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Pulse Wave Propagation and Interaction Phenomenon in a Large Number of Coupled van der Pol Oscillator Lattice

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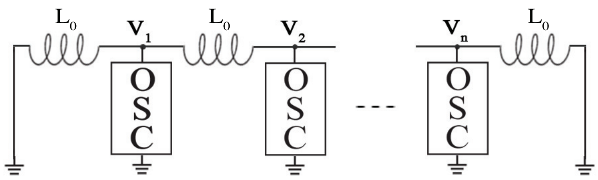
Abstract—In this paper, we investigate pulse wave propagation and pulse wave interaction phenomenon in a large number of coupled van der Pol oscillator lattice with hard-type nonlinearity. We demonstrate numerically that there exists a pulse wave which propagates with a constant speed, when ε (= a parameter which shows the degree of nonlinearity) exceeds a certain critical value. Moreover, We find that collision of these pulse waves result in various phenomena such as repulsion, unification, disappearance, standing waves, etc. depending on the value of ε.

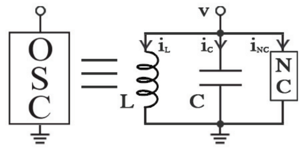
1. Introduction

Mutually coupled oscillators have been investigated by many authors for many decades in various areas of physics, mathematics and biology[1]-[3]. In particular, wave propagation phenomenon in a large number of coupled oscillator systems have been reported[4]-[6]. They are important not only in a pure nonlinear science viewpoint but also from the viewpoint of various applications, one of which includes biological information processing[7].

In this paper, we investigate wave propagation phenomenon observed in coupled van der Pol oscillator lattice with hard-type nonlinearity. The dynamics for weak nonlinear cases have been almost elucidated via averaging method[8]. But its dynamics for strong nonlinear case seems to be unexplored. Namely, for small value of ε (= a parameter showing the degree of nonlinearity), the behavior of this oscillator lattice obeys averaging theory, and only standing wave patterns are possible. However, when ε exceeds a certain critical value, there exist various kinds of traveling pulse waves. Yamauchi *et al.* investigated wave propagation phenomenon in an inductance-coupled van der Pol oscillator lattice with soft nonlinearity[4]. They demonstrated various propagating "phase" waves for ε beyond a certain critical value. In contrast, the pulse waves in our case are based on "oscillation amplitude", namely several adjacent oscillators forming a pulse oscillate with large amplitudes and others show no oscillation. The behavior after collision of two pulses strongly depends on the value of ε; i.e., repulsion, unification and disappearance, etc. can be seen.



(a) Coupled van der Pol oscillator lattice



(b) A van der Pol oscillator

Figure 1: An inductance-coupled van der Pol oscillator lattice

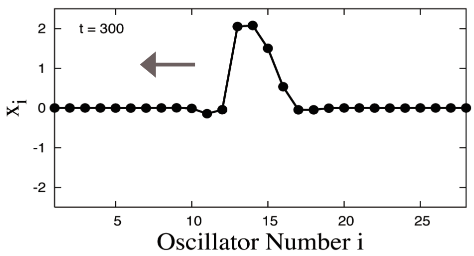
2. Circuit model

Figures 1 (a) and (b) present the inductance-coupled van der Pol oscillator lattice we adopt. Assuming the hard-type nonlinearity for NC, i.e., $i_{NC} = g_1V - g_3V^3 + g_5V^5$, $g_1, g_3, g_5 > 0$ in Fig.1(b), the circuit equation can be written by the following autonomous system after normalization[9]:

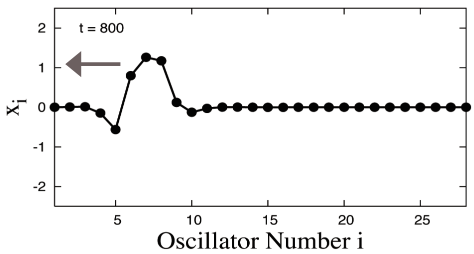
$$\begin{aligned}
\dot{x}_1 &= y_1 \\
\dot{y}_1 &= -\varepsilon(1-\beta x_1^2 + x_1^4)y_1 - (1+\alpha)x_1 + \alpha x_2 \\
\dot{x}_i &= y_i \\
\dot{y}_i &= -\varepsilon(1-\beta x_i^2 + x_i^4)y_i - (1+\alpha)x_i + \alpha(x_{i-1} + x_{i+1}) \\
\dot{x}_n &= y_n \\
\dot{y}_n &= -\varepsilon(1-\beta x_n^2 + x_n^4)y_n - (1+\alpha)x_n + \alpha x_{n-1}
\end{aligned} \tag{1}$$

$$, \ i = 2, 3, \cdots, n-1, \qquad (\cdot = d/d\tau, \cdot\cdot = d^2/d\tau^2)$$

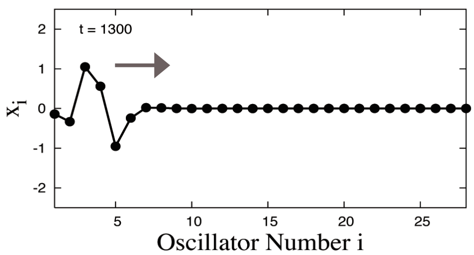
where n is the number of coupled oscillators. x_j denotes the normalized output voltage of the j-th oscillator, y_j denotes its derivative.



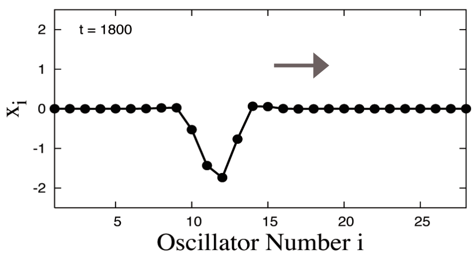
(a) t = 300



(b) t = 800



(c) t = 1300



(d) t = 1800

Figure 2: Snapshots of typical pulse wave propagation for $\alpha = 0.1, \beta = 3.1$ and $\varepsilon = 0.36$. Initial condition is $x_{15}(0) = 2.01, x_{16}(0) = 1.99$ and other variables are zero.

The parameter ε (> 0) shows the degree of nonlinearity. The parameter α ($0 \leqq \alpha \leqq 1$) is a coupling factor, namely $\alpha = 1$ means maximum coupling, and $\alpha = 0$ means no coupling. The parameter β controls amplitude of oscillation.

3. Wave propagation phenomenon

Pulse wave propagation phenomenon typically observed in Eq.(1) is depicted in Fig.2 for $n = 28$.[1] Each figure

[1] All numerical integrations are carried out by the fourth-order Runge-Kutta method with step size = 0.01.

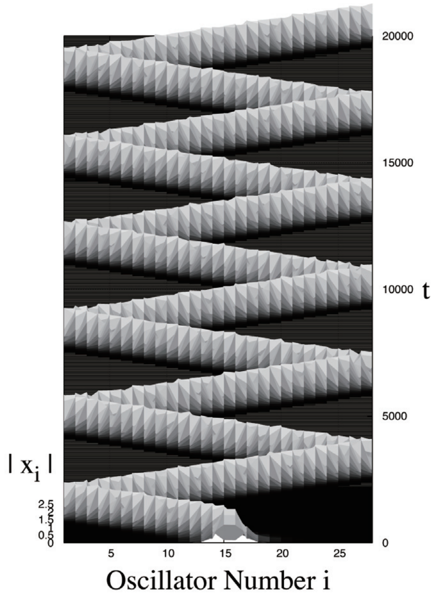
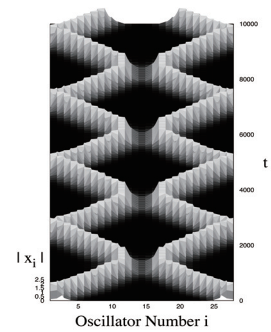


Figure 3: A bird's-eye view plot for typical pulse wave propagation

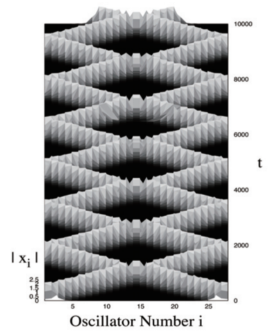
shows the snapshot at a certain time t. Here, we set the parameters as : $\alpha = 0.1, \beta = 3.1$ and $\varepsilon = 0.36$. It is noted that main body of the propagating pulse wave consists of several (almost 6) adjacent oscillators with large amplitude. The pulse wave propagation phenomenon can be observed when we set the initial condition of the x-component of two adjacent oscillators approximately to 2.0 and other variables (x_i, y_i) to zero. In our case, we set the initial condition of $x_{15}(0)$ to 2.01 and $x_{16}(0)$ to 1.99 and other variables to zero. Figure 3 presents a bird's-eye view plot of this pulse wave propagation where the absolute value of x_i is plotted. It is clearly seen that a pulse wave propagates with a constant speed. In this case, initial propagating direction is from right to left as drawn by an arrow in Fig.2(a). Which direction a pulse wave propagates is determined by initial condition. For example, when we set the initial condition of $x_{15}(0)$ to 1.99 and $x_{16}(0)$ to 2.01 and all other variables to zero, the propagating direction of pulse wave is from left to right. Although we take $n = 28$ throughout this paper, quantitatively the same phenomenon can be observed for $n = 100$. In this paper, we fix $\beta = 3.1$ as well as $\alpha = 0.1$, and we employ ε as changing parameter as shown in the next section.

4. Interaction among pulse waves

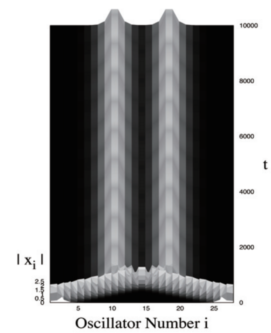
When we give large initial values to many oscillators at the same time, multiple pulse waves will emerge. These waves include propagating pulse waves, standing waves with periodic and quasi-periodic oscillation, etc. They collide at a certain time and start to interact with each other. In this section, we focus on the interaction among various propagating and standing waves.



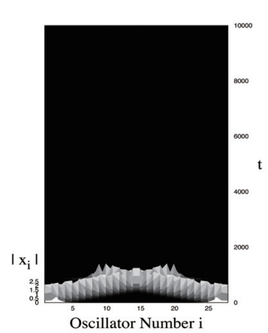
(a) $\varepsilon = 0.37$



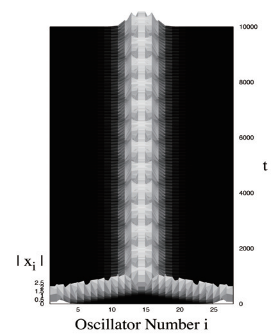
(b) $\varepsilon = 0.36$



(c) $\varepsilon = 0.35$



(d) $\varepsilon = 0.34$



(e) $\varepsilon = 0.33$

Figure 4: Bird's-eye view plots for interaction among two propagating waves for various values of ε for $\alpha = 0.1, \beta = 3.1$. Initial condition is $x_1(0) = x_2(0) = x_{27}(0) = x_{28}(0) = 2.0$ and other variables are zero.

For simplicity, we show interaction among two pulse waves. When we set the initial condition of $x_1(0), x_2(0), x_{27}(0)$ and $x_{28}(0)$ (, namely both ends of oscillator lattice) to 2.0 and other variables to zero, we observe interesting phenomena sensitive to the ε value. Figures 4(a)-(e) present some of the interaction phenomena according to various ε with initial conditions as stated above. In the following, we will explain the characteristics of each dynamics.

(a) When two pulse waves propagating in opposite direction collide, they repel in a symmetric manner for $\varepsilon = 0.37$.

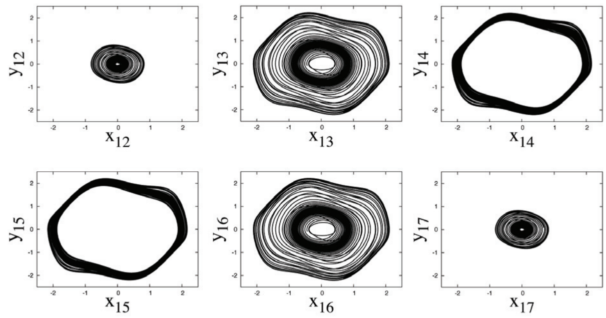


Figure 5: Quasi-periodic oscillations of the standing wave shown in Fig.4(e)

(b) Two pulse waves propagating in opposite direction pass through each other for $\varepsilon = 0.36$.

(c) After two pulse waves propagating in opposite direction collide, there exist two standing waves consisting of time-periodic oscillation for $\varepsilon = 0.35$. Each corresponding oscillator in the standing pulse waves is synchronized with the same phase.

(d) Two propagating waves annihilate after collision for $\varepsilon = 0.34$.

(e) There exists only one standing wave after collision for $\varepsilon = 0.33$. Each oscillator in the sanding pulse wave presents quasi-periodic oscillation as shown in Fig.5. Hence, this standing wave is different from that of case (c).

From our computer simulation, it seems that the same results as above can be observed for even arbitrary number of coupled oscillator lattice. In fact, we confirm the same results for the 100 coupled oscillator lattice. However, for odd number of coupled oscillator lattice, we only observe case (a) irrespective of the value of ε. We confirm this for the 27 and 99 coupled oscillator lattices.

Next, we show another type of pulse wave interaction among two waves. Figure 6 presents the interaction phenomenon observed from initial condition of $x_1(0), x_2(0), x_{15}(0), x_{16}(0)$ and $x_{17}(0)$ to 2.0 and other variables to zero for $\varepsilon = 0.36$. The dynamics is as follows. There are two pulse waves: one is a traveling pulse wave propagating from left margin to right. The other is a standing pulse wave around the 16th oscillator position (= original position) as shown in Fig.7(a). When they collide, the standing pulse wave slips away and stays around a 12th oscillator position (= moved position) as shown in Fig.7(b). And the reflected traveling pulse wave hits the standing pulse wave again, then the standing pulse wave moves to original position. Interestingly this phenomenon is repeated endlessly. We call this phenomenon the "standing wave slip" phenomenon.

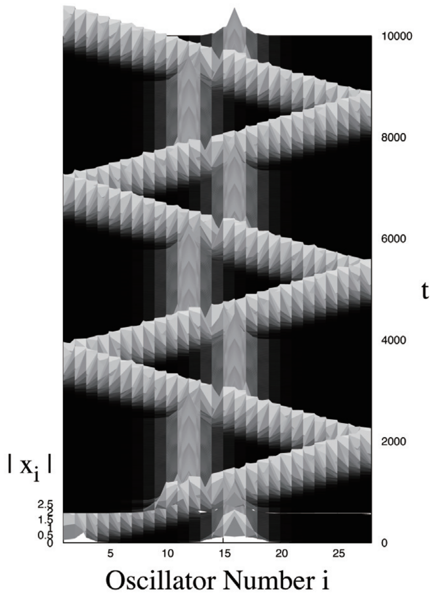
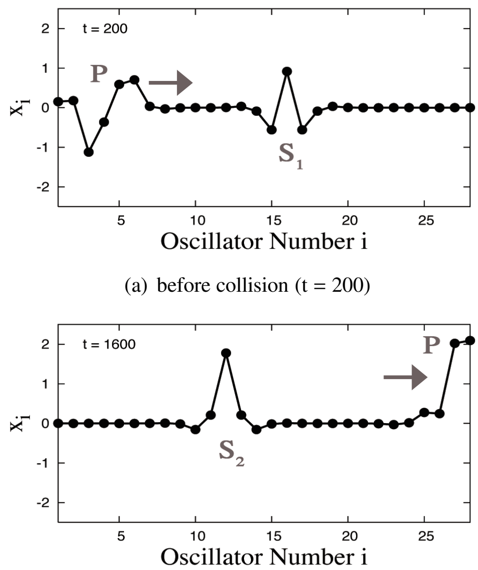


Figure 6: A bird's-eye view plot for the standing wave slip for $\alpha = 0.1, \beta = 3.1$ and $\varepsilon = 0.36$. Initial condition is $x_1(0) = x_2(0) = x_{15}(0) = x_{16}(0) = x_{17}(0) = 2.0$ and other variables are zero.



(a) before collision (t = 200)

(b) after collision (t = 1600)

Figure 7: Snapshots of the standing wave slip: P denotes the propagating wave, S_1 (S_2) denotes standing wave in original (moved) position.

5. Conclusions

We investigate the wave propagation phenomenon and its interaction observed in an inductance-coupled van der Pol oscillator lattice with hard-type nonlinearity. These wave propagation phenomena continuously exist, and its interactions present the rich interesting behaviors such as repulsion, unification and disappearance, etc. As a future problem, we will investigate this system more in detail.

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