + \Delta\omega)t - \theta_1) + \sin(\theta_2 - \theta_1), \\ \dot{\theta}_i &= \Omega + \sin(\theta_{i+1} - \theta_i) + \sin(\theta_{i-1} - \theta_i), \ (2 \leq i \leq N-1) \\ \dot{\theta}_N &= \Omega + A_{inj}\sin((\Omega - \Delta\omega)t - \theta_N) + \sin(\theta_{N-1} - \theta_N), \end{aligned} \tag{2}$$

where the end unlocking oscillators in Eq. (1) are replaced by the external injection signals $A_{inj}^{1,N} e^{i(\Omega \pm \Delta\omega)t}$, respectively. This modification is shown schematically in Fig. 1(b). Thus, the modification is not essential to the phase relationship considered here, and by this modification the analysis of Eq. (2) becomes much more tractable, as follows. First, a new variable $\phi_i \equiv \theta_i - \Omega t$ is introduced to Eq. (2), which yields:

$$\begin{aligned} \dot{\phi}_1 &= A_{inj}^1 \sin(\Delta\omega t - \phi_1) + \sin(\phi_2 - \phi_1), \\ \dot{\phi}_N &= A_{inj}^N \sin(-\Delta\omega t - \phi_N) + \sin(\phi_{N-1} - \phi_N), \\ \dot{\phi}_i &= \sin(\phi_{i+1} - \phi_i) + \sin(\phi_{i-1} - \phi_i), \quad (2 \leq i \leq N-1). \end{aligned} \tag{3}$$

For the long-term, slow movement of the phase relationship $\phi_i - \phi_{i+1}$, we reduce Eq. (3) by averaging the fast moving terms $A_{inj}\sin(\Delta\omega t - \phi_1)$ and $A_{inj}\sin(-\Delta\omega t - \phi_N)$, assuming $\Delta\omega t$ as the fast variable. In this averaging, a somewhat technical calculation is possible, using a nonlinear transformation of variables. This result is mathematically validated for large $\Delta\omega$ limit. Due to lack of space, we

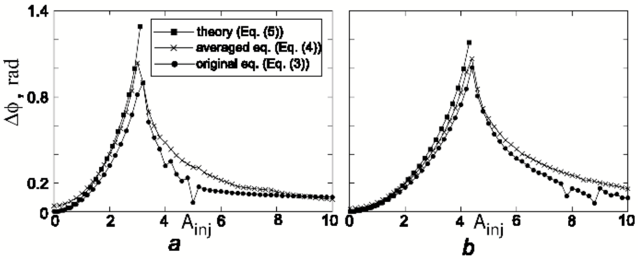

Fig. 3 Comparison of the phase progression $\Delta\phi$ obtained by Eqs.(3), (4), and (5), respectively.
(**a**) $\Delta\omega = 5.0$
(**b**) $\Delta\omega = 10.0$

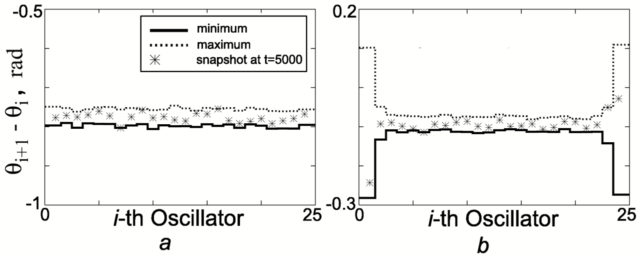

Fig. 4 Comparison of noise immunity
(**a**) Conventional method [2]
(**b**) Proposed method

herein omit the details. More general results will be reported elsewhere soon. Finally, Eq. (3) is averaged to yield:

$$\begin{aligned}
\dot{\phi}_1 &= \frac{(A_{inj}^1)^2}{2\Delta\omega} + \sin(\phi_2 - \phi_1), \\
\dot{\phi}_N &= -\frac{(A_{inj}^N)^2}{2\Delta\omega} + \sin(\phi_{N-1} - \phi_N), \qquad (4) \\
\dot{\phi}_i &= \sin(\phi_{i+1} - \phi_i) + \sin(\phi_{i-1} - \phi_i).
\end{aligned}$$

Interestingly, Eq. (4) takes the form of Eq. (1), and the phase progression $\Delta\phi \equiv \phi_i - \phi_{i+1}$ is explicitly given as

$$\Delta\phi = \sin^{-1}\left(\frac{A_{inj}^2}{2\Delta\omega}\right) \quad \text{where } A_{inj} \equiv A_{inj}^1 = A_{inj}^N, \qquad (5)$$

which explains the observed phase progression inside the array.

3.2. Simulation Results

Based on the closed formula (5) of the phase progression, a new beam-scanning control method is proposed. First, we check the consistency of the simulation results from Eq. (3), Eq. (4), and the closed formula of $\Delta\phi$ (5). Figure 3 shows typical examples of these three data obtained for $\Delta\omega = 5.0$ and 10.0 (in a normalized frequency), respectively, where they are verified to be in good agreement. Also, from systematic simulations, it is observed that the consistency becomes better as $\Delta\omega$ increases. However, $\Delta\omega$ becomes small, say, 0.1, the consistency decreases because $\dot{\phi}_1 \sim \Delta\omega$ (or $\dot{\phi}_N \sim -\Delta\omega$) is no longer large enough for validating the averaging results. Thus, the numerical simulations and the analytical results (4) and (5) suggest that the proposed method stably controls the linear phase progression by tuning A_{inj} and/or $\Delta\omega$.

The issues of robustness in the proposed method, i.e., noise immunity and the effect of asymmetric detuning frequencies, are considered. In Fig. 4, we show a comparison between the method of Liao and York [3] and the proposed method when low-phase noises are introduced to the oscillators. Extensive numerical simulations using random initial conditions are carried out according to [4], assuming the same white noise in both cases. A typical result, shown in Fig. 4, suggests that both methods have approximately the same noise immunity under the above condition.

The effect of asymmetric detuning frequencies has also been considered numerically, both for the method of Liao and York [3] (Eq. (1)) and for the proposed method (Eq. (4)). Numerical simulations are performed systematically for various A_{inj} and $\Delta\omega$, where asymmetric detuning is

realized by setting $\omega_1 = \Omega+\Delta\omega$ and $\omega_N = \Omega-\Delta\omega-\delta$ with $\delta = 0.5$, for instance. The results show that the amounts of distortion in the linear phase progression are approximately the same for both methods (data not shown due to space limitations). This suggests that both methods have approximately the same robustness as the asymmetric detuning frequencies. However, the proposed method effectively controls the phase progression by tuning the injection amplitude. In the above example using asymmetric detuning δ, the proposed method tunes the amplitudes A_{inj}^1 and/or A_{inj}^N according to Eq. (5), resulting in the perfect linear phase progression.

3.3. Phase Control by Impulsive Injection Signals

To model coupled oscillator arrays with impulsive injections, it is necessary to start from coupled quasi-optical, van der Pol-like oscillators [1]. Although it is possible to reduce the equation of this oscillator array with impulsive injections to the phase equation as in Eq. (1) we focus here on the original, coupled van der Pol oscillator model due to lack of space. Reduction to the phase equation will be reported else where.

The equation of the coupled van der Pol oscillator model takes the following form:

$$\begin{aligned}
\ddot{x}_1 &= \varepsilon(1 - {x_1}^2)\dot{x}_1 + \omega^2 x_1 + \kappa\dot{x}_2 + \kappa A_{inj} f_1(\omega_1 t), \\
\ddot{x}_N &= \varepsilon(1 - {x_N}^2)\dot{x}_N + \omega^2 x_N + \kappa\dot{x}_{N-1} + \kappa A_{inj} f_N(\omega_N t), \qquad (6) \\
\ddot{x}_i &= \varepsilon(1 - {x_i}^2)\dot{x}_i + \omega^2 x_i + \kappa(\dot{x}_{i-1} + \dot{x}_{i+1}), (2 \le i \le N-1),
\end{aligned}$$

where ω_1 and ω_N denote $\omega + \Delta\omega$ and $\omega - \Delta\omega$ respectively. In Eq. (6) ε, ω, $\Delta\omega$, and κ are the nonlinearity, the natural angular frequency, the deturning angular frequency, and the coupling strength of the oscillators, and f_1 and f_N represent the injection signal form to the oscillator 1 and N (at both ends of the array), respectively. It is noted that Eq. (6) is equivalent to the quasi-optical oscillators in [1], except for the injection terms $\kappa A_{inj} f_{1,N}$.

In this study, we have set the parameters ε, ω, κ, and $\Delta\omega$ as $\varepsilon = 0.1$, $\omega = 1.0$, $\kappa = 0.005$, $\Delta\omega = 0.1$ in order to consider 'weakly coupled','weak' nonlinear oscillators. However, the essential feature in the following simulation results is obtained for other choices of parameters.

Figure 4 shows the simulation results from Eq. (6) for the impulsive injections and for the sinusoidal injections, respectively. In this simulation, we have set $f_{1,N}$ as $f_{1,N} = (\omega \pm \Delta\omega)\text{pulse}((\omega \pm \Delta\omega)t)$ and $f_{1,N} = (\omega \pm \Delta\omega)\cos((\omega \pm \Delta\omega)t)$, respectively for the impulsive injection case and for the sinusoidal injection case. The reason why $(\omega \pm \Delta\omega)$ is multipled to the 'pulse' or 'cos' terms is due to fact that the injection signal (; current) is obtained as the time-derivative of a certain function (; voltage) in this particular example.

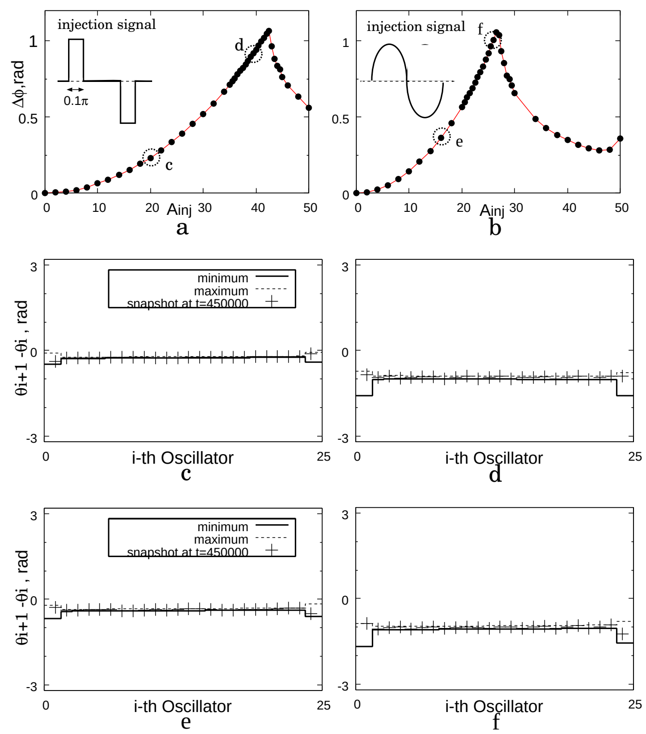

Fig. 5 Phase progression $\Delta\phi$ obtained by Eq. (6)
(a) Case for impulsive injection
(b) Case for sinusoidal injection
(c) Phase progression pattern observed for the impulsive injection signals ($A_{inj} = 20$)
(d) Phase progression pattern observed for the impulsive injection signals ($A_{inj} = 40$)
(e) Phase progression pattern observed for the sinusoidal injection signals ($A_{inj} = 16$)
(f) Phase progression pattern observed for the sinusoidal injection signals ($A_{inj} = 26$)

The form of 'pulse' in $f_{1,N}$ is shown in the inset of Fig. 4(a), where the pulse width and height are set to 0.1π and 1.0 respectively. It is noted that in all simulation results all oscillators are mutually frequency locked at around $\omega = 1.0$ and this clearly shows the oscillators at both ends of the array are unlocked to the injection frequency ω_1 and ω_N.

In Figs. 5(a) and 5(b), we observe the same tendency of increasing $\Delta\phi$ and sharp peaks at around $A_{inj} = 42.5$ and $A_{inj} = 26.8$ respectively.

This observation also matches to the theoretical result in 3.1 shown in Fig.3, suggesting the same mechanism shown in 3.1 and 3.2 exists in Eq. (6) with the impulsive injections.

In Figs. 5(c)–(f), phase progression patterns are respectively plotted for different four A_{inj}'s shown in Figs. 5(a) and 5(b). These shows the fluctuation of the phase difference $\Delta\phi$ is quite small inside the array for all cases. This suggests that even the unlocking impulse injections can control the phase progression pattern stably inside the array.

4. Conclusions

We proposed a novel control method for a linear phase pattern in coupled oscillator arrays, which provides more flexible and possibly robust control ability in coupled oscillators with small locking ranges. The phase pattern is clearly described in Eq. (5) and its validity and usefulness is confirmed by systematic simulations.

Acknowledgments

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